

UNIVERSITY OF ILLINOIS  
DIGITAL COMPUTER LABORATORY  
ILLIAC PROGRAM LIBRARY

Library Routine M 26 - 264

**TITLE:** Eigenvalues and Eigenvectors of a Symmetric Matrix  
(SADOI Only)

**TYPE:** Complete program

**NUMBER OF WORDS:**  $176 + (R\ 1) + (N\ 12) + (P\ 16) = 280$

**TEMPORARY STORAGE:** 0 - 30

**DURATION:** Approximately  $5n^3$  milliseconds per iteration

**PARAMETERS:** Four Parameters must be read in before the main routine and the matrix are read in.

S3: OOF 00gF where  $1 \leq g \leq 12$  is the number of decimal digits to be printed.

S4: OOF 00nF where n = order of matrix.

S5: OOF 00mF where m = 2n if eigenvalues and eigenvectors both are desired; otherwise m = n.

S6: OOF 00(311 + n(n+1)/2)F

**SYMBOLIC ADDRESSES USED:** (M26); (1); (2); (3); (4).

**LIBRARY ROUTINES USED:** (P16); (N12); (R1).

**LIMITATIONS:** If eigenvalues only are desired then n, the order of the matrix, must be  $\leq 37$ . If eigenvectors are desired as well then  $n \leq 21$ .

**SCALING:** The elements of the matrix should be scaled so that

$$\sum_{k=1}^n \sum_{j=1}^n a_{jk}^2 < \frac{1}{2}.$$

The eigenvalues and eigenvectors will be scaled by the same factor. However, in order to prevent overflow the eigenvectors are actually scaled down by an additional factor of  $10^{-1}$ .

**METHOD OF USE:**

1. Parameters must be read in first.
2. Read in Program tape; machine will stop on a 24(10J) SD.
3. Insert matrix tape in reader, the elements of the matrix being punched as signed decimal fractions, i.e., as + or - followed by up to 12 decimal digits. As only

half the off diagonal elements are needed (because of symmetry), the user need punch only  $a_{11}$ ;  $a_{21}$ ,  $a_{22}$ ;  $a_{31}$ ,  $a_{32}$ ,  $a_{33}$ ; etc.

i.e., we punch only  $\begin{pmatrix} a_{11} & & & & & \\ a_{21} & a_{22} & & & & \\ a_{31} & a_{32} & a_{33} & & & \\ \vdots & & & \ddots & & \\ a_{nl} & \dots & \dots & \dots & \dots & a_{nn} \end{pmatrix}$

Following  $a_{nn}$  an N terminating symbol must be punched. Start with black switch.

If there has been an arithmetic failure then two F's will be punched out and the machine will come to an OF stop at location  $(OLK)_{SD}$ . If the user, nevertheless, wishes to continue the computations he may do so by raising and lowering the white switch.

OUTPUT FORMAT:

If there has been no arithmetic failure then the first thing punched out will be an integer - the number of iterations required for convergence. This will be followed by the column of eigenvalues. The machine will then come to a 34 stop at  $(121)_{SD}$ . If eigenvectors are also desired then a black switch start will cause the eigenvectors to be punched as n columns of n components each (the eigenvectors being separated from one another by an extra line feed and carriage return). The machine will then come to an OF stop at  $(132)_{SD}$ .

For a complete description of the mathematical method see the write-up of M25.

DATE	May 6, 1959
PROGRAMMED BY	<i>W. A. Rosenblatt</i>
APPROVED BY	<i>J. N. Snyder</i>

LOCATION	ORDER	NOTES	PAGE 1
	00 22K		
0	00 1F		
	00 1F		
1	00 S4		
	00 S4		
2	00 (4)		
	00 (4)		
3	80 S5		
	00 S5		
4	20 F		
	00 F		
5	00 F		
	00 F		
6	40 F		
	00 F		
7	00 F		
	00 F		
8	J0 S6		
	74 S6		
	(M2) 00 K		
	(P16) 00 K		
	(R1) 00 K		
	00 K(M26)		
0	F5 27F		
	40 27F		
1	41 20F		
	41 21F		
2	41 4F		
	41 5F		
3	L5 4F		
	L4 22F		
4	40 4F		
	L0 23F		
5	32 100L		
	50 5F		

LOCATION	ORDER	NOTES	PAGE 2
6	L5 5F 74 5F		
7	00 38F 42 29F		
8	00 20F 46 29F		
9	L5 29F L4 24F		
10	46 130L L4 5F		
11	42 29L 46 94L		
12	42 95L 50 4F		
13	L5 4F 74 4F		
14	00 38F 42 29F		
15	00 20F 46 29F		
16	L5 29F L4 24F		
17	42 130L L4 4F		
18	46 96L 42 28L		
19	L0 4F L4 5F		
20	42 21L 42 22L		
21	42 96L L1 0		
22	36 97L L5 F		
23	40 7F L5 21F		

LOCATION	ORDER	NOTES	
24	50 7F 74 7F		
25	14 20F 40 20F		
26	S5 F 40 21F		
27	L5 7F 00 1F		
28	40 7F L5 F		
29	40 9F L5 F		
30	40 8F L0 9F		
31	40 10F L7 7F		
32	L2 10F 50 19F		
33	36 37L L5 7F		
34	66 10F S5 F		
35	10 1F 40 11F		
36	L5 26F 26 40L		
37	L5 10F 66 7F		
38	S5 F 10 1F		
39	40 11F 26 124L		
40	40 12F 50 11F		
41	75 11F 14 26F		

LOCATION	ORDER	NOTES	PAGE 4	M 26
42	40 1F S5 F			
43	40 F 50 43L			
44	22 (R1) 40 14F			
45	L3 12F 50 19F			
46	66 14F S1 F			
47	40 13F 22 118L			
48	40 1F 41 F			
49	22 49L 50 49L			
50	22 (R1) L5 12F			
51	L0 26F 32 56L			
52	L5 12F 36 54L			
53	L1 26F 22 54L			
54	L5 26F 50 19F			
55	66 14F S5 F			
56	22 58L 19 1F			
57	50 11F 74 13F			
58	00 1F 40 14F			
59	66 2F S5 F			

LOCATION	ORDER	NOTES	PAGE 5	M 26
60	40 3F			
	41 11F			
61	27 70L			
	L5 11F			
62	L0 23F			
	36 100L			
63	L4 23F			
	40 131L			
64	L5 5F			
	L0 11F			
65	32 67L			
	L5 4F			
66	L0 11F			
	32 68L			
67	22 69L			
	L5 22F			
68	46 131L			
	L5 22F			
69	42 131L			
	L5 131L			
70	L4 130L			
	40 130L			
71	46 74L			
	46 78L			
72	46 83L			
	42 76L			
73	42 80L			
	42 81L			
74	50 F			
	73 2F			
75	40 10F			
	50 F			
76	50 3F			
	74 F			
77	L4 10F			
	40 F			

LOCATION	ORDER	NOTES	PAGE 6
78	50 F 79 3F		
79	40 10F S5 F		
80	50 2F 74 F		
81	L4 10F 40 F		
82	22 82L L5 F		
83	40 F L5 11F		
84	L4 22F 40 11F		
85	L0 25F 32 61L		
86	LJ 13F 40 F		
87	L9 13F 40 1F		
88	50 7F 7J 14F		
89	40 10F S5 F		
90	50 8F 74 F		
91	L4 10F 40 10F		
92	S5 F 50 9F		
93	74 1F L4 10F		
94	40 F L5 8F		
95	L4 9F L0 F		



LOCATION	ORDER	NOTES	PAGE 7
96	40 F 41 F		
97	L5 5F L4 22F		
98	40 5F L0 4F		
99	32 2L 22 5L		
100	27 63L 41 15F		
101	41 16F L5 24F		
102	42 104L 46 104L		
103	22 103L L5 16F		
104	50 F 74 F		
105	L4 15F 40 15F		
106	S5 F 40 16F		
107	L5 104L L4 22F		
108	40 104L L0 30F		
109	32 103L L5 18F		
110	L0 16F L0 39F		
111	L4 17F L0 15F		
112	36 (2) L5 128L		
113	26 126L 19 34F		

LOCATION	ORDER	NOTES	PAGE 8
114	L2 F 36 116L		
115	92 904F 0F F		
116	L5 15F 40 17F		
117	L5 16F 40 18F		
118	26 L L5 13F		
119	L0 123L 36 121L		
120	LJ 13F 26 48L		
121	LJ 123L 40 2F		
122	22 50L 00 F		
123	3L 4095F LL 4095F		
124	F5 11F 10 1F		
125	26 40L 00 F		
126	40 112L L5 129L		
127	40 113L 26 116L		
128	36 (2) L4 20F		
129	40 F 19 34F		
130	00 F 00 F		
131	00 F 00 F		

LOCATION	ORDER	NOTES	PAGE 9
132	00 F		
	00 (3)		
133	00 F		
	00 F		
	00 K(1)		
0	50 (4)		
	50 L		
1	26 (NL2)		
	L5 21(NL2)		
2	10 20F		
	42 133(M26)		
3	42 6L		
	42 12(2)		
4	L5 4F		
	L0 5F		
5	36 (M26)		
	26 6L		
6	L5 28F		
	40 F		
7	L5 6L		
	L4 13L		
8	40 6L		
	F5 12L		
9	40 12L		
	L0 11L		
10	36 6L		
	26 (M26)		
11	80 F		
	00 S4		
12	00 F		
	00 F		
13	00 F		
	00 1S4		
	00 K(2)		
0	22 L		
	L5 27F		

LOCATION	ORDER	NOTES	PAGE 10
1	52 3F 50 1L		
2	26 (P16) 92 131F		
3	92 515F F5 19F		
4	40 19F 14 132(M26)		
5	40 132(M26) 42 7L		
6	10 133(M26) 34 10L		
7	22 7L 15 F		
8	50 S3 50 8L		
9	26 (P16) 22 2L		
10	92 135F 92 515F		
11	F5 12L 40 24L		
12	50 25L 7J F		
13	50 S3 50 13L		
14	26 (P16) 92 131F		
15	92 515F 15 12L		
16	14 12(1) 40 12L		
17	F5 27L 40 27L		
18	10 12(1) 32 19L		

LOCATION	ORDER	NOTES	PAGE 11	M 26
19	26 12L			
	L5 24L			
20	40 12L			
	41 27L			
21	F5 26L			
	40 26L			
22	L0 12(1)			
	32 23L			
23	26 10L			
	OF F			
24	00 F			
	00 F			
25	00 F			
	00 2000 0000 0000J			
26	00 F			
	00 F			
27 (3)	00 F			
	00 F			
	00 K(4)			
	2 (1)			
	26 LN			