UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE E 4-193

TITLE	Numerical Differentiation With Interpolation (DOI or SADOI)
TYPE	Closed
PURPOSE	Given x, successive values of $(h^r/r!)\, f^{(r)}(x)$ for arbitrary r are found, as well as the interpolated value of f(x).
NUMBER OF WORDS	103
DURATION	$(2d^2 + 3)(r + 1) + 2d(1 - r)$ milliseconds, where d = depth of interpolation r = number of derivatives desired (see Note 2).
ACCURACY	Depends on the mesh size and the depth of interpolation.
PRESET PARAMETERS	Prior to reading in this routine, memory locations 3 and 4 must contain the following: 3: 00 F 00 tF t = initial location for temporary storage 4: 00 F 00 eF e = difference between successive interpolates (see Note 1).
TEMPORARY STORAGE	0 through 15 plus $[(r + 1)(2d - r)/2] + 1$ locations beginning at t.
RESULTS	When the routine is left, $f(x)$, $hf^{(1)}(x)$, $(h^2/2!)\, f^{(2)}(x)$, ..., $(h^r/r!)\, f^{(r)}(x)$ are in memory locations t, t + 1, t + 2, ..., t + r, respectively.
ENTRY	With the argument x in A and the first word of this routine in memory location q, entry is made by:

	50 rF
p	50 pF
	26 qF
	00 nF
p + 1	00 mF
p + 2	00 dF

where n = number of subintervals over the interval [0,1] (see Description).

m = location of the first tabulated value $f(x_0)$.

DESCRIPTION

This routine employs Neville's interpolation formula and its derivatives, and computes the interpolate and derivatives of f(x) from one or more tables stored in the memory.

The domain of f must be scaled so that $0 \leq x \leq 1$. The mesh, therefore, will consistof the n + 1 equally spaced points $0 = x_0 < x_1 < x_2 < \ldots < x_n = 1$, and the table must consist of the n + 1 function values $f(x_i)$ at these points, where $-1 \leq f(x_i) < 1$. Given any argument x, subject only to $0 \leq x < 1$ (thus we may have $x = x_i$ for $0 \leq i < n$), this routine selects an equal number of points on each side of x whenever possible. For example, if n = 15 and d = 5 (thus requiring 5 points), then for

(a) $x = x_0$ the routine uses x_0, x_1, x_2, x_3, and x_4.

(b) $x_8 < x < x_9$ the routine uses x_6, x_7, x_8, x_9, and x_{10}.

(c) $x_{14} < x < x_{15}$ the routine uses x_{11}, x_{12}, x_{13}, x_{14} and x_{15}.

Internally, this routine employs no scaling devices

since all intermediate calculations must be in range provided a fine enough mesh is taken. The occurrence of overflow, therefore, indicates the mesh should be refined.

NUMERICAL METHOD

If we define $f_{i,i+1,\ldots,i+j-1,i+j}$ to mean $f(x_i, x_{i+1}, \ldots, x_{i+j-1}, x_{i+j})$ then Neville's method of successive linear interpolation is given by

$$f_{i,i+1,\ldots,i+j} = \frac{(x_{i+j} - x)\, f_{i,i+1,\ldots,i+j-1} - (x_i - x)\, f_{i+1,i+2,\ldots,i+j}}{x_{i+j} - x_i},$$

and the r^{th} derivative of this form is:

$$f^{(r)}_{i,i+1,\ldots,i+j} = \frac{(x_{i+j} - x)\, f^{(r)}_{i,i+1,\ldots,i+j-1} - (x_i - x)\, f^{(r)}_{i+1,i+2,\ldots,i+j}}{x_{i+j} - x_i} + \frac{f^{(r-1)}_{i+1,i+2,\ldots,i+j} - f^{(r-1)}_{i,i+1,\ldots,i+j-1}}{x_{i+j} - x_i}$$

where both expressions are applied successively with $j = 1,2,\ldots,d$. Replacing $x_{i+j} - x_i = jh$ by j/n and x by $x_k + x - x_k$ where $x_k \leq x < x_{k+1}$ and rearranging terms, these equations become:

$$f_{i,i+1,\ldots,i+j} = f_{i,i+1,\ldots,i+j-1}$$

$$+ (1/j)[(i-k)\,(f_{i,i+1,\ldots,i+j-1} - f_{i+1,i+2,\ldots,i+j})$$

$$- (nx - nx_k)\,(f_{i,i+1,\ldots,i+j-1} - f_{i+1,i+2,\ldots,i+j})]$$

$$(h^r/r!)\, f^{(r)}_{i,i+1,\ldots,i+j} = f^{(r)}_{i,i+1,\ldots,i+j-1}$$

$$+ (1/j)\,[(i - k)\,(f^{(r)}_{i,i+1,\ldots,i+j-1} - f^{(r)}_{i+1,i+2,\ldots,i+j})$$

$$- (nx - nx_k)\,(f^{(r)}_{i,i+1,\ldots,i+j-1} - f^{(r)}_{i+1,i+2,\ldots,i+j})$$

$$+ (f^{(r-1)}_{i+1,i+2,\ldots,i+j} - f^{(r-1)}_{i,i+1,\ldots,i+j-1})]$$

which are the forms used by this routine.

NOTES

(1) The routine will be automatically left whenever two successive interpolates differ by less than $e < 2^{-19}$. If it is desired to go to fixed depth, set e = 0.

(2) r and d must satisfy the inequality $r \leq d - 1$.

(3) If nothing more than the interpolate is desired, set r = 0.

Rt: 8/7/59
DATE September 19, 1955
CODED BY J. H. Fishel
APPROVED BY J. P. Nash

JHF/ge

LOCATION	ORDER	NOTES PAGE 1
	OOK (E4)	
0	40 F	Store argument x
	41 1F	
1	41 2F	
	41 4F	
2	41 5F	
	M5 83	
3	42 6L	Set add. for (p + 1)
	L4 100L	
4	42 7L	Set add. for (p + 2)
	L4 100L	
5	42 99L	Set Link
	10 20 F	
6	42 1F	
	L5 F	
7	42 2F	Set preset parameters
	L5 F	
8	46 3F	
	42 4F	
9	L0 100L	
	10 1F	
10	42 5F	(k - i)
	50 2F	
11	75 F	
	L0 5F	
12	40 6F	
	32 15L	
13	L5 5F	
	L0 100L	
14	40 5F	
	F5 6F	
15	26 12L	
	L4 4F	Determine x_i
16	40 8F	
	F5 2F	

LOCATION	ORDER	NOTES PAGE 2
17	L0 8F	
	40 8F	
18	36 22L	
	L5 6F	
19	L0 100L	
	40 6F	
20	F5 5F	
	40 5F	
21	F5 8F	
	22 17L	
22	S5 60L	
	40 7F	$(mx - mx_k)$
23	L5 6F	
	00 20F	
24	L4 3F	
	46 26L	
25	41 6F	
	S5 85L	
26	L5 F	Form the set $f(x_i)$
	40 S3	
27	F5 6F	
	40 6F	
28	L0 4F	
	36 31L	
29	L5 26L	
	L4 101L	
30	40 26L	
	26 26L	
31	L5 2L	
	42 26L	
32	41 2F	
	41 3F	Set Counters
33	41 8F	
	41 F	

LOCATION	OPERATION	NOTES PAGE 3
34	L5 4F	
	L0 100L	
35	40 4F	Has desired depth been reached?
	L3 4F	
36	32 68 L	
	F5 F	
37	40 F	
	L1 1F	
38	L4 2F	
	32 4L	Has desired derivative been found?
39	L5 5L	
	46 55L	
40	F5 4F	
	40 9F	
41	40 14F	
	L4 8F	Advance counters
42	40 8F	
	F5 2F	
43	40 2F	
	L5 9F	
44	40 10F	
	L5 8F	
45	40 3F	
	L5 2F	
46	40 12F	
	49 13F	
47	F5 6L	
	L4 3F	
48	42 66L	
	L0 100L	
49	42 55L	
	42 7AL	
50	00 20F	Set addr. in formula
	46 66L	

LOCATION	ORDER	NOTES PAGE 4
51	46 74L	
	L5 2L	
52	L4 3F	
	L0 10F	
53	42 71L	
	L4 100L	
54	00 20F	
	46 71L	
55	22 60L	
	41 F	
56	F5 55L	
	40 55L	
57	22 57L	Set j^{th} derivatives equal to
	L5 14F	0 before beginning j^{th} depth.
58	L0 100L	
	40 14F	
59	32 55L	
	L5 22L	
60	46 55L	
	L5 12F	
61	L0 100L	
	40 12F	
62	32 64L	Will this be an interpolation
	F5 54L	or differentiation?
63	42 70L	
	L5 25L	
64	42 76L	
	41 15F	
65	L1 5F	
	40 14F	
66	L5 F	
	L0 F	
67	40 11F	
	L3 11F	

LOCATION	ORDER	NOTES PAGE 5
68	L4 102L	Test difference of successive
	32 69L	interpolates against e
69	40 13F	
	50 11F	
70	71 7F	interpolate or differentiate
	26 71L	
71	L4 F	
	L0 F	
72	50 14F	
	74 11F	
73	66 F	
	S5 82L	
74	L4 F	
	40 F	
75	F5 15F	
	40 15F	
76	L0 4F	Has given order of interp. or
	32 82L	diff. been completed?
77	L5 66L	
	L4 101L	
78	40 66L	
	L5 71L	
79	L4 101L	
	40 71L	
80	L5 74L	Advance adds. in formula
	L4 101L	
81	40 74L	
	F5 14F	
82	22 65L	
	L5 3F	
83	L0 10F	
	40 3F	Advance counters
84	F5 10F	
	40 10F	

LOCATION	ORDER	NOTES PAGE 6
85	26 47L	
	L5 54L	
86	42 70L	
	L5 73L	
87	42 76L	
	L1 13F	
88	36 34L	Are successive interpolates
	F5 2L	< e?
89	42 93L	
	L5 6F	
90	40 2F	
	22 97L	
91	L5 2L	
	L4 2F	
92	00 20F	
	46 93L	
93	L5 F	Arrange interp. and derivs. for
	40 F	exit.
94	L5 6F	
	L0 100L	
95	40 6F	
	L4 2F	
96	40 2F	
	F5 93L	
97	40 93L	
	L5 1F	
98	L0 100L	
	40 1F	
99	36 91L	
	26 F5	Link
100	00 F	
	00 1F	
101	00 1F	
	00 1F	

LOCATION	ORDER		NOTES PAGE 7
102	00 F		
	00 S4		*

JWF/ge