## UIVITRSITI OF ILLIHOIS

 DIGITAL CONPUTER LABORATORY STIATISITICAL IIRRARYKSL $1.90-288$

TITLE:
TYPE:
SYMBOLS:

CAPACITY:

MHIHOD OF USE:

Oblimax Rotation of Factors
Entire program
d - decimal places in results
v - number of variables
f - number of factors
$x-$ printing directive
The printing directive, $X$, can be any value from zero through 15 (See section on OPTIONS SPECIFIED BY THE DIRECTIVE, $X$ ).

For any $X$, the limits are:
$\mathrm{d} \leq 12 ; \quad \mathrm{V} \leq 128 ; \mathrm{f} \leq 40$.
If $X=4,5,6,7,12,13,14$, or 15 , the following additional reatrictions apply:
$\mathrm{P} \leq 20 ; \quad \mathrm{V}+4 \mathrm{f} \leq 166$.
A. Initial machine run Stops

1. Master tape 2407 J
2. Parameter tape 24pFO
3. Factor tape, run with bl. down OF

If the black switch is placed in mid-position, the machine will stop at 34092 at the end of each iteration. If the process has not converged, but the allotted time has run out, intermediate results can be obtained at 34092 by moving the white switch up and down. The problem can be finished subsequently using Alternative B.

To read another parameter tape to begin a new problem at stop OF, move the white switch up and down.

## B. Subsequent machime run with a transform

1. Naster tape 2407 J
2. Parameter tape 240 FO
3. Transform tape, move wh. sw. up and down 340L8
4. Factor tape, run with bl. down OF

Alternative $B$ need not be a subsequent machine run.
If by some other means, a transform and reference vector structure are available, the amount of time required will be reduced depending upon how close the transform is to the one calculated by this routine.
PUNCHITC OF THE RESULTS:
Each section is labeled with an appropriate heading. The first section consists of successive criterion values, one after each iteration. (See Note 2) What follows depends upon the value of the printing directive, $X$. The results will consist of all or part of the following:

T Transformation matrix ( $V_{0} T=V_{r} ; V_{0}$ is the factor matrix to be transformed). $T$ will be punched either by rows or by columns depending upon $X$. The elements of $T$ are punched as signed fractions scaled by $10^{-1}$ with an $N$ symbol at the end of each row or colum.
$V_{r}$ Reference vector atructure. $V_{r}$ will be punched by rows or by colums depending upon $X$. The elements of $\nabla_{r}$ are punched as signed fractions scaled by $i^{-1}$ with an 1 symbol at the end of each row or column and a $J$ at the end of the matrix.
$C_{r}$ Reference vector correlations ( $T \mathbf{T}=\mathbf{C}_{\mathbf{r}}$ ). $\mathbf{C}_{\mathbf{r}}$ is symmetric and is punched in triangular form by rows. Each element is a signed fraction scaled by $10^{-1}$.
$C_{f}$. Primary factor correlations ( $D C_{r}^{-1} D=C_{f}$ ). $C_{f}$ is punched in triangular form by rows with each element a signed fraction scaled by $10^{-1}$.
$\mathrm{V}_{\mathrm{f}} \quad$ Primary factor pattern $\left(\mathrm{V}_{\mathrm{o}} T^{-1}=\mathrm{V}_{\mathrm{f}}\right) . \mathrm{V}_{\mathrm{f}}$ is punched by rows with an $N$ symbol at the end of each row and a $J$ at the end of the matrix. The elements are signed fractions scaled by $10^{-1}$.

D A diagonal matrix of the reciprocals of the square roots of the diagonal elements of $\mathrm{C}_{\mathrm{r}}{ }^{-1}$. $D$ is punched as a column with the zeros omitted. An NJ is punched at the end of the column. The elements are unscaled.
$D^{-1}$ Inverse of the diagonal matrix, D. The diagonal elements only are punched in a column with an NJ at the end of the column. Decimal points indicate the scaling.

THE TRANSFORM TAPE:

THE FACTOR TAPE:

THE PARAMETIER TAPE:

If alternative $B$ which requires a transformation is used, the transform must be punched by columns with an $N$ at the end of each column. The elements must be punched as signed fractions scaled by $10^{-1}$. In addition the transform must be consistent with $V_{o}$ and $V_{r}$ such that $V_{0} T=V_{r}{ }^{\circ}$
The set of factors to be rotated to oblique simple structure by this routine must be punqhed by columns with an $N$ symbol at the end of each column. The elements must be punched as signed fractions scaled by $10^{-1}$.

The parameter tape consists of four unsigned integers (representing the number of decimal places, the number of variables, the number of factors, and the directive) separated by fifth-hole characters and punched in the following order:
d space $v$ space $I$ space $X$ space.
The options in the results are specified by the value of X which is explained in the next section.
OPIIONS SPECIFIED BY THE DIRECTIVE, $X$ :
X Output will be:
0 Criterion values only.
$1 T$ and $V_{r}$ by columns. $T$ and $V_{r}$ are required by columns if the problem is to be continued aubsequently using Alternative $B$.
$2 T$ and $V_{r}$ by rows.
3 T and $\mathrm{V}_{\mathrm{r}}$ by both rows and columns.
$4 C_{r}, C_{f}, V_{f}, D^{-1}$, and $D$.
5 T and $\mathrm{V}_{\mathrm{r}}$ by columns; $\mathrm{C}_{\mathrm{r}}, \mathrm{C}_{\mathrm{f}}, \mathrm{V}_{\mathrm{f}}, D^{-1}$, and D .
$6 T$ and $V_{r}$ by rows; $C_{r}, C_{f}, V_{f}, D^{-1}$, and $D$
7 T and $\mathrm{V}_{r}$ by both rows and columen; $\mathrm{C}_{\mathrm{r}}, \mathrm{C}_{\mathrm{P}}, \mathrm{V}_{f}, D^{-1}$, and $D$.

If the values of $X$ given above are increased by 8 , the oblimax rotation process will be suppressed. Thus, if a transform, $\mathrm{T}^{*}$, and a reference vector structure, $\mathrm{V}_{\mathrm{r}}{ }^{*}$, are formed by some other rotation procedure, the oblimax routine can still be used to form $C_{r}{ }^{*}, C_{f}^{*}$, $V_{f}^{*}, D^{*-1}$, and $D^{*}$. For example:
$\frac{\mathrm{x}}{14} \frac{\text { Output will be: }}{\mathrm{T}^{*}}$
$14 \mathrm{~T}^{*}$ and $\mathrm{V}_{\mathrm{r}}^{*}$ by rows; $\mathrm{C}_{\mathrm{r}}^{*}, \mathrm{C}_{\mathrm{f}}^{*}, \mathrm{~V}_{\mathrm{f}}^{*}, \mathrm{D}^{*-1}$, and $\mathrm{D}^{*}$. THE OBLTMAX PROCEDURE: This routine transforms a set of factor vectors, $g_{j}$, to a new set, $h_{j}$, such that the function,

$$
K=\frac{\sum \sum h_{i j}^{4}}{\left(\sum \sum h_{i j}{ }^{2}\right)^{2}} \quad \begin{aligned}
& 1=1,2, \ldots . v \\
& j=1,2, \ldots . f
\end{aligned}
$$

is maximized. The purpose of this transformation is to attempt analytically to rotate the factors such
that they satisfy the subjective criteria for simple structure of L. L. Thurstone. (See Multiple Factor Analysis, $p p .319-410,1947$ )

The user of this routine is cautioned not to accept the results blindly, for they do not satisfy Thurstone's criteria exactly. There are occasions, moreover, when the routine fails to rotate some few of the vectors. (See Note 3)

It would be elegant to solve directly for the transformation, $T$, but unfortunately no solution to this problem has been found. Instead oblimax takes two vectors at a time, solves for the rotational angles, transforms these vectors, and then selects another pair until all $f(f-1)$ pairs have been rotated. This is called one iteration. This process is repeated iteratively until the criterion function, $K$, no longer increases.

It should be pointed out that maximizing all possible pairs is not the same operation as maximizing the criterion, K. For example, it is possible for a particular vector to be shifted in one direction by one pairing and to be shifted back with another pairing. It is also possible that $K$ will become smaller as a result of a particular iteration. In general, however, the criterion $K$ is well behaved and approaches steadily to a maximum.

For any pair of vectors, say $a_{i}$ and $b_{i}$, the solution is as follows:

$$
K_{a b}=\frac{\sum \sum\left(a_{i} \cos \phi_{j}+b_{i} \sin \phi_{j}\right)^{4}}{\left[\sum \sum\left(a_{i} \cos \phi_{j}+b_{i} \sin \emptyset_{j}\right)^{2}\right]^{2}} \quad \begin{aligned}
& i=1,2, \ldots v \\
& j=1,2 .
\end{aligned}
$$

$$
=\frac{\sum \Sigma\left(a_{i}+b_{i} x_{j}\right)^{4}}{\left[\sum \sum\left(a_{i}+b_{i} x_{j}\right)^{2}\right]^{2}} \quad x_{j} \text { are tangents of the } \begin{aligned}
& \text { rotational angles. }
\end{aligned}
$$

If the derivative of $K_{a b}$ is set equal to zero, this results in a quartic equation in $X$ with the following coefficients:

$$
\begin{aligned}
& x^{4}: \sum a b \sum b^{4}-\sum b^{2} \sum a b^{3} \\
& x^{3}: \sum a^{2} \sum b^{4}+2 \sum a b \sum a b^{3}-3 \sum b^{2} \sum a^{2} b^{2} \\
& x^{2}: 3 \sum a^{2} \sum a b^{3}-3 \sum b^{2} \sum a^{3} b \\
& x^{1}: 3 \sum a^{2} \sum a^{2} b^{2}-2 \sum a b \sum a^{3} b-\sum b^{2} \sum a^{4} \\
& x^{0}: \sum a^{2} \sum a^{3} b-\sum a b \sum a^{4}
\end{aligned}
$$

The four solutions to the quartic equation are tangents, two of which will maximize $K_{a b}$. When a value for $X$ is found, the sign of the second derivative of $K_{a b}$ is inspected to determine if the particular X is a maximum or a minimum. After two maxima are found, the routine forms a small transform ( $2 \times 2$ ). This transform must first be adjusted so that the columns of $T, t_{a}$ and $t_{b}$, will remain normalized. This effectively converts the tangent transform. back to a sine-cosine transform, and then the postmultiplication of the columns of $T$ and the vector pair is performed.

The range of values for $X$, the solutions to the quartic equation, is from negative infinity to positive infinity. In fact, near the end of the process when some pairs have become maximized and no further movement is expected, the solutions are: $-1,0,+1$, and infinity. The two maxima are zero and infinity. After normalization, this results in an identity matrix. To circumvent this scaling difficulty, there are two complete
quartic solutions -- one for the tangents, $X$, and one for the cotangents, $1 / \mathrm{X}$. Whenever the value for X begins to converge to a value greater than +1 or less than -1 , the routine jumps to the other section.

It is not at all unusual for the routine to fail to find two maxima for a particular pair of vectors. When this happens, quite often a solution in the next iteration will be found, for in the meantime each vector has been changed by being paired with ( $f-1$ ) other vectors. Failures occur most often in the earlier iterations. Usually all of the vector pairs have been maximized many times by the time the criterion has reached a maximum. (See Note 3)

DURATION:

1. Read master tape: 120 sec .
2. Read data tape: vf(.009+.004 d) sec.
3. Approximate calculation time per iteration:
$f^{2}(.230+.007 f+.010 \mathrm{v}) \mathrm{sec}$.
4. Printing time when $X=5$ or 6:
$\left[5 f+.354 f^{2}+.012 f^{3}+.033 d\left(f^{2}+v f\right)\right] \sec$.
There is no simple way to estimate the number of iterations, 1 , for this routine to converge. In general, i will vary from less than $f$ to more than $3 f$. For short problems, this lack of precision in time estimation will cause no concern.

For the larger problems where $v$ and especially $f$ are large, it is suggested that the problem be run for a definite amount of time (fixed number of iterations). If the problem has not converged, it can be interrupted (See Method of Use) and resumed on a subsequent machine run. To use Alternative $B$, both $T$ and $V_{r}$ are required by columns. The directive, $X$, should hence be an odd number.

NOTE 1:

HOTE 2:

NOTE 3:

NOTE 4:

NOTE 5:

NOTE 6:

A stop on FFO14 after reading the master tape indicates a sum check failure. Clear the machine and try rereading the master tape.

The criterion values are unscaled. If a succeeding $K$ value differs from the previous one by less than . 0000005 , the criterion function is considered to be maximized.

In between successive $K$ values a record is kept of the successes and failures in finding two maxima. A two-hole delay indicates a success and a figure shift indicates a failure. A figure shift in the same place in the sequence after each iteration indicates that a particular factor pair is never maximized: In general the figure shifts disappear in the later iterations. A stop on FFOOS indicates the $C_{r}$ matrix is singular. Unless some peculiar error occurs such as reading the same factor twice as part of the $V_{0}$ matrix, this kind of stop should never occur. If $X=4,5,6,7,12,13,14$, or 15 , the limit on the number of factors, $f$, must be less than 21. A stop on FFO15 indicates the limit has been exceeded. The results that have been punched on tape are correct. To begin a new problem, raise the white switch up and down. In the special case where $f=2$, only 1 iteration is required for the routine to converge.



| LOC | ATION |  | ORDER | NOTTES | PAGE 21.90 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Abs . | Rel. | Sym. |  |  |  |
| 131 | 2 |  | 00550F 26140F 00K |  |  |
|  |  |  |  |  |  |
|  | 0 | (M3) | 50140F 50L 26(y1) 003260F 00230F 26140F OOK | $\begin{aligned} & \text { from } 145 F \\ & \text { in Set II } \end{aligned}$ | Drum Set III: Print $T, \mathrm{~V}_{\mathrm{r}}, \mathrm{C}_{\mathrm{r}}$. |
|  | 1 |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
| 134 | 0 | (M4) | 50140F 50L | from 15(Pl2) | Drum Set IV; Form $\mathrm{C}_{\mathrm{r}}{ }^{-1}$ |
|  | 1 |  | 26 (Y1) 003500F | in Set III |  |
|  |  |  | 00160F 26140F |  |  |
|  |  |  | OOK |  |  |
| 137 | 0 | (M5) | 50140F 50L | from 148F | Drum Set V: Print $C_{f}, V_{f}$ <br> D, $D^{-1}$ |
|  | 1 |  | 26 (Y1) 003670F | in Set IV |  |
|  | 2 |  | 00210F 26140F |  |  |
|  |  |  | 00140K |  |  |
| 140 | 0 |  | 193F 401F | from 2 (MI) | Drum Set I |
|  |  |  | L510L 426L |  |  |
|  |  |  | 41F 814F |  | Read parameters: |
|  |  |  | 50F 74(10) |  | D, V, F, X |
|  |  |  | S5F 40F |  |  |
|  |  |  | 914F 363L |  |  |
|  |  |  | L5F 40F |  |  |
|  |  |  | F56L 426L |  |  |
|  |  |  | L51F 001F |  |  |
|  |  |  | 401F 362L |  |  |
|  | 10 |  | $961 F 006 F$ |  | Print title: |
|  |  |  | 92135F 9259F |  |  |
|  |  |  | 92135F 92259F |  |  |
|  |  |  | 92578F 92195F |  | OBLIMAX ROTATION |
|  |  |  | 92962F 92514F |  |  |
|  |  |  | 92643F 92387F |  |  |
|  |  |  | 92451F 92961F |  |  |
|  |  |  | 92258F 92578F |  |  |
|  |  |  | 92322F 92387F |  |  |











\begin{tabular}{|c|c|c|c|c|c|}
\hline LOCA \& IION \& \& ORDER \& \& NOTES PAGE \(12 \quad 1.90\) \\
\hline Abs. \& Rel. \& Sym. \& \& \& \\
\hline \multirow[t]{2}{*}{363} \& 90
91
0 \& (NT) \& \[
\begin{aligned}
\& \text { L7(AO) 102F } \\
\& 401 F \text { L4F } \\
\& 402 F \text { L32F } \\
\& 3210(C 4) \quad 415 F \\
\& \text { L3F } 3617(Q C) \\
\& \text { L31F } 3617(Q T) \\
\& 26(Q T) 00 F \\
\& 00 K \\
\& \text { K5F } 425 L \\
\& \text { F5(2) } 42(2) \\
\& \text { LO(TR2) } 366 \mathrm{~L} \\
\& 41(1) \text { L55F } \\
\& 0023 F 405 F \\
\& 50(0) 22 F
\end{aligned}
\] \& \[
\text { from } 2\left\{\left.\begin{array}{l}
Q T, Q C \\
C T, C C
\end{array} \right\rvert\,\right.
\] \& \begin{tabular}{l}
\[
\begin{aligned}
\& \left|A_{0}\right| \neq 0, \\
\& \left|A_{4}\right|+\left|A_{0}\right| \neq 0
\end{aligned}
\] \\
Replace \(X\) with \(X \times 2^{23}\) \\
after 20 trials
\end{tabular} \\
\hline \& 6 \& \& \[
\begin{aligned}
\& 921 \mathrm{~F} 2210(\mathrm{C} 4) \\
\& 00 \mathrm{~K}
\end{aligned}
\] \&  \& Punch 5th hole delay for fail after 10 sets of 20 trials. \\
\hline 370 \& 0

5 \& (x) \& | K5F 425L |
| :--- |
| 505F 755F |
| 404 F 504 F |
| 755F 403F |
| 504F754F |
| 402F 22F |
| OOK | \& from $\left\{\begin{array}{l}\mathrm{QT}, \mathrm{QC} \\ \mathrm{CT}, \mathrm{CC} \\ \mathrm{QUT}, \mathrm{QUC} \\ \mathrm{LT}\end{array}\right.$ \& \[

\left.$$
\begin{array}{ll}
5: & x \\
4: & x^{2} \\
3: & x^{3} \\
2: & x^{4}
\end{array}
$$\right\} \quad|x|>.001
\] <br>

\hline 376 \& 0

9 \& (DVT) \& | K5F 429L |
| :--- |
| 503F 75 (A4) |
| $002 \mathrm{FL4}$ (A1) |
| 401F 50(A3) |
| 754 F 40 F |
| 001F L4F |
| I41F 401F |
| 505F 75 (A2) |
| $001 F$ L4]F |
| 40(4) 22F | \& from $\left\{\begin{array}{l}\text { QT, } \\ \mathrm{CT}, \\ \text { QUT } \\ \text { LT }\end{array}\right.$ \& Tangent derivative

$$
(4)=4 A_{4} X^{3}+3 A_{3} x^{2}+2 A_{2} X+A_{1}
$$ <br>

\hline
\end{tabular}











| LOCA | mion |  | ORDER |  | TTRS PAGE $22 \quad 1.90$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Abs. | Rel. | Sym. |  |  |  |
| 160 | 10 |  | 92259F 92195P |  |  |
|  |  |  | 923867 92961 F |  |  |
|  |  |  | 92835F 925788 |  |  |
|  |  |  | 92962F 92450F |  |  |
|  |  |  | 92643 F 92770 F |  |  |
|  |  |  | 92706F 509L |  |  |
|  |  |  | 26(P2) 41(1) |  | Print title |
|  |  |  | L5(D) 0020 F |  |  |
|  |  |  | 464(P4) 41(2) |  |  |
|  |  |  | L5(AV) 4211 (P3) |  |  |
|  |  |  | 4213(P4) F5 (0) |  |  |
|  |  |  | 40 (3) 5015 L |  |  |
|  |  |  | 26 (P4) 9259F |  | Print $\mathrm{T} \times 10^{-1}$ by columns |
|  |  |  | 92135 F 41 (1) |  |  |
|  |  |  | L5(F) 40 (2) |  |  |
|  | 19 |  | L5(A) L4(V) |  |  |
|  | 20 |  | 4211 (P3) 4213 (P4) |  |  |
|  |  |  | 92131F 5021L |  |  |
|  | 23 |  | ${ }^{26}$ (P4) 98834 F |  | Print $\mathrm{V}_{\mathrm{r}} \times 10^{-1}$ by columns |
|  |  |  | 9259 F 26 (P5) |  |  |
|  |  |  | OOK |  |  |
|  | 0 | (P2) | K5F 4224L | from 150,13(P5) | Print title: |
| 164 |  |  | ${ }^{921355} 929515 \mathrm{~F}$ |  | A transform |
|  |  |  | ${ }^{923874} 92965{ }^{\text {P }}$ |  | B Refrerence vector |
|  |  |  | 92322F 92258F |  | STRUCTURE |
|  |  |  | 92387F92770F |  |  |
|  |  |  | 92706F 92898F |  |  |
|  |  |  | 92578F92958F |  |  |
|  |  |  | 92643F92131F |  |  |
|  |  |  | 92515F 92195F |  |  |
|  |  |  | 92965F 92258 F |  |  |
|  | 10 |  | 92194F 92898 F |  |  |
|  |  |  | 92194F 92258 F |  |  |
|  |  |  | 92194 F 99770 F |  |  |








\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multicolumn{3}{|l|}{LOCATION} \& ORDER \& \& NOTES \& PAGE 29 \& 1.90 <br>
\hline Abs. \& Rel. \& Sym. \& \& \& \& \& <br>
\hline 140 \& 0

8

8 \& \& | 41(1) 41 (2) |
| :--- |
| L58L L4 (F) |
| 42 (Q4) F4 (0) |
| 421 (Q4) L5 (F) |
| 427 0020F |
| 466L 50 (0) |
| JOF L56L |
| 26(M14) OOF |
| 26(M5) 00115 ( |
| OOK | \& from 3(M4)

M14) \& \& \& <br>
\hline 149 \& 0

10 \& (Q2) \& | L510L 425L |
| :--- |
| 50(1) 75(193) |
| K5F L4 (N) |
| L4 (DR) 404L |
| OOF OOF |
| 40F 40 F |
| F54L 404L |
| F55L 425L |
| LO (Q4) 364L |
| F5(1) 42 (1) |
| 2221(M14) 001 |
| 00K | \& \[

15(M14)
\] \& Auxili

Read a \& | for M14 |
| :--- |
| of $\mathrm{C}_{\mathrm{r}} \mathrm{fr}$ | \& drum <br>

\hline 160 \& 0

10. \& (Q3) \& \begin{tabular}{l}
L510 (Q2) 424 L <br>
40(2) 75(193) <br>
S5F L4 (N) <br>
I4 (DS) 405L <br>
OO1F L5F <br>
OOF OOF <br>
F55L 405L <br>
F54L 424L <br>
LO1 (Q4) 324L <br>
F5 (2) 42 (2) <br>
26106(M14) OOF

 \&  \& Auxili \& 

II for ML <br>
umn $C_{r}^{-1}$
\end{tabular} \& drum <br>

\hline
\end{tabular}




| LOCAT | TION |  | ORDER |  | TES $\quad$ PAGE $32 \quad 1.90$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Abs. R | Rel. | Sym. |  |  |  |
| $203$ | 20 <br> 30 <br> 40 |  |  |  | $\begin{aligned} & \text { (3) } \sqrt{s_{i j}} c_{r}^{i j} \sqrt{s_{s j}} \times 10^{-1} \\ & (4) \sqrt{c^{i j}} s_{i} \sqrt{c^{i j}} \end{aligned}$ <br> Print $C_{f} \times 10^{-1}$ |






