## 5 $\overline{5} 5$ 5IGMA 5/7 FUNCTIONAL MATHEMATICAL PROGRAMMING SYSTEM

Reference Manual

SCIENTIFIC DRTA 5TSTEMS


# FUNCTIONAL MATHEMATICAL PROGRAMMING SYSTEM REFERENCE MANUAL for 

 SDS SIGMA 5/7 COMPUTERSPRELIMINARY EDITION

9016 09A

April 1969

## 505

## RELATED PUBLICATIONS

Title

| SDS Sigma 5 Computer Reference Manual | 900959 |
| :--- | :--- |
| SDS Sigma 7 Computer Reference Manual | 900950 |
| SDS Sigma 5/7 Batch Processing Monitor (BPM) Reference Manual | 900954 |
| SDS Sigma 5/7 Batch Processing Monitor (BPM) Operations Manual | 901198 |

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## 1. INTRODUCTION

This manual describes the Functional Mathematical Programming System (FMPS) for SDS Sigma 5/7 computers. FMPS is a mathematical programming system composed of functions for solving linear programming (LP) problems. The manual is designed for the user who is familiar with mathematical programming theory and application. Chapter 1 provides general information about FMPS features. These features include:

- Subroutines, called "procedures", for solving linear programming problems.
- A user-oriented control language for sequencing operations, controlling exception conditions, and adjusting tolerances.
- The flexible design of communication files and format options, and the ability, at the level of each major function, to direct the output stream to magnetic tape (in addition to the printer), permitting FMPS to be used as a free-standing package, or as part of a userdesigned optimization package.

Chapters 2 and 4 discuss basic concepts and basic procedures, respectively, of FMPS that are applicable to all operating modes. FMPS control language statements are described in detail in Chapter 3. Chapter 5 presents data formats and data deck organization. Chapter 6 outlines procedures used in the linear programming operating mode, and Chapter 7 describes procedures used in the separable programming operating mode. When these procedures are identical in both modes, they are repeated in Chapter 7 for user convenience. Appendix A describes parametric programming and ranging procedures (an optional extension to the basic system); Appendix B is a list of error messages; and Appendix $C$ presents an FMPS LP mode sample run.

## PROCEDURES

FMPS procedures and their functions are given in Table 1 below. (Basic FMPS operating procedures are given in Chapter 4.)

Table 1. FMPS Procedures

| Procedure | Purpose |
| :--- | :--- |
| INPUT | Reads matrix data from cards or tape in <br> standard FMPS format or in various <br> SHARE formats such as LP 90/94, <br> UNIVAC 1108 LP, or CDC CDM4. |
| OUTPUT | Displays the input or current matrix in <br> various formats. |
| Reads correction data for modifying the <br> matrix. |  |

Table 1. FMPS Procedures (cont.)

| Procedure | Purpose |
| :---: | :---: |
| CRASH | Creates an initial basis structure for the current matrix and performs preliminary validity checks on the matrix. |
| OPTIMIZE, INVERT | Performs the actual linear programming solution. |
| SOLUTION | Displays the solution values in various formats. |
| ERRORS | Displays the computation errors incurred during the solution process for the primal and dual problems. |
| CONDITION | Prints out the communications region contents. |
| GET | Retrieves information about a row or column and alters the strategy in the control language. |
| BASISOUT | Punches or files (FILE parameter) the current basis structure and bounds status. |
| SAVE | Saves the contents of the communications region, the various internal work areas, and all internal files (MATRIX, INVERSE, etc.) on the tape file RESTART. |
| BASISIN | Inputs a new basis or modifies the existing basis. |
| RESTORE | Restores (from file RESTART) the data areas and internal files saved by SAVE. |
| PARARHS, PARAOBJ | Performs post-optimal parametric analysis of the solution with respect to the right-hand-side and objective function. (Refer to Appendix A.) |
| RANGE | Performs post-optimal range analysis. (Refer to Appendix A.) |
| LOADLIST | Loads a list of row labels and/or column labels to be used as selection lists or masks during the OUTPUT, SOLUTION, and/or RANGE procedures when selective output is desired. |

## CONTROL LANGUAGE

The sequence of operations executed in an FMPS run is controlled through statements, written in a user-oriented control language, that

- Initialize and, if desired, modify tolerances during execution.
- Assign input/output devices at the FMPS level.
- Preprogram action to be taken in case of exception or error conditions.

In the following chapters of the manual, certain conventions have been adopted for defining FMPS commands. Capital letters indicate command words that are required in the literal form shown. Lower case letters are figurative representations of parameters. Command parameters enclosed by braces (\{\}) indicate a required choice. Bracketed ([ ]) parameters are optional. The format of the FMPS control language closely resembles the FORTRAN language. A procedure is activated by using the CALL statement as shown below,

```
CALL procedure [(argument)]
```

where CALL is followed by the name of the procedure and, if required, a list of arguments enclosed by parentheses to be used by the procedure. For example, the statement

## CALL OUTPUT (BYROWS)

causes the input matrix to be listed by rows.
Initialization and modification of tolerances are performed by means of assignment statements. Reserved names have been assigned to each tolerance available to the user. For example, the statement

FDJZT $=1.0 \mathrm{D}-6$
assigns to the DJ zero tolerance the value 0.000001 . Other examples of tolerances available to the user are FMPIVT (minimum pivot clearance during optimization) and ILINES (number of lines to be printed per page).

Provision is made for user working-storage variables. The language allows execution of simple arithmetic such as

where
FWD3 is a user working-storage variable.
325 is the label of a statement in the control program as in a FORTRAN program.

Reserved variable names have been assigned for the handling of exception interrupts. For example, the statement

can be used to cause statement 460 to be executed if unboundedness occurs during optimization or parametric procedures. Assignments are dynamic and can be modified under program control during the course of execution.

## COMMUNICATION REGION

An area of computer memory called the communications region (CR) contains all variables with reserved names (such as FDJZT, ILINES, KUBS, etc.). FMPS initializes these variables to standard values; therefore, it is not necessary to initialize them in the control program if the standard values are appropriate.

## FILES

Data is carried in disc or tape files. Their purpose is to hold FMPS data in a format allowing maximum processing speed. The standard FMPS files are MATRIX, INVERSE, UTIL1, and UTIL2. These files carry the matrix, its inverse, and various intermediate information (UTILI and UTIL2). In addition, the RESTART file may be used for intermediaie dumping of the run status. The DEVICE and ATTACH procedures must be used to define Data Control Blocks (DCBs) through which files are to be used and to assign these files to these devices. (See Chapter 4 for a detailed description of these procedures.) The files are internal to FMPS and are not intended to be used as input or output files by user-designed programs.

## INPUT DATA

Data can be input to FMPS from cards or tapes, in either card image format, or FORTRAN unformatted WRITE format. (FORTRAN unformatted WRITE format provides for better data packing when using user-written matrix generators.) Input data for FMPS is accepted by the following procedures: INPUT, REVISE, LOADLIST, and BASISIN.

## OUTPUT

Most FMPS procedures create printer output. The OUTPUT, SOLUTION, and BASISOUT procedures write output on magnetic tape in addition to the printer if the user so chooses. The magnetic tape output for OUTPUT and

SOLUTION is in FORTRAN unformatted WRITE format, which provides a compact data format for interface with user-designed report writers. The BASISOUT procedure produces either punched cards or card images on magnetic tape. Both are suitable for subsequent reloading by the BASISIN procedure. As with input files, a symbolic unit for each output file must be declared by means of the ATTACH procedure.

Users need not be concerned with the format of the FMPS internal files since INPUT and OUTPUT transfer data to or retrieve data from them in a user-oriented format. However, note that the user must assign DCBs for the internal files at the beginning of the run.

To provide a convenient method for abstracting the output results (whether they are written on tape or printed), the OUTPUT, SOLUTION, and RANGE procedures include many optional parameters. For example, OUTPUT provides for listing the matrix by rows, by columns, in matrix tableau format, or in coded format. (In coded format, coefficients are symbolized by letters showing the sign and magnitude of the coefficients.) Similarly, the RANGE procedure can be made selective with respect to the type of variable printed, that is, printing only the basic, only the nonbasic, or both. Furthermore, RANGE can select individual items of information for printing.

All three procedures can be made selective with respect to the individual rows and/or columns to be printed, that is,

1. Print only specified rows.
2. Print all rows except specified rows.
3. Print all rows which match specified masks.
4. Print all rows except those which match specified masks.

Similar options are available independently for columns.

## SELECTION LISTS

Selection lists consist of names (rows and/or columns) and/ or masks (rows and/or column names with an asterisk matching any character in the row or column name in the corresponding position in FMPS internal files). Since the same selection list usually applies over an entire run, a single procedure, LOADLIST, is used to load the rows-andcolumns selection lists.

Items selected are controlled by optional arguments. For example

causes the solution to be written on the user file SOLFILE as well as on the printer, outputting only the rows included in row selection list LISTR, the columns not included in column selection list LISTC, the row name and its slack activity for rows, and the column name and its activity for columns. One selection list may be used to control output items during several procedures such as OUTPUT, SOLUTION, and RANGE. Such procedures have an optional parameter indicating whether the information to be output is to be controlled by the selection list. The list need be loaded only once. In some procedures such as RANGE, reduction of output and calculations will result in sizable savings in execution time.

## 2. FMPS FUNDAMENTALS

This chapter describes in detail some basic elements of FMPS such as variables and constants available in the control language, internal files, selection lists, and the structure of communication files.

## CONSTANTS

The FMPS control language uses three types of constants in Arithmetic statements and as parameters in procedural CALL statements. They are: integer, floating-point, and character.

## INTEGER

A number written without a decimal point is called an integer constant. An integer constant is composed of one to seven decimal digits. It may be preceded by a plus sign, a minus sign, or a blank. If unsigned, it is assumed to be positive. It may not contain any embedded blanks. Sample valid and invalid integer constants are shown in the tables below.

## VALID INTEGER CONSTANTS

0
100000
-54
$+1$

## INVALID INTEGER CONSTANTS

-i7 35 Contains an embedded blank
100,000
Contains a comma

## FLOATING-POINT

A number with a decimal point, optionally followed by a decimal exponent (written as the letter D followed by a signed or unsigned one- or two-digit integer constant) is called a floating-point constant. The magnitude of a real constant must be compatible with that allowed by FORTRAN for the machine being used. However, only eight significant digits are allowed. A floating-point constant may be preceded by a plus sign, a minus sign, or a blank. Embedded blanks are not allowed. The first table shown below gives correct floating-point constants and their real magnitudes. The second table shows invalid representations of floatingpoint constants.

```
VALID FLOATING-POINT CONSTANTS
\begin{tabular}{ll}
-3.49 & -3.49 \\
1.47 D 3 & 1470
\end{tabular}
    .47D3 !470.
    -. 23D-4 -.000023
    0.0 zero
    .2D+2 20.
```

INVALID FLOATING-POINT CONSTANTS

| .123456789 D 1 | Will be truncated to eight <br> significant digits |
| :--- | :--- |
| $1,217.2$ | Contains a comma |
| 1.7 D 2 | Contains a blank between |
| $1.3 E 4$ | D and 2 |
|  | E not valid - must use D |

## CHARACTER

A string of from one to eight characters, enclosed by single quotation marks, is called a character constant. (The single quotation mark is represented by a 5-8 punch on the card.) Character constants, sometimes called literals, may be composed of alphabetic, numeric, special, or blank characters. The quotation marks are not part of the character constant, but are used to delimit it. The quotation mark itself is the only special character not allowed within the body of the character constant. Correct character constants are shown directly below, incorrect examples in the second table.

## VALID CHARACTER CONSTANTS

```
'ROWS'
```

'THE END'
'2+3'
'DOG/CAT'

INVALID CHARACTER CONSTANTS

| 'CPERATION' | Only eight characters are <br> allowed |
| :--- | :--- |
| 'ABD | Second quotation mark missing <br> Em' $\mathrm{EC'}^{\prime}$ |
|  | Embedded quotation mark not <br> allowed |

## VARIABLES

Variables (storage references) are symbolic names of either locations within the control program (user working-storage variables), or locations in the FMPS communication region (CR variables).

All storage within FMPS is identified by type. The four types of variables, each identified by its leading character, are shown in Table 2 below.

Table 2. Types of Variables

| Code | Type |
| :---: | :--- |
| I | Integer |
| F | Floating-Point |
| A | Alphanumeric |
| K | Interrupt |

User-created variables are distinguished from CR variables by their second character, which must be a W. Also, usercreated variable names may contain a maximum of four characters, while $C R$ variable names may contain a maximum of eight characters. User-created variable names containing more than four characters will be truncated to four. The user may create a total of 50 integer and $K$-type variables and a total of 50 floating-point and alphanumeric variables. Each distinct type is discussed below.

## INTEGER

Each integer (I-type) variable is a single precision word containing a single precision integer value. Integer variables may assume any of the values of an integer constant. An I-type variable may be used in an Arithmetic statement, an IF statement, a WRITE statement, or as a parameter in a procedure CALL statement. Table 3 contains a list of all $C R$ integer variables and an explanation of each.

Some sample integer variables are shown in the following tables.

## VALID INTEGER VARIABLE NAMES

IFREQI CR variable for inversion iteration frequency
IWBG User working-storage variable
IW3 User working-storage variable
INVALID INTEGER VARIABLE NAMES
IU5 $\quad$ Not a valid CR variable name nor a valid user working-storage name since second character is not W
KROW Integer names must begin with I

## FLOATING-POINT

Each floating-point (F-type) variable is a double precision word and contains a double precision floating-point value. A floating-point variable may assume any of the values of a floating-point constant. It may be used in an Arithmetic statement, an IF statement, a WRITE statement, or as a parameter in a procedure CALL statement. Table 4 contains a list of all floating-point $C R$ variables and an explanation of each.

Table 3. Integer (I-type) CR Variables

| CR <br> Variables | Initialized Value | Explanation |
| :---: | :---: | :---: |
| IDNFSOL | 0 | Number of feasible solutions found for the integer problem. |
| IDULSTOP | 0 | Controls the brake on DUAL in MIP operating mode. If IDULSTOP is nonzero, DUAL will run to a feasible solution to the (possibly reduced) problem every IDULSTOP major iterations. |
| IESWT | 0 | The console jump switch to interrogate. IESWT must be 0-8. If zero, no switch is tested. If IESWT is $1-8$, and the jump switch is on, KESWT interrupt will occur. |
| IFREQA | 0 | Iteration frequency interrupt for OPTIMIZE, PARAOBJ, and PARARHS. If IFREQA is 0 , no interrupt will occur. Otherwise, the KFREQA interrupt will occur every IFREQA iterations. |
| IFREQI | 0 | Iteration frequency interrupt for inversion. In the iterating procedures OPTIMIZE, PARAOBJ, and PARARHS, the KINV interrupt will occur every IFREQI iterations (IFREQI >0). |
| IIWGHT | 0 | Infeasibility weighting switch. When IIWGHT is 1, the reciprocal of the amount of infeasibility is used as a weighting factor. When IIWGHT is -1 , the amount of each infeasibility is used as a weighting factor. When IIWGHT is 0 , all infeasibilities are given equal weight. |
| ILOGC | 0 | Iteration logging frequency on console typewriter. |
| ILOGP | 0 | Iteration logging frequency on standard printing device. |
| ILOGSS | 0 | On/Off switch for printing column selection messages during pricing of matrix. |
| ILINES | 50 | Maximum number of lines to be printed on a page. |
| INCAND | 0 | Number of profitable candidates from which one is selected during pricing of the matrix. For example, if INCAND is 5 , then from each group of 5 profitable columns, the most profitable is selected. If INCAND is 0 , the system will attempt to choose the optimum set. |
| ININF | 0 | Current number of infeasible variables in the basis. |

Table 3. Integer (I-type) CR Variables (cont.)

| CR <br> Variables | Initialized Value | Explanation |
| :---: | :---: | :---: |
| INVTIME | 0 | Switch controlling the KINV interrupt timing routine in the PRIMAL procedure. If INVTIME is 0 , the timing routine is active and causes KINV interrupts at times such that the total optimization time tends to be minimum. If INVTIME is -1 , the timing routine is not active. |
| IPARAM | 0 | Parametric programming mode indicator. If IPARAM is -1 , PARAOBJ is in effect, if IPARAM is 1 , PARARHS is in effect, and if IPARAM is 2 , PARARIM is in effect. |
| IPASS | 2000 | Number of assignments allowed during solution of the integer subproblem in MIP mode before the KASS interrupt occurs. |
| IPFES | 2000 | Number of feasible solutions allowed to the integer subproblem in the MIP mode before the KPFES interrupt occurs. |
| IPSOLTN | 0 | After solution of an integer subproblem in MIP operating mode, IPSOLTN will be nonzero if there was a change in the integer solution and wil! be zero if the integer solution has remained the same. |
| ITCNT | 0 | Current iteration count. |
| ITIME | 0 | The length of time, in minutes, before the KTIME interrupt will occur. The KTIME interrupt does not occur if KTIME is set to zero. Whenever the KTIME interrupt occurs, KTIME is set to zero. Time for KTIME is measured from the time of the last initialization of ITIME. |

Table 4. Floating-Point (F-type) CR Variables

| CR <br> Variables | Initialized Value | Explanation |
| :---: | :---: | :---: |
| FABSZT | 1. OD-12 | Absolute zero tolerance. Any computed number is replaced by zero if its absolute value is less than FABSZT. |
| FCMPDJ | 0.5 DO | Factor used in determining effective $D J$ when infeasible, that is, $D J E=F C M P D J * D J+(1.0-F C M P D J) * D J I$ <br> where DJE is Effective DJ, DJ is True DJ of column, and DJI is DJ based on infeasibility removal qualities of column. |
| FDJZT | 1.0D-07 | DJ zero tolerance. If the absolute value of the reduced cost (DJ) is less than FDJZT, it is considered zero. |
| FEPSILON | 0.0 | The value used to replace zero right-hand-side elements of inequalities on degenerate problems. If the constraint is of the less-than type, a zero RHS element is replaced with FEPSILON. If the constraint is of the greater-than type, a zero RHS element is replaced with -FEPSILON. |
| FINFZT | 1. OD-07 | Infeasibility zero tolerance. If the absolute value of the amount of infeasibility is less FINFZT, the variable is considered feasible. |
| FMINVT | 1. 0D-09 | Minimum inversion pivot tolerance. During INVERT, in the nontriangularized portion, an element is not considered as potentially pivotal unless its absolute value is greater than FMINVT. |
| FMPIVT | 1.0D-08 | Minimum pivot tolerance. During any optimization procedure (here, INVERT is not considered an optimization procedure), an element is not considered as potentially pivotal unless its absolute value is greater than FMPIVT. |
| FOBJVAL | 0.0 | Current objective function value. |
| FOBJWT | -1.0 | Objective function weight: -1.0 for maximization, 1.0 for minimization. |

Table 4. Floating-Point (F-type) CR Variables (cont.)

| CR <br> Variables | Initialized <br> Value | Explanation |
| :--- | :--- | :--- |
| FRDIFT | 4096.0 | Relative difference tolerance. This tolerance represents a power of 2, that is, <br> $2.0^{* *} 12$ is 4096. If the difference of two numbers is in the low-order twelve <br> bits, the numbers are considered identical. Any user-specified value must be <br> a power of 2, such as 8192.0 or 16384.0 . <br> Relative zero tolerance. If the absolute value of the summation of a series of <br> numbers divided by the absolute value of the largest sum or number is less than <br> FRELZT, the summation is considered to be zero. <br> Current sum of infeasibility. Each infeasibility is summed in absolute terms. |
| FSINF 0.0 <br> FTHETAC 0.0 <br> FTHETACM 0.0 <br> FTHETACP 0.0 <br> FTHETAR 0.0 <br> FTHETARM 0.0 <br> FTHETARP 0.0$\quad$Maximum value of THETA for PARAOBJ. <br> The incremental value for THETA during PARAOBJ for which the KSOLTN inter- <br> rupt will occur. <br> Initial value of THETA for PARARHS. <br> Maximum value of THETA for PARARHS. |  |  |

Correct and incorrect floating-point variable names are shown in the tables below.

## VALID FLOATING-POINT VARIABLE NAMES

FMPIVT $\quad \begin{aligned} & \text { CR variable for minimum pivot tolerance } \\ & \\ & \text { for optimization }\end{aligned}$.
FW01 User working-storage variable
FW5D User working-storage variable

## INVALID FLOATING-POINT VARIABLE NAMES

FDOG Not a valid CR variable name nor a valid user working-storage name since second character is not W

AW07
Floating-point names must begin with F

## ALPHANUMERIC

Each alphanumeric (A-type) variable is a double precision word and contains up to eight characters. An alphanumeric variable may assume any of the values of a character constant. It may be used in a simple Arithmetic statement, in an IF statement, in a WRITE statement, or as a parameter in a procedure CALL statement. Table 5 contains a list of all alphanumeric CR variables and an explanation of each, followed by tables showing valid and invalid alphanumeric variables.

## VALID ALPHANUMERIC VARIABLE NAMES

ARHS CR variable for name of current right-hand-side

AWLD User working-storage variable
AW07 User working-storage variable
INVALID ALPHANUMERIC VARIABLE NAMES
$\begin{array}{ll}\text { AMESS } & \begin{array}{l}\text { Neither a valid CR variable name nor a } \\ \text { valid user working-storage name since } \\ \text { second character is not a } W\end{array} \\ \text { NAME } \quad \begin{array}{l}\text { Alphanumeric names must begin with A }\end{array}\end{array}$

## INTERRUPT

During the execution of a mathematical programming system, many conditions arise which require some form of corrective action. Although much thought is generally given to the corrective action to be taken, no particular action is suitable under all circumstances. The interrupt processing concept in FMPS has been developed to facilitate initiation of appropriate corrective action when it is required.

For each condition requiring corrective action or for any point where greater user flexibility is desired, a CR interrupt variable is reserved. The function of each variable is to serve as a pointer to a control language statement or group of statements that will perform the corrective active or procedural steps desired by the user and allow for the resumption or exiting of the procedure causing the interrupt.

FMPS will initialize all interrupt variables to perform standard recovery techniques. The user, through the use of the ASSIGN command, may reset any interrupt variable to perform his own sequence of commands.

An interrupt (K-type) variable may assume the value of any valid statement number. The user working-storage K-type
variable may be used in a GO TO statement, an ASSIGN statement, or a WRITE statement. Conversely, a K-type $C R$ variable may only be referenced in a WRITE statement or an ASSIGN statement. The K-type CR variable is a single precision word containing a pointer to a control language sequence of instructions to be executed if an interrupt in a procedure occurs. Table 6 contains a list of all interrupt variables and an explanation of each. Sample K-type variables are shown in the tables below.

VALID K-TYPE VARIABLE NAMES

| KMAJER | CR major error interrupt variable used by <br> many procedures |
| :---: | :--- |
| KWST | User working-storage variable |
| INVALID K-TYPE VARIABLE NAMES |  |$\quad$| KQUIT |
| :--- |
| IWAL a valid CR variable name nor a user |
| valid working-storage name since second |
| character is not a W |

## FILES

FMPS includes two types of files:

- INTERNAL FILES For intermediate storage during FMPS procedures (magnetic tape or disc)
- COMMUNICATION FILES

Table 7 lists required and optional files for operating in the linear programming (LP) or separable programming (SEP) operating mode. This table also indicates the input/ output device type (sequential such as tape, or randomaccess such as disc) that is required, preferred, or optional.

Table 5. Alphanumeric (A-Type) CR Variables

| CR <br> Variables | Initialized <br> Value | Explanation |
| :--- | :--- | :--- |
| ADATA | None | Contains the name of the data deck for data reading procedures such as INPUT, <br> REVISE, etc. Also used by data-outputting procedures (such as BASISOUT) to <br> name output data deck. It specifies the name that appears on the NAME card <br> of image input. (Refer to Chapter 5 for general data formats). |
| AOBJ | None | Contains name of objective function row. <br> APBNAME <br> APOBJ |
| Contains name of problem. |  |  |
| APRHS | None | Contains name of PARAOBJ change row. |
| ARHS | None | Contains name of PARARHS change column. |

Table 6. Interrupt (K-Type) CR Variables

| CR <br> Variables | Initialized <br> Value | Explanation |
| :--- | :--- | :--- |
| KFREQA | None | Iteration frequency A interrupt. This interrupt will occur when IFREQA iterations <br> occur. <br> Inversion interrupt. This interrupt will occur when IFREQI iterations occur or <br> an inversion is required. |
| KINV | None | Terminate Run |
| KIOER | Tnput/output device error interrupt. |  |
| KMAJER | Norminate Run | Major error interrupt. |
| KMINER | Ninor error interrupt. |  |
| KNFS | None | No feasible solution interrupt. <br> KSOLTN |
| KTIME | NolUTION print interrupt. |  |
| KUBS | None | Elapsed time interrupt. This interrupt will occur when ITIME minutes have elapsed. |
| Unbounded solution interrupt. |  |  |

Table 7. Internal and Communication Files

| Required Internal Files |  |  |
| :---: | :---: | :---: |
| File Name | Device Type | Description of File |
| MATRIX | Sequential or Random-Access | Contains the internal representation of the matrix processed by INPUT. |
| INVERSE | Preferably Random-Access | Contains the internal representation of the product form of the inverse. |
| UTIL 1 | Sequential or Random-Access | A utility file used by many procedures for scratch storage. |
| UTIL2 | Sequential or Random-Access | A utility file used by many procedures for scratch use. |
| Optional Internal Files |  |  |
| RESTART | Sequential | Used by the SAVE procedure for storing all files for later resumption of run. Used by the RESTORE procedure for restoring the machine to the state at the time the SAVE procedure prepared the file. |
| Optional Communication Files |  |  |
| 'filename' | Sequential | Any user-defined file used for internal communication between FMPS and user's programs. Several such files can be used. The quote marks are part of the name of the file. |

## INTERNAL FILES

Within each operating mode of FMPS, a minimum number of internal files is required. Each internal file has been assigned a unique preempted name, and these names will be referred to throughout this manual. The user is required to attach the required files to appropriate DCBs (see Chapter 4).

## STORAGE REQUIREMENTS FOR INTERNAL FILES

The number of words of disc storage required by the MATRIX file is specified by the following equation.

$$
2.25(5 \mathrm{M}+\mathrm{NSP}+4 \mathrm{~N}+\mathrm{NNZ}+4 \mathrm{NRHS}+\mathrm{NNZRHS})
$$

## where

$M$ is the number of rows in the matrix.
NSP is the number of slack prices.
$\mathrm{N} \quad$ is the number of columns in the matrix.
NNZ is the number of nonzero elements in columns.

NRHS is the number of right-hand-sides.
NNZRHS is the number of nonzero elements in right-hand-side(s).

The number of words of disc storage required by the INVERSE file is specified by the following equation

$$
4.5(M * 1.25 \text { ANNZ })
$$

where
$M \quad$ is the number of rows in the matrix.
ANNZ is the average number of nonzero elements in a matrix column.

The number of words of disc storage required by files UTILI and UTIL2 is the same as for the MATRIX file.

These estimates for disc storage may vary during certain procedures. For example, during REVISE, the storage requirement for the INVERSE file is generally twice that of the MATRIX file.

For large problems, it may not be possible to assign all files to disc storage during preliminary phases such as INPUT and REVISE. Since it is desirable to have the files on disc during the iterating procedures (OPTIMIZE, INVERT, etc.), it is suggested that the user assign all files to magnetic tape during the INPUT/REVISE phase. Following this, he may call the CONDITION and SAVE procedures.

The CONDITION output will list the current storage requirements (in words) for each file and the maximum storage required to date. The current size of the MATRIX file can
be used for its disc storage requirements as well as for UTIL and UTIL2. The current storage requirements stated for the INVERSE file cannot be used for disc estimating since the iterating procedures have not yet been used.

For maximum efficiency, the following priority should be given in assigning files to disc for the iterating procedures.

| Priority | Procedure |
| :---: | :---: |
| 1 | INVERSE |
| 2 | MATRIX |
| 3 | UTILI |
| 4 | UTIL2 |

## COMMUNICATION FILES

Communication files are the means of communication between FMPS and user-written programs. FMPS input procedures accept data from a standard card reading device or, optionally, from communication files. FMPS output procedures retrieve data from internal files and prepare printed reports. Optionally, the data may be written on a communication file.

To provide a mutually-convenient form of communication, such files are structured to be read or written with FORTRAN READ or WRITE statements. By using FORTRAN input/ output as the basic means of communication, the user can write his own specific matrix generators and report writers in FORTRAN.

The following table identifies the FMPS procedures that include the option of accepting input from communication files or of writing output on communication files.

Table 8. Procedures Using Communication Files

| Procedure <br> (in LP mode) | Card Format | FORTRAN <br> Format |
| :--- | :---: | :---: |
| LOADLIST | Yes | No |
| INPUT | Yes | Yes |
| OUTPUT | No | Yes |
| REVISE | Yes | No |
| SOLUTION | No | Yes |
| BASISOUT | Yes | No |
| BASISIN | Yes | No |

The following paragraphs describe basic communication file structure and the means by which FORTRAN READ and WRITE statements may be used to access the data.

## CARD FORMAT FILES

All data decks that may be read or written on a CARD file are organized as described in Chapter 5. Each data deck is preceded by a $\bar{N} A \dot{M} M E$ card which identifies the data, and each data deck is terminated with an ENDATA card.

Whenever a procedure requires input data, the input device, whether card reader or CARD file, is searched for a NAME card with an identification field (columns 15 to 22) that matches the current contents of communication region variable ADATA.

Whenever a procedure produces a data deck (that is, BASISOUT), NAME and ENDATA cards are also produced. If the output device is other than the card punch, that is, a CARD file, the card file is positioned to the logical end-of-file and the new data deck is written. The logical end-of-file is assumed to be a NAME card with zzzzzzzz in the identification field.

Procedures such as INPUT, REVISE, and LOADLIST require data input. Whether the input is from cards, card images on magnetic tape, or in FORTRAN unformatted WRITE format, the following conventions apply:

1. The data must be preceded by a name record identifying the data record, and the data must be followed by an ENDATA record.
2. In the control program, the CR variable ADATA must be initialized with the name of the data set to be loaded before the procedure requiring input is called. For example, the following sequence,
```
ADATA = 'MATRIXI'
CALL INPUT
```

causes the card data set with the name MATRIX1 to be loaded by the INPUT procedure.
3. The card data sets must be placed after the END statement of the control program. The card data sets must follow each other in the sequence of input.
4. Input records on magnetic tape can occur in any sequence. FMPS will rewind the input tape, if necessary, to locate the desired set of data if the tape was positioned beyond the record to be loaded.
5. For proper operation, it is necessary that all input files include as the last record a NAME record with the name zzzzzzzz and an ENDATA record. This constitutes the logical end-of-file for FMPS.
6. When writing output on magnetic tape, FMPS automatically supplies the NAME and ENDATA records. The name is copied from the current CR variable ADATA which must be initialized to the desired name by the user before executing the output. If the tape includes data prior to the output operation, the new output data is appended to the current data and a logical end-of-file (NAME zzzzzzzzz and ENDATA ) is added. Decks punched by FMPS also include the NAME and ENDATA records.

7．The INPUT procedure includes the option of reading card decks or magnetic tape reels prepared for other linear programming packages such as LP 90／94， 1108 LP，and CDM4．When reading such data from cards， NAME and ENDATA must precede and follow the input data．When reading from magnetic tape，the NAME and ENDATA records must not be present on the tape．

## FORTRAN FORMAT FILES

A FORTRAN format file consists of a series of unformatted， FORTRAN－written records on tape．Each record contains 60 double precision（DP）words．The structure of each rec－ ord is shown in Figure 1．The first three DP words are used to identify the record．

The first DP word contains the name of the procedure gen－ erating the record or the name of the procedure for which this record is input．N14，the left half of the first DP word，contains the first four characters of the name，and N58，the right half of the first DP word，contains the last four characters of the name．

The second DP word contains the subname of the record． SN14，the left half of the second DP word，contains the first four characters of the subname．SN58，the right half of the second DP word，contains the last four char－ acters of the subname．

The third DP word contains the record number and index of the word last used in the record．RN，the left half of the third DP word，contains the record number． RN is used to signal the end of a series of records．

As an example，if three 60 －word records were required to contain the information， RN in the first record would be -1 ，in the second -2 ，and in the third 3．Therefore，if $R N$ is negative，it indicates that there is more of the same kind of information in the next record．When RN is posi－ tive，it indicates that this is the end of records containing the stated information．ILAST，the right half of the third DP word，contains the index of the last item in the record． ILAST is always less than or equal to 60．The fourth through the sixtieth DP words contain the information in groups of three DP words．

## DATA STORAGE ON RECORDS

Conventions for storage of data on records are outlined below．
1．All names（character strings of eight characters or less） are stored with the first four characters of the name in the left half of a DP word and the last four characters in the right half of the DP word．
2．Floating－point values are stored as double precision floating－point．
3．Integers are stored in the left half（most significant）of a DP word．

As with CARD communication files，all input data must be preceded by a NAME record．In addition，output will be
preceded by a NAME record that contains the contents of CR cell ADATA．The format of a NAME record is shown in Figure 2.

The last record on a communication file will be a NAME record whose name is zzzzzzzz（supplied by the user）．
Each time information is written on a FORTRAN communi－ cation file，the tape is positioned to the zzzzzzzz name record and the zzzzzzzz record is overwritten with a new NAME record containing the contents of CR cell ADATA． The information is then written followed by a new NAME zzzzzzzz record．
Record formats produced by SOLUTION are shown in Figure 3．Record formats for the INPUT procedure are shown in Figure 4.


Figure 1．FORTRAN CommunicationFile Record Structure

| （1） | — DOUBLE PRECISION $\longrightarrow$ |  |
| :---: | :---: | :---: |
|  | NAME | ちちちち |
| （2） | AAAA | AAAA |
| （3） | 1 | 3 |
| （4） | Rema | nder |
|  |  |  |
| （60） | record | nused |

Figure 2．Format of a NAME Record


Figure 3. Record Formats Produced by SOLUTION


| Sprices |  |  | Name of Slack <br> Name of Cost Row <br> Slack Price |
| :---: | :---: | :---: | :---: |
| 1 | INPU | T666 |  |
| 2 | SPRI | CES |  |
| 3 |  |  |  |
| 4 | SLKN | AME 1 |  |
| 5 | COST | ROW |  |
| 6 |  |  |  |
| 7 | ALKN | AME2 |  |
| 8 | LOST | ROW6 |  |
| 9 |  |  |  |
| : |  |  |  |
| 58 | SLKN | AMEM |  |
| 59 | COST | ROW5 |  |
| 60 |  |  |  |


Ranges

Range Column Name
ounds

Endata


Figure 4. Record Formats for INPUT

## 3. FMPS CONTROL LANGUAGE STATEMENTS

## INTRODUCTION

An FMPS run always includes a set of cards that specify the operations to be executed. These cards are grouped together in a control program. Rather than using fixed-formatcontrol cards, FMPS uses control statements that are compiled by FMPS at the beginning of the run.

## STATEMENT TYPES

The control language for FMPS was designed to be a subset of the FORTRAN language. There are five basic types of statements:

1. The procedural CALL statement, which loads and transfers control to one of the FMPS procedures. This type of statement is analogous to a FORTRAN subroutine call.
2. Arithmetic statements, which evaluate simple arithmetic expressions.
3. Program flow control statements, such as ASSIGN, GO TO, EXIT, RETURN, and IF, which transfer control to a statement other than the next one in sequence.
4. The WRITE statement, which displays any user or common-storage variable on the standard output device. The TITLE statement provides a heading for each page of output.
5. Delimiting statements, which indicate the end of the control program. The END statement is a message to the compiler that there are no more statements to be processed. It is not executable. The STOP statement is executable and indicates that execution of the control program is to terminate.

## CARD FORMAT

The card format for the FMPS control language is identical to that of FORTRAN.

Column 1 is used to indicate a comment card. A $C$ punched in column 1 indicates that the rest of the card is a comment, and is not processed. The comment card will appear on the listing produced by the compiler. Comment cards may be used freely to give information or improve readability.
Any statement, other than an END statement, may be given a statement (step) number. A step number is any unsigned integer between 1 and 9999. It may be placed anywhere in columns 2-5 of the card.

Column 6 is reserved to indicate a continuation card. As many continuation cards as are needed may be used, but
they can only be used to continue the parameter list of a procedure CALL statement. They may not be used with any other kind of statement. Any nonblank character punched in column 6 will indicate that the card is a continuation of the parameter list from the previous card. A statement may begin in column 7 or anywhere thereafter.

Columns 73-80 are ignored, and may be used for sequence numbers if the user wishes. A summary of card format is shown below.
$[C][$ step $]\left\{\begin{array}{l}\text { nonblank } \\ \text { blank }\end{array}\right\}$ statement $\quad$ [sequence numbers]

## CONTROL LANGUAGE STATEMENTS

CALL The procedure CALL statement causes the specified procedure to be loaded into memory, control to be transferred to the procedure, and the set of parameters specified in the argument list to be communicated to it. The procedure CALL statement has the form

CALL pname [(parameter 1, parameter 2, . . .)]
where
pname is the name of the FMPS procedure to be executed.
parameter 1, ... represents the values to be transmitted to the procedure. Parameters may be constants, variables (either CR variables or user working-storage variables), or keywords. Some procedures have no parameters associated with them. The parameters are always enclosed by parentheses and separated by commas.
Correct and incorrect procedure CALL statements are shown below.

## VALID PROCEDURE CALL STATEMENTS

CALL OPTIMIZE
CALL ENTER (LP)
CALL ATTACH ('FILEI', 'F:F1')
Note that the CALL ATTACH procedure above could be written as

```
    AWD4 \(=\) 'FILEI'
    AWOI = 'F:FI'
    CALL ATTACH (AWD4, AW01)
INVALID PROCEDURE CALL STATEMENTS
    ENTER(LP) CALL must be specified
    CALL ENTER 'LP' Missing parentheses
```

CALL ATTACH ('PROBFILE' 'FILETAPE') Parameters not separated by commas

The parameter list of a procedural CALL may make use of a continuation card as in

```
    CALL ATTACH ('PROBFILE',
X'FILETAPE', CARD, NEW)
```

Note that a field must not be broken in the middle, and that the preceding card must end with a comma.

The examples shown below illustrate improper continuation cards for procedure CALL statements.

## INVALID CONTINUATION CARDS FOR PROCEDURE CALL STATEMENT

| CALL ATTACH | At least one parameter |
| :---: | :--- |
| X('PROBFILE', 'FILETAPE') | must be on first card |
| CALL ATTACH ('PROBFILE' | Preceding card must |
| X,'FILETAPE', CARD NEW) | end with a comma |

## ARITHMETIC

The Arithmetic statement is used to initialize or set all storage-reference variables (CR or user working-storage) except interrupt (K-type) variables. The Arithmetic statement has the form

where
srsym is either a CR or user working-storage variable.
arithex is an arithmetic expression of the form

> variable

$$
\text { constant } \left.\begin{array}{c}
+ \\
\text { variable } \\
- \\
* \\
/
\end{array}\right\} \text { constant }
$$

variable $\left\{\begin{array}{c}+ \\ - \\ * \\ /\end{array}\right\}$ variable
and in which variable refers to either a CR or a user working-storage variable.

Mixed mode is allowed between integer and floating-point computations, but all alpha computations must not mix modes. An arithmetic expression that contains a
floating-point number will be done in double precision floating-point arithmetic.

Compare the following tables of valid and invalid Arithmetic statements.

## VALID ARITHMETIC STATEMENTS

```
ARHS = 'ALOY''
FWO1 = FWO1 + 1
IWNM = 79.0
FW01 = FW01 * IWNM
ILOGP = IWNM/79
```

INVALID ARITHMETIC STATEMENTS
KWO1 = $100 \quad$ K-type cells cannot be defined with an Arithmetic statement

ARHS $=$ FWOI Mixed mode not allowed with alpha type

IWNM $=$ FWOI * [WOI +4 Invalid form of arithmetic expression

ASSIGN The ASSIGN statement is used to initalize or set an interrupt (K-type) variable. It has the form ASSIGN stmtno TO $k x x x$
where
stmtno is any valid statement number (1-9999) appearing in the control language program.
$k x x x \quad$ is a $K$-type $C R$ or user working-storage variable.

The following two statements are correct uses of ASSIGN.

VALID ASSIGN STATEMENTS
ASSIGN 100 TO KMAJER ASSIGN 20 TO KWOI

This list shows incorrect uses of the ASSIGN statement.

INVALID ASSIGN STATEMENTS

| ASSIGN SEVEN TO KWD1 | Statement number must <br> be an integer constant |
| :--- | :--- |
| ASSIGN 100 TO IW01 | Assignment must be <br> made to a K-type vari- <br> able only |

GO TO The GO TO statement causes the unconditional transfer of control to the statement specified by the
statement number after GO TO. The GO TO statement has the form

```
GO TO{{{\begin{array}{l}{\mathrm{ stmtno}}\\{kxxx}\end{array}}
```

where
stmino is any valid statement number (1-9999) appearing in the control language program.
$k x x x \quad$ is a $K$-type user working-storage variable that has been defined by an ASSIGN statement.

The two lists below present correct and incorrect uses of GO TO.

VALID GO TO STATEMENTS
GO TO 100
GO TO KWOI
INVALID GO TO STATEMENTS
GO TO A A is not a K-type user workingstorage variable
GO TO KMAJER KMAJER is a K-type CR variable, not a user working-storage variable

HF The IF statement makes a conditional transfer of control to the statement specified by a statement number. It may be used in the construction of loops. If has the form
IF (srsym .op. $\left\{\begin{array}{l}\text { srsym } \\ \text { constant }\end{array}\right\}$ ) GO TO stmtno
where
srsym is either a CR or user working-storage variable.
constant is a valid constant.
op enclosed by periods, is a two-letter code that represents one of the following conditions.

| $\frac{\text { Code }}{}$ | Condition |
| :--- | :--- |
| GT | Greater than |
| GE | Greater than or equal |
| LT | Less than |
| LE | Equal |
| EQ | Not equal |
| NE |  |

stmtno is any valid statement number (1-9999) appearing in the control language program.

When IF is executed, the expression within the parentheses is evaluated first. If it is true, control is transferred to the specified statement number. If it is not true, control is passed to the next statement in the program sequence.

Mixed mode is allowed if integer and floating-point quantities are involved. Mixed mode is not allowed if an alpha quantity is used.

The sample IF statements below are correct.

## VALID IF STATEMENTS

IF (FOBJWT . GT. IW4I)GO TO 30
IF (ARHS .EQ. 'ROWS') GO TO 150
These IF statements are incorrect.
INVALID IF STATEMENTS
IF (ARHS . EQ. FWOI) GO TO 20 Mixed mode is not allowed if alpha quantity involved

IF (IWOI LT 7) GO TO 10 LT must be enclosed in periods

IF (FW75) 10, 20, 30 This form of IF statement is not allowed in this control language

RETURN The RETURN statement is used to return control to a procedure that has created an interrupt. When an interrupt occurs, control will be given to the statement whose number has been assigned to the corresponding $C R$ interrupt (K-type) variable for that particular condition. After the number, it may be desired to return to the procedure that caused the interrupt. The RETURN statement has the form

RETURN

An example of interrupt processing using a RETURN statement is shown below.
ASSIGN 150 TO KINV
IFREQI $=50$
CALL OPTIMIZE
=
150 CALL INVERT
RETURN

Note that OPTIMIZE will interrupt for an INVERT every 50 iterations. Control will be transferred to statement 150 which is a CALL for INVERT, and following the INVERT, control will be transferred to OPTIMIZE via RETURN.

EXIT The EXIT statement is a special type of statement used in the FMPS control language. Like the RETURN statement, the EXIT statement is concerned with interrupt processing. After receiving an interrupt, it may not be desirable to return to the procedure causing the interrupt. The EXIT statement may be used to exit the procedure and to continue processing with the statement following the
procedure CALL statement that triggered the interrupt. EXIT has the form


An example of interrupt processing using an EXIT statement is given below.

```
ASSIGN 200 TO KNFS
CALL OPTIMIZE
200 CALL OUTPUT (BYROWS, ROWS, LISTI)
    EXIT
```

Note that if no feasible solution condition is encountered by OPTIMIZE, control is transferred to statement 200 to output the infeasible rows, and the following EXIT statement will cause control to be transferred to the statement after CALL OPTIMIZE.

WRITE The WRITE statement (not to be confused with the standard FORTRAN WRITE statement) may be used to display the current value of any $C R$ or user working-storage variable on the system output device. The variable name and its value are printed. The WRITE statement has the form
WRITE srsym
where
srsym is either a CR or user working-storage ref-
erence symbol.

Notice that only one symbol may be referenced on a WRITE statement.

Some uses of WRITE are shown below.

$$
\begin{array}{ll}
\text { AW01 = 'EXAMPLE' } & \text { Printout will contain } \\
\text { WRITE AW01 } & \text { AW01 = EXAMPLE } \\
\text { FW07 }=.2365 \text { D3 } & \text { Printout will contain } \\
\text { WRITE FW07 } & \text { FW07 }=236.5
\end{array}
$$

TITLE This statement, which is a special FMPS control language statement, provides a page heading on each page of the output produced by execution of the control program. The TITLE statement has the form

TITLE heading
where
heading is a string of literal alphanumeric characters that terminate by column 72.

The title is printed out as shown below
title this is the title.

STOP The STOP statement terminates execution of the control program. The STOP statement has the form


END The END statement is a nonexecutable statement that defines the end of a source program for the compiler and must be the last statement of every program. Since the END statement is not executable, it should have a statement number. END has the form


## SAMPLE FMPS PROGRAM

Figure 5 shows an example of a typical FMPS control language program.

```
C DEFINE PAGE TITLE
    TITLE FMPS CONTROL LANGUAGE EXAMPLE
C ENTER LINEAR PROGRAMMING OPERATING MODE
    CALL ENTER(LP)
C INITIALIZE MAJOR AND MINOR ERROR INTERRUPTS
    ASSIGN 1000 TO KMAJER
    ASSIGN 1010 TO KMINER
    CALL DEVICE('DISCl',DISC,'B')
    CALL DEVICE('DISC2',DISC,'C')
    CALL DEVICE('DISC3',DISC,'D')
    CALL DEVICE('DISC4',DISC,'E')
C
C ATTACH INTERNAL FILES MATRIX, INVERSE,UTILI, UTIL2 TO THE SYMBOLIC
C DISC UNITS DISC1, DISC2, DISC3, DISC4
    CALL ATTACH(MATRIX, 'DISC1')
    CALL ATTACH(INVERSE, 'DISC2')
    CALL ATTACH(UTILI,'DISC3')
    CALL ATTACH(UTIL2,'DISC4)
C
C DEFINE NAME OF INPUT DATA DECK
    ADATA = 'PLANTI'
C INPUT THE LP MATRIX
    CALL INPUT
C DEFINE NAME OF RHS AND OBJECTIVE FUNCTION ROW
    ARHS = 'RHS1'
    AOBJ = 'COSTROW'
C OUTPUT BYROWS, THE NON-ZERO ELEMENTS OF INPUT MATRIX
    CALL OUTPUT(BYROWS)
C
C INITIALIZE OPTIMIZE INTERRUPTS KINV,KNFS,KUBS
    ASSIGN 2000 TO KINV
    ASSIGN 2100 TO KNFS
    ASSIGN 2200 TO KUBS
C SET INVERSION FREQUENCY TO 100
    IFREQI = 100
C OPTIMIZE INPUT MATRIX
    CALL OPTIMIZE
C OUTPUT THE OPTIMAL SOLUTION
    CALL SOLUTION
C TERMINATE RUN
    STOP
C
C PROCESS MAJOR ERROR INTERRUPT BY TERMINATING RUN
    1000 STOP
C
C PROCESS MINOR ERROR INTERRUPT BY EXITING PROCEDURE CAUSING IT
    1010 EXIT
C
C
    PROCESS INVERT INTERRUPT BY CALLING INVERT AND RETURNING TO
    L INVERT
    RETURN
C
C PROCESS NO FEASIBLE SOLUTION INTERRUPT BY OUTPUTING THE INFEASIBLE
C ROWS, PUNCHING THE CURRENT BASIS STRUCTURE, AND TERMINATING RUN
    2100 CALL OUTPUT(BYROWS,ROWS,LISTI)
    2110 CALL BASISOUT
    STOP
C
C PROCESS UNBOUNDED SOLUTION INTERRUPT BY OUTPUTING THE UNBOUNDED
C COLUMN, PUNCHING THE CURRENT BASIS, AND TERMINATING RUN.
    2200 CALL OUTPUT(BYCOLS,COLS,LISTU)
    GO TO 2110
C END OF CONTROL PROGRAM
    END
```

Figure 5. Sample FMPS Control Language Program

## 4. BASIC FMPS PROCEDURES

This chapter describes those FMPS procedures that are available under all FMPS operating modes. These operating procedures perform the following functions.

- Establish the operating mode.
- Define input/output devices.
- Assign files to input/output devices.
- Define selection lists.

FMPS operating procedures and their functions are given in Table 9 below.

Table 9. FMPS Operating Procedures

| Procedure | Purpose |
| :--- | :--- |
| ENTER | Establish the operating <br> mode. <br> DEVICE <br> Defines storage media for <br> run. |
| ATTACH | Attaches symbolic files to <br> DCBs. |
| LOADLIST | Inputs names and/or masks to <br> be used as a selection list. |

## OPERATING PROCEDURES REPERTOIRE

Each of the procedures outlined in Table 9 above will be explained in detail in the following paragraphs.

ENTER The ENTER procedure establishes the operating mode for FMPS. Therefore, it must be the first procedure used. The mode may not be changed during a run. The following list contains codes for parameters currently available for ENTER. One of the following parameters must be specified.

| Parameters | Explanations <br> LP |
| :--- | :--- |
| programming operating mode. |  |
| SEP | FMPS establishes the separable <br> programming operating mode. |

The following interrupt may occur through misuse of ENTER.
$\frac{\text { Interrupt }}{\text { KMAJER }}$

Causes

1. Unrecognizable parameter.
2. Operating mode already establ ished.

DEVICE The DEVICE procedure defines magnetic tapes and RAD files to be used as storage media during the FMPS run. This procedure contains parameters informing FMPS
of the data control block (DCB) to be used with the file or tape and the organization of the file (consecutive-sequential or keyed direct-access). This data is given to BPM via the !ASSIGN control command; the DEVICE procedure passes it to FMPS.

Symbolic units must be defined by a call for DEVICE before FMPS files can be attached to them. A symbolic unit may be defined only once during a run.

## DATA CONTROL BLOCKS

The data control blocks for use with FMPS are included in the system at installation. Nominally the system is built to the maximum of 10 DCBs whose names are F:1,F:2, . . , $\mathrm{F}: 10$. Thus, !ASSIGN cards for a run are restricted to these DCBs. In addition, the F:1 DCB is preempted by FMPS in the storage of the control language programs. However, any of the remaining DCBs may be assigned to either tape or RAD. RAD DCBs may be organized sequentially or as direct-access. The internal FMPS file INVERSE should always be a RAD file and as such must be a keyed direct-access file. Note that the !ASSIGN control command designates the physical location (RAD or tape) of the data transmitted via a DCB.

## DEVICE ARGUMENT

The DEVICE procedure requires three arguments, as in
CALL DEVICE ('symbolic unit' $\left\{\begin{array}{l}\text { TAPE } \\ \text { DISC' }\end{array}\right\}$ ' $\stackrel{D}{\mathrm{D} C B}$ key')
where
'symbolic unit' specifies the symbolic unit defined by DEVICE to which internal and communication files may be attached.

TAPE indicates that the file or tape was specified as consecutive-sequential on the !ASSIGN card.

DISC Indicates that the file was specified as keyed direct-access on the ! ASSIGN card.
'DCB key' is one of the following codes that specify the DCB name to be used.

| Code | DCB Name |
| :---: | :---: |
| 'B' | DCB F:2 |
| ${ }^{\prime} \mathrm{C}$ | DCB F:3 |
| : | : |
| 'J' | DCB F:10 |

For example, the procedural call CALL DEVICE ('INVS', ' $C$ ') would define symbolic unit 'INVS' to be a RAD file with keyed direct-access organization, to be driven via the F:3 DCB.

ATTACH The ATTACH procedure attaches symbolic files to DCBs. There are two classes of files that must be attached. The first class consists of files reserved for internal use by FMPS. All internal files have preempted names recognizable as keywords such as MATRIX, INVERSE, efc. (refer to Table 7). The second class of files consists of files used for communications between the user and FMPS. The user assigns symbolic names (eight or less characters enclosed by quotation marks) to communication files.

When attaching FMPS internal files to DCBs, ATTACH requires the use of two parameters. For example,

## CALL ATTACH (INVERSE,'SYMBI')

assigns internal file INVERSE to the symbolic unit 'SYMBI'.
When attaching communication files to symbolic units ATTACH requires the use of four parameters. The third parameter (which is not required for internal FMPS files) describes the mode of the file. The mode may be specified as CARD, implying 80 -column card image format, or FORTRAN, implying standard communication format. The fourth parameter, OLD or NEW, specifies whether the tape has previously been prepared by a program (or FMPS) and contains information to be preserved (OLD), or whether the tape is a new tape without information to be saved on it (NEW). If the NEW parameter is specified, FMPS writes a pseudo end-of-file record at the beginning of the tape (NAME zzzzzzzz, ENDATA). If it is an outputfile, it is defined as NEW. It is imperative that, if a communication file (whether CARD or FORTRAN) is defined, NEW or OLD follow the file definition.

Symbolic files may be reattached to different DCBs during a run. If the INVERSE file is reattached, an INVERT call must be made following the latest ATTACH. A common use of the reattach facility is in connection with the RESTART file. For example

CALL ATTACH (RESTART, = 'TAPEI')
CALL RESTORE
CALL ATTACH (RESTART, 'TAPE2')
$\vdots$
CALL SAVE
Also, the statement
CALL ATTACH('OUTFILE', 'COMMTAPE', FORTRAN, NEW)
assigns communication file 'OUTFILE' to DCB 'COMMTAPE' in standard communication format.

The following interrupt may occur within ATTACH.

$$
\begin{array}{ll}
\frac{\text { Interrupt }}{\text { KMAJER }} & \frac{\text { Causes }}{\text { 1. Symbolic unit not defined, }} \\
& \begin{array}{l}
\text { 2. Internal FMPS file assigned as } \\
\text { communication file. }
\end{array}
\end{array}
$$

3. Unrecognizable parameter.
4. Internal random-access file assigned to sequential-access device.
5. Communication file not specified as OLD or NEW.

LOADLIST The LOADLIST procedure is responsible for the input of a list of names and/or masks from cards or communication files to be used as a selection list during output of procedures such as SOLUTION, OUTPUT, etc.

The first parameter of the procedure defines which of two lists, LISTR or LISTC, is to be loaded. LISTR is the list used to contain the names and/or masks for row selection or exception. LISTC is the list used to contain the names and/or masks for column selection or exception.

The names in a list correspond to the name of a row or column in the matrix. Masks are used to represent classes of rows or columns that have unique character configurations in their names. A mask is composed of eight characters. The characters in the mask are matched, position by position, with a row or column name. If all positions match, then that row or column name is considered part of the selection list. If one or more characters within the mask are an asterisk(*), that position(s) will match with the corresponding position(s) of any row or column name. For example,

```
CRUDE***
```

is a mask that considers any row or column name having CRUDE as its first five characters as part of the selection list.

Input to LOADLIST is from card images on the standard card reading device unless the FILE parameter is specified, in which case the third parameter must be the name of the file on which the data resides. The data format for the LOADLIST procedure is described in Chapter 5.
The communication region variable ADATA must be initialized before the call for LOADLIST. It contains the name of the data deck for data reading procedures such as INPUT, REVISE, etc. ADATA is also used by data outputting procedures, such as BASISOUT, to name output data deck. It specifies the name that appears on the NAME card of image input. (Refer to Chapter 5 for general data formats.)
The parameters available to LOADLIST are:

Parameter
LISTR

LISTC

FILE Specifies that data is on file 'filename' (card format only).

| Parameter | $\frac{\text { Explanation }}{\text { 'filename' }} \quad$Symbolic name of file, including <br> quotation marks, on which data <br> resides. |
| :--- | :--- |

Interrupt quotation marks, on which data resides.

The FILE and 'filename' parameters are optional.
The following interrupts may occur within LOADLIST.

| $\frac{\text { Interrupt }}{\text { KMAJER }}$ | $\frac{\text { Causes }}{\text { 1. Unrecognizable parameter. }}$ |
| :--- | :--- |
|  | 2. Undefined 'filename'. |

## Causes

3. NAMES or MASKS data not grouped together.
4. Unrecognizable data indicator.

Irrecoverable input/output error on file.

Core memory area exceeded by list. Remainder of data cards ignored.

## 5. DATA CARD FORMATS AND DECK ORGANIZATION

This chapter describes data card formats and data deck organization applicable for the various procedures (INPUT, REVISE, BASISIN/BASISOUT, and LOADLIST) in all FMPS operating modes. It also describes acceptable nonstandard data formats.

## STANDARD CARD AND DECK FORMATS FOR INPUT

The data file for the INPUT procedure contains four types of cards in all cases.

1. NAME card
2. Indicator cards
3. Data cards
4. ENDATA card

Comment cards, identified by an asterisk (*) in column 1, may be inserted anywhere in a data deck.

## NAME CARD

The first card of a data deck is always a NAME card. The NAME card gives a user-specified name to the data decks so that the data may be uniquely identified from the control program. NAME has the following format.

| Columns |  | Description |
| :--- | :--- | :--- |
| 1-4 |  | NAME: card identification. |
| $5-14$ |  | Blank |
| $15-22$ | User-assigned name: from one to eight <br> characters in length. |  |
| $23-80$ | Blank |  |

## INDICATOR CARDS

The INPUT data deck consists of data cards grouped according to the type of data they contain. A group of cards containing the same type of data is called a chapter. The first card of a chapter is always an indicator card, which identifies the type of data in that chapter. The optional and required types of data appearing in a data deck for the INPUT procedure are:

| Data Type |  | Status |
| :--- | :--- | :--- |
|  |  |  |
| ROWS |  | Required |
| SPRICES |  | Optional |
| COLUMNS | Required |  |
| RHS |  | Required |
| RANGES |  | Optional |
| BOUNDS |  | Optional |

The format of indicator cards is given below.

| Columns | Description <br> $1-7$ <br> Data type: one of the six types shown <br> above. |
| :--- | :--- |
| $8-80$ | Blank |

## DATA CARDS

Data cards are divided into six fields. The type of data card determines the content of each field, but all data cards follow the same general format. The six fields of a data card are outlined below.

| Columns | Description <br> Blank or *. If asterisk is present, it indi- <br> cates that this is a comment card, which <br> may be inserted anywhere in the data <br> deck. |
| :--- | :--- |
| $2-3$ | Field 1: code for type of row constraint <br> or type of bound (see ROWS and <br> BOUNDS cards). |
| $50-12$ | Field 2: name of from one to eight alpha- <br> numeric and special characters. |
| Field 3: same as field 2 above. |  |

## ENDATA CARD

The ENDATA card, which simply indicates that the end of the data deck has been reached, has the following format:

| Columns | Description <br> $1-6$ |
| :--- | :--- |
|  | ENDATA |
| $7-80$ | Blank |
|  | DATA DECK ORGANIZATION |

Figure 6 shows the organization of a complete INPUT data deck. Note that the dashed lines indicate optional cards and decks.


Figure 6. Data Deck Organization for INPUT

## ROWS DATA CARDS

ROWS cards specify the name to be assigned to the rows of the matrix, as well as the type of constraint (equality or inequality) represented by the row. The ROWS data card format is shown below.

| Columns | Description |  |
| :---: | :---: | :---: |
| 2-3 | Field 1: type of constraint as specified by the following codes: |  |
|  | Code | Meaning |
|  | 6 N or N 6 | No constraint (change or objective row) |
|  | 6 G or $\mathrm{Gb}^{\text {b }}$ | Greater than or equal to |
|  | 6 L or Lb | Less than or equal to |
|  | 6E or Eb | Equality |

Columns Description

5-12 Field 2: name of the row, where blanks are considered part of the name.

Field 3: blank
Field 4: blank
Field 5: blank
Field 6: blank

## SPRICES DATA CARDS

SPRICES (slack prices) cards specify the price or prices to be associated with the slack vector of a row. The slack prices must be specified by slack: that is, when one price is given for a slack, any other prices for the same slack must be entered before the next slack is referenced. The slack prices must be entered in the same order as the slack name appears in the rows section. The SPRICES data card format is shown below.

| Columns | Description <br> $2-3$ |
| :--- | :--- |
| Field 1: blank |  |
| Field 2: name of the slack vector, which <br> is identical to the name of the row with <br> which it is associated. |  |
| $25-22$ | Field 3: name of the cost row to which <br> the price is associated. |
| $40-47$ | Field 4: value of the slack price. |
| $50-61$ | Field 6: optional and used like field 4. |

## COLUMNS DATA CARDS

COLUMNS cards specify the names to be assigned to the columns (structural variables) in the LP matrix and define the actual values of the matrix elements in terms of column vectors. The matrix elements must be specified by column; that is, when one element is given, all other nonzero elements in that column must also be entered before another column is mentioned. Zero entries should not be specified, since they will be filled in automatically by the system. The COLUMNS data card format is shown below.

## Columns Description

2-3 Field 1: blank
5-12
Field 2: name of the column that is to contain the elements specified in the field that follow.

RHS CARDS
RHS cards specify the names of the right-hand-side constraint vectors or change vectors (used in parametric programming). They define, in terms of column vectors, the values of these elements. The right-hand-side elements must be specified by RHS; that is, when one element is given, all other nonzero elements in that RHS must also be entered before another RHS is mentioned. The RHS data card format is shown below.

| Columns | Description |
| :---: | :---: |
| 2-3 | Field 1: blank |
| 5-12 | Field 2: name of the right-hand-side (RHS) vectors or change vectors. |
| 15-22 | Field 3: name of the row in which an element is to be entered. |
| 25-36 | Field 4: value of the element to be entered in the row and in the RHS of field 2. |
| 40-47 | Field 5: optional and used like field 3. |
| 50-61 | Field 6: optional and used like field 4. |

## RANGES DATA CARDS

Range constraints are used when a row is to represent both a greater-than inequality and a less-than-or-equal-to inequality. When none of the rows have such double limits, range constraints are not used.
One of these limits is given in the normal manner when both upper and lower limits are desired. The type of row constraint is specified in the ROW data, and one limit (upper or lower) is specified in the RHS data. The other limit specified in this section of the data is the allowable magnitude by which the right-hand-side may vary from the value previously specified.

If $b_{i}$ is the value given in the RHS section, the range $r_{i}$ is specified as follows:

| Type <br> of Row | Resultant Upper <br> Limit on Right- <br> Hand-Side | Resultant Lower <br> Limit on Right- <br> Hand-Side |
| :---: | :---: | :---: |
| $G$ | $b_{i}+r_{i}$ | $b_{i}$ |
| $L$ | $b_{i}$ | $b_{i}-r_{i}$ |

The set of ranges is defined as a column vector with a name specified by the user. Only one vector of ranges will be loaded by the INPUT procedure. If more than one is present, the additional vectors will be punched in REVISE format.

The RANGES data card format is shown below.

| Columns |  |
| :--- | :--- |
| Description |  |
| 2-3 |  |
| Field 1: blank |  |

## BOUNDS DATA CARDS

BOUNDS data cards impose limits on the values which the activities, or "structural variables", may assume. If none of the variables are bounded, this section of input is not needed.

When bounds are desired, they are entered as a row vector with a name specified by the user. Bounds are automatically set at 0 and $+\infty$ for all columns not specified in a BOUNDS card. Only one vector of bounds will be loaded by the INPUT procedure. However, if more than one is present, the additional vectors will be punched in REVISE format.

Within a given bounds row vector, the column (structural variable) names must appear in matrix order (that is, the same order in which column names appear in the COLUMNS section).

The user may specify both an upper and a lower bound, a lower bound only, or an upper bound only. When a single bound is specified, the other bound will remain as $+\infty$ or 0 . When both upper and lower bounds on a single variable are desired, they must be entered on separate cards. Possible combinations are:

$$
\begin{aligned}
& L O-U P \\
& L O-P L
\end{aligned}
$$

Since an upper bound of $+\infty$ is automatically generated, PL cards are ignored by INPUT.

To fix a variable at zero, the code FX with a value of zero must be used.

Lower bound values may be positive or negative; upper bound values must be positive.

The BOUNDS data card format is shown below.

| Columns | Description |
| :---: | :---: |
| 2-3 | Field 1: type of bound as specified by the following codes: |
|  | Code Meaning |
|  | LO Lower bound |
|  | UP Upper bound |
|  | FX Fixed value |
|  | FR Free variable ( $-\infty$ to $+\infty$ ) |
|  | PL Upper bound is $+\infty$ |
| 5-12 | Field 2: name of the row of bounds. |
| 15-22 | Field 3: name of the column with which the variable to be bounded is associated. |
| 25-36 | Field 4: value of the bound for an LO, UP, or FX card; otherwise blank. |
| 40-47 | Field 5: blank. |
| 50-61 | Field 6: blank. |

## NONSTANDARD CARD FORMATS FOR INPUT

Three nonstandard input formats are acceptable to the INPUT procedure when the parameter SHARE is used. They are:

1. LP/90/94 LP
2. UNIVAC 1108 LP
3. CDM4 LP

LP/90/94 SHARE FORMAT
The INPUT format when using LP/90/94 LP is
CALL INPUT (SHARE, 'LP90')
where the LP90 parameter must be enclosed by single quotation marks.

## LP/90/94 CHAPTERS

The following chapters of input information will be processed when using LP/90/94.

| ROW ID | FIRST B |
| :--- | :--- |
| BASIS | NEXT B,kkkk |
| MATRIX | EOF |

## RHS NAMES

FMPS assigns the RHS name from the contents of columns 7 to 12 of the data cards for the FIRST B or NEXT B chapter. If these columns are blank for the FIRST B chapter data cards, the name *Bl弓łb (where 6 represents a blank) will be assigned to this RHS. If columns 7 to 12 are blank for the NEXT B chapter data cards, the RHS vectors will be named *Bkkkk, where kkkk are characters copied from the NEXT $B, k k k k$ header card.

## BASIS DATA CHAPTER

When the BASIS chapter header is encountered by the INPUT procedure, its data is punched on cards in a format acceptable to the BASISIN routine. No further processing of BASIS data occurs, but the punched cards can be loaded as a part of the FMPS input to a subsequent run. The BASIS data chapter can appear in any order relative to the other chapter headings in the input stream.

ORDER OF INPUT
The following data chapters are directly processed upon input and must appear in the order listed.

| Data Type | Status |
| :---: | :---: |
| 1. ROW ID | Required |
| 2. MATRIX | Required |
| 3. FIRST B | Required |
| 4. NEXT B,kkkk | Optional |

## CARD FORMAT

ROW ID. The first card of the ROW ID chapter is a ROW ID indicator card. The card format is shown below.

Columns Description
ROW 6 ID: where the characters $ね$ represent a blank. This parameter is present on the first ROW ID card only; columns 1 to 6 are blank on all other ROW ID cards.

Row type: where the type is specified by one of the following codes.

| Code | Row Type |
| :--- | :--- |
| + | Less than or equal to |
| - | Greater than or equal to <br> 0 |
| 6 Indicates a Free Row (for |  |
| example, Cost Row) |  |


| Columns | Description |
| :---: | :---: |
| 13-18 | Row name. |
| 24 | Row type. |
| 25-30 | Row name. |
| 36 | Row type. |
| 37-42 | Row name. |
| 48 | Row type. |
| 49-54 | Row name. |
| 60 | Row type. |
| 61-66 | Row name. |

MATRIX. The first card of the MATRIX chapter is a MATRIX indicator card. The MATRIX data is entered column by column (all coefficients pertinent to one column must be grouped together) as shown in the format outline below. Note that only one coefficient can be defined per data card.

| Columns | Description |
| :---: | :---: |
| 1-6 | MATRIX. This parameter is present on the first MATRIX card only; columns 1 to 6 are blank on all other MATRIX cards. |
| 7-12 | Column name. |
| 13-18 | Row name. |
| 19-30 | Coefficient value; assumed format is F12.6. |

FIRST B. The first card of the FIRST B chapter is a FIRST B indicator card. This card has FIRSTBB punched in columns 1 to 7. The data format is identical to that for MATRIX. If columns 7 to 12 are blank on the data cards, the column (right-hand-side) will automatically be named *Blbங6.

NEXT B,kkkk. The first card of the NEXT B,kkkk chapter is a NEXT B,kkkk indicator card. This card has NEXT B,kkkk punched in columns 1 to 11 . The data format is identical to that for MATRIX; if columns 7 to 12 are blank on the data cards, the column (right-hand-side) is automatically named *Bkkki, where the characters kkkk are copied from the indicator card.
BASIS. The first card of the BASIS chapter is a BASIS indicator card. BASIS data cards contain up to five pairs of names, as shown below.

| Columns | Description <br> BASIS. This parameter is present on the <br> first BASIS card only; columns i to 5 are <br> blank on all other BASIS cards. |
| :--- | :--- |
| $7-12 \quad$ | Variable to enter the basis. |


| Columns | Description |
| :--- | :--- |
| $13-18$ | Variable to be excluded from the basis. |
| $19-24$ | Variable to enter the basis. |
| $25-30$ | Variable to be excluded from the basis. |
| $31-36$ | Variable to enter the basis. |
| $37-42$ | Variable to be excluded from the basis. |
| $43-48$ | Variable to enter the basis. |
| $49-54$ | Variable to be excluded from the basis. |
| 55-60 |  |
| 61-66 | The enter the basis. |
| EOF card has EOF punched in columns 1 to 3. |  |

## UNIVAC 1108 SHARE FORMAT

The INPUT format when using UNIVAC 1108 LP is
CALL INPUT (SHARE, '1108')
where the 1108 parameter must be enclosed by single quotation marks.

## UNIVAC 1108 CHAPTERS

The following chapters of input information will be processed when using UNIVAC 1108.

```
DELETE
ROW ID
BASIS
MATRIX
FIRST B
NEXT B,kkkk
SPRICES
EOF
ENDATA
```

A maximum of 100 column or row names may be input as part of the DELETE data. A minor error interrupt will occur if this number is exceeded, and only the first 100 names will be used.

RHS NAMES

RHS names are formed in the same manner as described for LP/90/94 data above.

## ORDER OF INPUT

The following data chapters are directly processed upon input and must appear in the order listed.

| Data Type | Status |
| :---: | :---: |
| 1. DELETE | Optiona |
| 2. ROW ID | Required |
| 3. MATRIX | Required |
| 4. FIRST B | Required |
| 5. NEXT B,kkkk | Optiona |
| 6. SPRICES | Optiona |

The BASIS chapter is optional and may appear anywhere in the input deck. It is processed in the same manner described for LP/90/94. If the SPRICES chapter is present in the input data and is to be used, the argument 'SPRICES' must be present in the CALL INPUT argument list, as in

> CALL INPUT (SHARE, 'll08', 'SPRICES')
when the input source is the card reader, the SPRICES chapter must be placed directly after the ROW ID chapter in the data deck. When the input source is tape, the SPRICES chapter may appear at the end.

If SPRICES is used, AOBJ must be set (through the control language) to the name of the cost row for which the slack prices apply. This must be done before the call to INPUT.

## CARD FORMAT

DELETE. The first card of the DELETE chapter is a DELETE indicator card. This card has DELETE punched in columns 1 to 6 , and contains up to eleven name fields in columns $7-12,13-18, \ldots, 67-72$. All blank fields are ignored.

ROW ID, MATRIX, FIRST B, NEXT B,kkkk, and BASIS. These data formats are identical to the corresponding data formats for LP/90/94 SHARE.

SPRICES. The first card of the SPRICES chapter is a SPRICES indicator card. This card has the format shown below.

| Columns |  |
| :--- | :--- |
|  | Description <br> SPRICES. This parameter is present on <br> the first SPRICES card only; column 1 <br> 5 are blank on all other SPRICES cards. |
| $7-12$ | Row (slack) name. |
| $19-30$ | Slack price: assumed format is F12.6. |

Pairs for which both fields are blank are ignored. Inclusion of variable names which do not correspond to any variable in the matrix will cause an error comment during subsequent processing of the punched BASIS cards, but will not cause this run to be discontinued.

EOF. The EOF card has EOF punched in columns 1 to 3 .
ENDATA. The ENDATA card has ENDATA punched in columns 1 to 6.

## CDM4 SHARE FORMAT

The INPUT format when using CDM4 LP is

where the CDM4 parameter must be enclosed by single quotation marks.

## CDM4 CHAPTERS

The following chapters of input information will be processed when using CDM4.

```
ROW ID
MATRIX
FIRST B
RHS
BASIS
NEWRHS
SECOND
ENDRHS
EOR
EOF
```


## RHS NAMES

FMPS will introduce a new RHS vector in the input matrix for every redefinition of the RHS vector in the input data. Upon input, the original RHS vector is automatically named *B0001; the first revised RHS vector, *B0002; the second revised vector, *B0003, etc. Any of the vectors can be specified for solution by assigning its name to the ARHS communication cell, for example, $\mathrm{ARHS}={ }^{1 *} \mathrm{BOOO2}{ }^{\prime}$.

## ORDER OF INPUT

The following data chapters are directly processed upon input and must appear in the order listed.

| Data Type | Status |
| :---: | :---: |
| 1. ROW ID | Required |
| 2. EOR | Optional |
| 3. MATRIX | Required |
| 4. EOR | Optional |
| 5. FIRST B OR RHS | Required |


| Data Type | Status |
| :--- | :--- |
| 6. EOR OR ENDRHS | Optional |
| 7. NEWRHS OR SECOND | Optional |
| 8. EOR OR ENDRHS | Optional |
| 9. EOI $=$ ENDATA | Required |

The BASIS chapter is optional and is treated in the same manner as it is in LP/90/94 format.

## CARD FORMAT

All data formats for CDM4 SHARE are identical to those specified for LP/90/94 except ROW ID.

The first card of the ROW ID chapter is the ROW ID indicator card. This card has the format shown below.

| Columns | Description |
| :---: | :---: |
| 1-6 | ROW末 ID: This parameter is present on the first ROW ID card only; columns 1 to 6 are blank on all other ROW ID cards. |
| 12 | Row type: where the type is specified by one of the following codes. |
|  | Code Row Type |
|  | $+\quad$ Less than or equal to |
|  | - Greater than or equal to |
|  | 0 Equal to |
|  | b Indicates a Free Row (for example, Cost Row) |
| 13-18 | Row name. |
| 24 | Row type. |
| 25-30 | Row name. |
| 36 | Row type. |
| 37-42 | Row name. |
| 48 | Row type. |
| 49-54 | Row name. |
| 60 | Row type. |
| 61-66 | Row name. |

Row types and names on ROW ID data cards are interpreted as outlined below.

1. If columns 19 to 24 or columns 12 to 18 , or both, of the data card are blank, the card is ignored.
2. If columns 19 to 24 and columns 12 to 18 of the data card are nonblank, the data is read as follows:

Column 12 Row type.
Columns 13-18 Row name.

## NAME AND ENDATA CARDS

Data may be read from cards or tape. When read from cards, the data must be preceded by a standard NAME card and must end with an ENDATA card. When read from tape, no NAME or ENDATA card is required.

## OUTPUT

The input data may include NAME cards other than the ones mentioned above. FMPS will ignore the NAME card and its associated data. However, a listing of this ignored data is produced on the output medium. It is listed shifted to the right beginning in print position 30.

The chapter headings, but not the associated data, which are processed by FMPS are listed on the output medium leftjustified as they are read from the input stream.

## SLACK INDICATORS ON ROWS CARDS

The row type is coded as shown for the ROW ID indicator card above. If cost rows are not specified with a blank slack indicator, the REVISE procedure must be called following the INPUT procedure to define the cost rows as nonrestraining.

## REVISE DATA CARDS

In the control language program, a procedure REVISE modifies data previously processed by INPUT.

Essentially, the REVISE data deck is identical to the INPUT data deck. It is composed of the same six chapters of data: ROWS, SPRICES, COLUMNS, RHS, RANGES, and BOUNDS. However, only those chapters to be actually changed are included. Within each chapter, four types of revisions are possible:

```
MODIFY
DELETE
BEFORE
AFTER
```

These revisions are stated on data cards similar to those used for INPUT. First, the chapter to be revised is identified by a chapter indicator card. Kinds of changes to be made are then specified by REVISE control cards (MODIFY, DELETE, BEFORE, and AFTER) and by actual data cards composing the changes. This sequence is repeated for each section to be revised. The use of REVISE is subject to the following conditions.

1. Modifications may be made in any order subject to the rule forbidding splitting of modifications in agiven vector.
2. If an existing nonzero element is to be changed to zero, it must be defined with the value of zero in the REVISE data deck.
3. Any new vector to be added must be given a name that is different from the name given to any old vector, even if that vector is to be deleted.
4. If an E-, L-, or G-type row is modified into an N-type row, range elements in the row are automatically removed.
5. A modified row or bound element must be entirely redefined, that is, a row must have its type of constraint specified. A bound element must have both its lower and upper limits specified even if only one is modified.
6. To keep each individual modification in core, the REVISE deck should not include more than 100 data cards for any individual revision type (MODIFY, DELETE, etc.) within a chapter. If the deck is too large, the KMAJER interrupt is taken. If revisions are extensive enough to require more than 100 data cards for any individual revision type within a chapter, the revision data should be separated into individual decks of proper size, and one call for REVISE should be made for each deck. NAME and ENDATA cards must be inserted before and after each deck.
7. If a row is added by using BEFORE or AFTER in the ROWS section, values are entered in this row for existing columns by using MODIFY.

## ROWS CARDS FOR REVISE

MODIFY The MODIFY chapter indicator card signifies that the row definition cards that follow redefine the existing type of row. The command word MODIFY is punched in columns 2 to 7 , as in

MODIFY

DELETE The DELETE chapter indicator card signifies that the data cards that follow contain the names of existing row (punched in columns 5 to 12) are to be deleted. DELETE is punched in columns 2 to 7 , as in

DELETE

BEFORE The BEFORE chapter indicator card signifies that row definition cards that follow are to be inserted before the row named in the indicator card (specified in columns 15 to 22). If no row is specified, the rows will be inserted before the first row. BEFORE is punched in columns 2 to 7. Hence, the card takes the form

row named in the indicator card (specified in columns 15 to 22). If no row is specified, the rows will be inserted after the last row. AFTER is punched in columns 2 to 7 . Hence, the card takes the form

## AFTER name

## SPRICES CARDS FOR REVISE

Slack prices for any new rows must be defined immediately following the SPRICES chapter indicator. The format of the data cards is the same as required by INPUT. Do not use BEFORE or AFTER indicators.

MODIFY The MODIFY indicator card signifies that the following data cards define new slack prices for existing slacks. All prices for an existing slack must be redefined, even if only one price is modified. MODIFY is punched in columns 2 to 7 , as in

## MODIFY

## COLUMNS CARDS FOR REVISE

MODIFY The MODIFY indicator card signifies that the following data cards redefine coefficients in existing columns and/or places coefficients in new rows of existing columns. All modified coefficents for the same column must be grouped together. The command word MODIFY is punched in columns 2 to 7 , as in

## MODIFY

DELETE The DELETE indicator card signifies that the following data cards contain the names (in columns 5 to 12) of existing columns to be deleted from the matrix. DELETE is punched in columns 2 to 7 , as in


BEFORE The BEFORE indicator card signifies that the following data cards define new matrix columns that are to be inserted in the matrix before the existing column named in the indicator card (specified in columns 15 to 22 ). If no column is specified, the new columns will be inserted before the first existing column. BEFORE takes the form,


AFTER The AFTER indicator card signifies that the following data cards define new matrix columns that are to be inserted in the matrix after the existing column named in
the indicator card (specified in columns 15 to 22 ). If colums 15 to 22 are blank, the new columns will be inserted after the last existing column. AFTER is punched in columns 2 to 6 . The form of the AFTER command is
AFTER name

## RHS CARDS FOR REVISE

Revisions to the RHS chapter are the same as for the COLUMNS chapter with the exception that the name field (columns 15 to 22) of the BEFORE and AFTER indicator card refers to names of the RHS vectors.

## RANGES CARDS FOR REVISE

Range values for new rows must be first. They may be introduced by BEFORE or AFTER, but neither is necessary.

MODIFY The MODIFY indicator card signifies that the following data cards redefine a range value on an existing row. MODIFY is punched in columns 2 to 7 , as in

## MODIFY

DELETE The DELETE indicator card signifies that the following cards contain (in columns 5 to 12) the name of the row that is to have its range value removed. DELETE is punched in columns 2 to 7 , as in

## DELETE

## BOUNDS CARDS FOR REVISE

MODIFY The MODIFY indicator card signifies that the data cards that follow redefine the bounds on existing columns. Note that the bounds on any column must be restated completely. For example, if only the lower bound was being changed, any upper bound on that column must be restated. MODIFY is punched in columns 2 to 7 , as in


DELETE The DELETE indicator card signifies that the following data cards contain (in columns 5 to 12) the name of the existing column for which all bounds will be removed. DELETE is punched in columns 2 to 7 , as in

DELETE

BEFORE The BEFORE indicator card signifies that the data cards that follow define the bounds for new columns. The BEFORE card should be identical to the BEFORE card that defined the new columns in the COLUMNS chapter. BEFORE has the form


AFTER The AFTER indicator card signifies that the data cards that follow define the bounds for new columns. The AFTER card should be identical to the AFTER card that defined the new columns in the COLUMNS chapter.

## BASISIN/BASISOUT DATA CARDS

Data for the BASISIN procedure is the same as the output from the BASISOUT procedure. As with all data decks, the data is preceded by a NAME card and terminated by an ENDATA card. The general form of the data card is shown below.

| Columns | Description |  |
| :---: | :---: | :---: |
| 2-3 | Field 1: two-letter indicator that specifies one of the following actions. |  |
|  | Code | Action |
|  | XU | Remove the variable named in Field 3 from the basis and set it at upper bound. Put the variable named in Field 2 in the basis. |
|  | XL | Remove the variable named in Field 3 from the basis and set it at lower bound. Put the variable named in Field 2 in the basis. |
|  | UL | Set the variable named in Field 2 at upper bound. Field 3 is ignored. |
|  | LL | Set the variable named in Field 2 at lower bound. Field 3 is ignored. |

5-12 Field 2: name 1.
15-22 Field 3: name 2.
25-36 Field 4: not used.
40-47 Field 5: not used.
50-61 Field 6: not used.
LL indicators are not necessary if the MODIFY parameter is not used on BASISIN since all variables wili be automatically initialized to lower bound. BASISOUT will not output any LL indicators.

## LOADLIST DATA CARDS

As with all data decks, LOADLIST data is preceded by a NAME card and terminated by an ENDATA card.

## INDICATOR CARDS

The LOADLIST data deck consists of data cards grouped according to the type of data (names or masks) they contain. A group of cards containing the same type of data is called a chapter. The first card of a chapter is always an indicator card which identifies the type of data in that chapter. Indicator cards contain only one word (NAMES or MASKS, beginning in column 1) which specifies the type of data cards that follow.

## DATA CARDS

Data cards are divided into ten 8 -column fields. Field 1 is always blank. The ten fields of a data card are outlined below.

| Columns |  | Description |
| :--- | :--- | :--- |
| 1-8 |  | Field 1: blank |
| 9-16 |  | Field 2: name or mask. |
| 17-24 |  | Field 3: name or mask. |

Columns Description

33-40

41-48
49-56
57-64
65-72
73-80

25-32 Field 4: name or mask.
Field 5: name or mask.
Field 6: name or mask.
Field 7: name or mask.
Field 8: name or mask.

Field 9: name or mask.
Field 10: name or mask.

## NAMES DATA CARDS

NAMES cards specify the names of rows or columns in the selection list. Each data card contains up to nine names in Fields 2 to 10 . Field 1 is always blank. If a field other than 1 contains all blanks, it is ignored.

MASKS DATA CARDS

MASKS cards specify the masks for selecting rows or columns. Each data card contains up to nine masks in Fields 2 to 10. Field 1 is always blank. If a field other than 1 contains all blanks, it is ignored.

## 6. LINEAR PROGRAMMING OPERATING MODE

Use and operation of procedures in the linear programming mode will be described in this chapter. The procedures are presented in four logical phases.

1. Input
2. Optimization
3. Output
4. Preservation and Restoration
(Parametric programming, an optional procedure available for use in the linear programming operating mode, is described in Appendix A.)

## INPUT PHASE

The input phase consists of two procedures, INPUT and REVISE. An outline of each is given in Table 10 below.

Table 10. Input Procedures

| Procedure | Purpose |
| :--- | :--- |
| INPUT | Initially states the LP <br> matrix. |
| REVISE | Makes revisions to the <br> LP matrix. |

INPUT The INPUT procedure specifies a linear programming matrix to FMPS. This procedure reads the input data and converts it into a compact internal representation on file MATRIX. The following internal files (see Table 7) must be defined before the call for INPUT.

1. MATRIX
2. INVERSE
3. UTILI
4. UTIL2

Also, if INPUT's data are on file, the user's communication file must be defined too.

The input file may consist of more than one reel of tape. The primary input unit must be defined through the DEVICE and ATTACH procedures. The second unit will be the next reel specified in the BPM assign control command. The occurrence of a tape end-of-file on the input tape causes switching to the alternate input tape.

For example, consider the case where input consists of three reels of tape, numbered 104, 59, and 73. The user provides ASSIGN statements to mount tapes 104, 59, and 73 on the primary input unit in that order. He also provides
a DEVICE and ATTACH statement to define the primary input unit, as in

```
!ASSIGN F:6, (DEVICE, MT), (INSN, 104,59,73). . .
:
CALL DEVICE ('TAPE6',TAPE,'F')
:
CALL ATTACH ('MYFILE', 'TAPE6',FORTRAN, OLD)
.
CALL INPUT (FILE,'MYFILE')
```

The data deck setup for the INPUT procedure is shown in Chapter 5.

The INPUT procedure will also accept input in the SHARE formats of other LP systems. These include 1108 LP data, LP/90/94 data, and CDM4 LP data. Chapter 5 contains detailed information about SHARE input formats.

The following CR variables must be initialized before the call for INPUT.

| CR Variable $\quad$ | Explanation <br> Contains the name of the data deck for <br> data reading procedures such as INPUT <br> and REVISE. Also used by data out- <br> putting procedures such as BASISOUT <br> to name output data deck. |
| :--- | :--- |
| APBNAME $\quad$The name to be assigned to the LP <br> problem. |  |

Optional parameters for INPUT are given below.

| Parameter | Explanation <br> Indicates that the input is in SHARE <br> format and not in standard FMPS format. <br> If this parameter is not present, stan- <br> dard FMPS format is assumed. |
| :--- | :--- |
| '1108' | Input is in UNIVAC 1108 LPSHARE for- <br> mat. The quotation marks are required. |
| 'LP90' | Input is in LP/90/94 SHARE format. The <br> quotation marks are required. |
| 'CDM4' | Input is in CDM4 SHARE format. The <br> quotation marks are required. |
| 'SPRICES' | Indicates that the slack prices chapter <br> is present in the input data and is to be <br> used. Used only with SHARE. |
| FILE | Indicates that the input data are to be <br> found on file 'filename'. If the param- <br> eter is not used, INPUT data are as- <br> sumed to be on the standard card input <br> device. |


| Parameter | $\frac{\text { Explanation }}{\text { 'filename' }} \quad$The symbolic name of the communica- <br> tion file on which the input data re- <br> side. The quotation marks are required. |
| :--- | :--- |

The following interrupts may occur within INPUT.
$\frac{\text { Interrupt }}{\text { KMAJER }}$

KMINER

KIOER
Causes

1. Invalid parameter.
2. Input data not found.
3. Minimum required input not found (ROWS, COLUMNS, and RHS).
4. Undefined files.
5. Rows chapter exceeds available memory.
6. FILE 'filename' undefined.
7. Duplicate columns. The duplicate column is ignored.
8. Duplicate element. The duplicate element is ignored.
9. Invalid indicator in ROWS or BOUNDS chapter.
10. Invalid combination of indicators in BOUNDS chapter.
11. Columns out of sort in BOUNDS chapter.
12. An irrecoverable input/output er- ror has occurred.
13. Insufficient storage allocated for internal files.

REVISE The REVISE procedure modifies a matrix according to the input data from the standard card input device or from an internal communication file. Any element of the matrix can be modified, deleted, or inserted.
REVISE requires that the matrix to be revised be currently loaded in the MATRIX file, and that all of the standard FMPS internal files be defined. Initial loading of the matrix may be performed by INPUT or RESTORE. Matrix information is not destroyed or modified during execution of any other procedure except for CRASH (see "Optimization Phase" later in this chapter), which may alter the bound status of certain variables and set certain equations nonrestraining if the MODIFY parameter is used. CR variable ADATA contains the name of the REVISE data deck or identification record name if the data is on file.

Calling the REVISE procedure causes the problem to be initialized to a slack basis. If REVISE is called at a stage of the problem where the basis is not a slack basis, it may be desirable to preserve the current basis (BASISOUT) prior to the call for REVISE, and to reinstate the current basis following the call for REVISE (BASISIN and INVERT).

The data card format is the same as for INPUT. Refer to Chapter 5 for information about data deck setup.

Optional parameters for REVISE are given below.

Parameter Explanation
FILE Indicates that the input data for REVISE are on the file 'filename'.
'filename' The symbolic name of the communication file on which the input data resides.

The following interrupts may occur within REVISE.
Interrupt Causes
KMAJER

1. Invalid parameter.
2. Input data not found.
3. Undefined files.
4. ROWS chapter exceeds available memory.
5. No matrix exists to REVISE.

KMINER 1. Duplicate columns. The duplicate column is ignored.
2. Duplicate element. The duplicate element is ignored.
3. Invalid indicator in ROWS or BOUNDS chapter.
4. Invalid combination of indicators in BOUNDS chapter.
5. Columns out of sort in BOUNDS chapter.

KIOER 1. An irrecoverable input/output error has occurred.
2. Insufficient storage allocated for internal files.

## OPTIMIZATION PHASE

The optimization phase contains three procedures, OPTIMIZE, INVERT, and CRASH. An outline of each is given in Table 11 below.

Table 11. Optimization Procedures

| Procedure | Purpose |
| :---: | :--- |
| OPTIMIZE | Attempts to find an optimal, feasible <br> solution to the existing matrix. |
| INVERT | Restates the product form of the in- <br> verse in terms of the minimum num- <br> ber of transformation required to <br> state the basis. |
| CRASH | Attempts to find a better initial <br> basis. |

OPTIMIZE The OPTIMIZE procedure attempts to find an optimal feasible solution to the linear programming model. If the model has no feasible solution or the solution is unbounded, OPTIMIZE causes the KNFS or KUBS interrupts to occur.

While the model is infeasible, OPTIMIZE uses a composite pricing (PI) vector. (Infeasibility is defined as the amount by which a basis variable is below its lower bound or above its upper bound.) The function of the composite PI vector is either to maintain or to move toward optimality while achieving feasibility. CR cell FCMPDJ is the compositing factor which determines the balance between the drive for optimality and/or feasibility. As an example, a value of 0.5 for FDMPDJ implies a balanced driving force between optimality and feasibility, while a value of 0.0 implies total disregard for optimality. When a balanced driving force is requested, OPTIMIZE systematically reduces FCMPDJ by 0.125 if the drive for feasibility is insufficient.

CR variable IIWGHT is used to weight individual infeasibilities. The standard setting for IIWGHT is 0 , which implies that all infeasibilities are given equal weight. If IIWGHT is set to -1 , individual infeasibilities are weighted by the amount by which they are infeasible. If IIWGHT is set to +1 , individual infeasibilities are weighted by the reciprocal of the amount by which they are infeasible.
Setting IIWGHT equal to -1 during part of the first phase of OPTIMIZE (the phase which attempts to eliminate all infeasibilities) may help reduce the number of iterations required to arrive at a feasible solution. However, this may also cause the problem to cycle. Therefore, it is recommended that the use of IIWGHT $=-1$ be limited to a given number of iterations or to a time period. This is done by initializing CR variables IFREQA or ITIME and setting IIWGHT to zero or to +1 for the remainder of this phase of OPTIMIZE.

CR variable FEPSILON may be used to perturb zero RHS elements on degenerate problems. For "less-than" constraints, zero RHS elements are replaced with FEPSILON. For "greater-than" constraints, zero RHS elements are replaced with -FEPSILON.

Problems for which the OPTIMIZE iteration log shows a zero ACTIVITY value for a large number of iterations may benefit from such perturbation. This is effected by the following control program statements.

```
FEPSILON \(=1.0 \mathrm{D}-5\)
CALL OPTIMIZE
FEPSILON \(=0.0\)
CALL OPTIMIZE
```

The communication region variables utilized by OPTIMIZE are listed below. Of all the variables in the list, only ARHS, AOBJ, and FOBJWT must be initialized by the user prior to calling OPTIMIZE.

| CR Variable $\quad$ | Explanation |
| :--- | :--- |
| ARHS | Name of the right-hand side. |
| AOBJ | Name of the objective row. |

CR Variable
FOBJWT

FCMPDJ

INCAND

IIWGHT

FEPSILON

FDJZT

FINFZT

FMPIVT

Explanation
The weight given to the objective function. Must be +1.0 for minization, -1.0 for maximization.
Factor used in determining effective DJ when infeasible, as in

$$
\begin{aligned}
D J E= & \text { FCMPDJ * DJ + }(1.0 \\
& - \text { FCMPDJ) * DJI }
\end{aligned}
$$

where
DJE is the effective DJ of the column.

DJ is the true DJ of the column.

DJI is the DJ based on infeasibility removal qualities of column.

Number of profitable candidates from which one is selected during pricing of the matrix. For example, if INCAND is 5, then from each group of five profitable columns, the most profitable is selected. If INCAND is zero, the system will attempt to choose the optimum set.

Infeasibility weighting switch, according to codes shown below.

| $\frac{\text { Code }}{-1}$ | $\frac{\text { Meaning }}{\text { Weight by amount of }}$ <br> infeasibility. |
| :--- | :--- |
| 0 | All infeasibilities given <br> equal weight. |
| +1 | Weight by reciprocal of <br> amount of infeasibility. |

The value used to replace zero right-hand-side elements of inequalities on degenerate problems. If the constraint is of the less-than type, a zero RHS element is replaced with FEPSILON. If the constraint is of the greater-than type, a zero RHS element is replaced with -FEPSILON.

DJ zero tolerance. If the absolute value of the reduced cost (DJ) is less than FDJZT, it is considered zero.

Infeasibility zero tolerance. If the absolute value of the amount of infeasibility is less FINFZT, the variable is considered feasible.

Minimum pivot tolerance. During any optimization procedure (here, INVERT is not considered an optimization procedure), an element is not considered as potentially pivotal unless its absolute value is greater than FMPIVT.

| CR Variable | Explanation <br> ILOGC <br> Iteration logging frequency on con- <br> sole typewriter. |
| :--- | :--- |
| ILOGSS | Iteration logging frequency for stan- <br> dard printing device. <br> On/Off switch for printing column <br> selection messages during pricing of <br> matrix. |
| IFREQI | Iteration frequency interrupt for in- <br> version. The KINV interrupt will <br> occur every IFREQI iterations <br> (IFREQI $\geq 0$ ). |
| IFREQA | Iteration frequency interrupt. If <br> IFREQA is 0, no interrupt will occur. <br> Otherwise, the KFREQA will occur <br> every IFREQA iterations. |
|  | The length of time, in minutes, before <br> the KTIME interrupt will occur. The <br> KTIME interrupt does not occur if <br> KTIME is set to zero. Whenever the |
| KTIME interrupt occurs, KTIME is |  |
| set to zero. Time for KTIME is mea- |  |
| sured from the time of the last initial- |  |
| ization of ITIME. |  |

The following interrupts may occur within OPTIMIZE.

| Interrupt | Causes |
| :---: | :---: |
| KMAJER | 1. AOBJ or ARHS undefined. |
|  | 2. No matrix to optimize. |
| KIOER | 1. Unrecoverable I/O error. |
|  | 2. INVERSE file capacity exceeded. |
| KNFS | No feasible solution. |
| KUBS | Unbounded solution. |
| KINV | 1. Inversion frequency (IFREQI) to be satisfied. |

2. Correcting numerical errors.
3. Inverse exceeding file storage.
4. Clock control active. Corrective action requires calling the INVERT procedure.

| Interrupt | $\underline{\text { Causes }}$ |
| :--- | :--- |
| KFREQA | User iteration frequency (IFREQA) <br> satisfied. |

KTIME User-specified time increment reached.
Some possible difficulties that may occur during optimization, and some suggested cures are given below.

## DEGENERACY

If many RHS coefficients are zero, the problem may be degenerate. Degenerate problems are characterized by an inability to reduce infeasibilities beyond a certain number during phase one, or an excessive number of iterations to arrive at the optimal solution.

The cure is crashing before calling for OPTIMIZE. Use of the MODIFY parameter in the call for CRASH is recommended. However, since this causes modification of the matrix data, one may have to save (using the SAVE procedure) the current matrix before calling for CRASH (MODIFY), preserve the optimal basis after optimization (BASISOUT), reload the original matrix by means of RESTORE, reload the optimal basis (BASISIN), and invert to the optimal basis (INVERT). This in effect cancels any changes made by CRASH to the matrix and allows subsequent execution of PARARHS or the use of an alternate RHS vector.

Another cure is to use RHS perturbation (FEPSILON).

## PIVOT REJECTIONS

Exception messages printed by the OPTIMIZE and INVERT procedures indicate pivot rejections. Subsequently, the problem may become pseudo-infeasible, or pseudo-unbounded, or may become pseudo-optimal during phase two of OPTIMIZE. Also, the numerical accuracy may be impaired.

Generally, occasional pivot rejections during the OPTIMIZE procedure have no adverse effects. Pivot rejections during INVERT may result in some of the abnormalities listed above.

The following actions may correct pivot rejections:

1. Raise the value of the FABSZT and/or of the FRELZT tolerances: this tends to eliminate small terms from the matrix, thus making it more unlikely for a pivot to be small enough to be rejected. During computations, round-off errors may cause certain zero elements in the transformed matrix to be computed as very small values. Hence, the FABSZT and FRELZT tolerances should be set large enough so that resulting pseudo-values will not be chosen as pivot terms. Care must be taken not to use too large a value; since this could eliminate valid elements.
2. Lower the value of FMPIVT and FMINVT: during OPTIMIZE and INVERT, pivoting on very small elements may cause loss of numerical accuracy. To avoid this, elements
smaller than FMPIVT and FMINVT are rejected as pivot elements. Values that are too large for these tolerances may result in ignoring valid pivot terms, thereby causing unboundness or preventing feasibility.
3. Eliminate poor scaling of the matrix: scaling is adequate when the matrix coefficients are within two or three orders of magnitude of each other.

INVERT The INVERT procedure establishes the productform inverse for the currently specified basis. To minimize the number of elements in the inverse and, therefore, reduce numerical rounding error and computation time, INVERT uses the most modern techniques in triangularization and sub-triangularization. INVERT may be called either explicitly by the user or as the result of the KINV interrupt.

Periodic calls to INVERT from OPTIMIZE help preserve numerical accuracy and reduce total optimization time. Such calls are automatically executed at suitable time intervals. Setting CR variable INVTIME to a negative value inhibits these automatic calls.

CR variable IFREQI, if set to a positive nonzero value, controls the maximum number of iterations that can occur between occurrences of the KINV interrupt. Exceptional conditions, such as the INVERSE procedure exceeding file storage, or loss of accuracy during OPTIMIZE, PARARHS, or PARAOBJ procedures, may also cause the KINV interrupt to occur.

In general, operating with INVTIME $=0$ and $\operatorname{IFREQI}=0$ gives the best speed and accuracy. CR region variable -FMINVT is used by INVERT as the minimum pivot tolerance. Elements are not considered pivotal if their value is smaller than FMINVT. FMINVT should be initialized to a value smaller than the value used for FMPIVT, the minimum pivot tolerance for OPTIMIZE.

The following interrupts may occur within INVERT.

| $\frac{\text { Interrupt }}{\text { KMAJER }}$ | $\frac{\text { Causes }}{\text { 1. No matrix defined. }}$ |
| :--- | :--- |
|  | 2. No basis to invert to. |
| KIOER | Irrecoverable input/output error. |

CRASH The CRASH procedure attempts to find an initial basis structure that reduces infeasibility, reduces degeneracy, and that contains variables that must be basic at solution. In addition, any row that has no feasible solution is pointed out and a KNFS interrupt occurs.

In the following LP equation,

$$
\sum_{A_{i j}} X_{j} \pm S_{i}=R H S_{i}
$$

the sign of the slack coefficient $S_{i}$ is positive for equations of the type "less than" or "equal to", and negative for equations of the type "greater than". Both $\dot{A}_{i j}$ and $S_{i}$ are referred to as elements. $R H S_{i}$ is the right-hand-side coefficient.

The following messages may be printed during CRASH.

```
ROW xxxxxxxxx DOMINATING. ROW SET NON-
RESTRAINING (FREE).
```

This message is produced when row $x \times x x x x x x$ has a zero RHS and either no plus elements or no negative elements. Since this equation constrains all of the columns having elements in it to zero, CRASH will also fix all those columns at lower bound. This is equivalent to having specified the row as N (nonrestraining) in the ROWS chapter during INPUT.

## SLACK ON ROW xxxxxxxx SET FREE.

This message is produced when the slack for row $x \times x \times x \times x \times$ is the only plus element in the row. Therefore, the slack for this row must be basic. This is equivalent to having specified the row as N (nonrestraining) in the ROWS chapter during INPUT.

```
COLUMN yyyyyyyy SET FREE IN ROW xxxxxxxx.
```

This message is produced if the element in column yyyyyyyy is the only plus element in equality row $x x x x x x x x$ and the RHS for this row is positive or zero, or if the element in column yyyyyyyy is the only minus element if row $x \times x \times x \times x x$ and the RHS for this row is zero. Column yyyyyyyy is entered into the basis in row $x \times x \times x x x x$. This is equivalent to having specified the column as FR (free) in the BOUNDS chapter during INPUT.

## COLUMN yyyyyyyy FIXED AT LOWER BOUND.

This message is produced whenever a column has an element in a dominating row implying that it must be nonbasic. This is equivalent to having specified the column as FX (fixed at lower bound) in the BOUNDS chapter during INPUT.

A summary line is printed stating the number of rows set free (slack on rows must be basic), the number of columns set free (columns that must be basic), the number of fixed columns (columns that must be nonbasic), and the number of rows that have no feasible solution.

INVERT is automatically called by CRASH to invert to the basis described by CRASH.

If it is desired to have the free and fixed status applied to the MATRIX, the parameter MODIFY on the call for CRASH will effect this.

Crashing often results in a significant speed increase in the OPTIMIZE procedure if the problem is degenerate and MODIFY is specified. The CRASH execution time is generally negligible compared with the OPTIMIZE time.

If the right-hand-side parametric procedure is to be used later in the run, or if a successive case is run which is obtained from the current case by use of the REVISE procedure or by using other right-hand-sides, and the

MODIFY parameter is specified, the following sequence of operations is necessary.

1. Save the problem before calling for CRASH (call SAVE).
2. Save the optional basis after reaching the solution (CALL BASISOUT, FILE, 'filename').
3. Restore the original matrix (call RESTORE).
4. Restore the optimal basis (CALL BASISIN, FILE, 'filename').

Note that if parametric programming is to be used later in the run or other right-hand-sides are to be used, MODIFY should not be used since the free and fixed status assigned by CRASH will not be valid for another right-hand-side or for PARARHS.

The optional parameter for CRASH is given below.

| Parameter | Explanation <br> Indicates that the free and fixed <br> status of variables is to be made <br> permanent in the MATRIX. |
| :--- | :--- |

The following communication region variables must be initialized by the user prior to the call for CRASH.

| CR Variable | Explanation |
| :--- | :--- |
| ARHS | Name of the right-hand-side. |
| AOBJ | Name of the cost row. |

The following interrupts may occur within CRASH.

| Interrupt | Causes |
| :---: | :---: |
| KMAJER | 1. AOBJ or ARHS undefined. |
|  | 2. No matrix to optimize. |
| KIOER | 1. Irrecoverable input/output error. |
|  | 2. File capacity exceeded. |
| KNFS | No feasible solution. |
|  | OUTPUT PHASE |

The output phase contains five procedures, OUTPUT, SOLUTION, ERRORS, CONDITION, and GET. An outline of each is given in Table 12.

Table 12. Output Procedures

| Procedure | Purpose |
| :--- | :--- |
| OUTPUT | Displays the matrix in various <br> forms. |
| SOLUTION | Reports the solution values. |

Table 12. Output Procedures (cont.)

| Procedure | Purpose |
| :--- | :--- |
| ERRORS | Examines errors in the solution. |
| CONDITION | Displays the condition of vari- <br> ous FMPS regions and files. |
| Retrieves solution information <br> in the control language. |  |

OUTPUT The OUTPUT procedure displays the entire matrix or a selected subset on the standard printing device, or files on the internal communications device. OUTPUT displays the entire original matrix in tabular form on the standard printing device. Referring to the LP equation formulations below,

$$
\begin{aligned}
& A_{i j} X_{i} \pm S_{i}=R H S \\
& C_{i} X_{j} \longrightarrow \text { Maximum }
\end{aligned}
$$

The OUTPUT procedure displays the values of the following elements:

## Coefficients $A_{i j}$

Coefficient $S_{i}$ (value of 1 for the slack variable)
Right-Hand-Side values RHS
Cost coefficient $C_{i}$
The options of OUTPUT (described in Table 13) control the following display options:

1. Grouping of the coefficients: the coefficients can be grouped and displayed for each variable (matrix column), or for each equation (matrix row), or can be displayed on the printer form in such a way that they form the entire matrix when the printer pages are separated and reassembled together in a certain manner. The grouping by rows is generally the most compact way of displaying large LP matrices. The grouping in tableau format is only practical for small problems (less than 200 variables).
2. Representation of the coefficient values (numerical value) or symbol for order of magnitude.
3. Applicability of selection lists: output may be made to include or exclude all coefficients for specified rows or for rows the names of which match specified row masks or both, or for specified columns or for columns the names of which match specified column masks. If desired, row and column selection lists may be used in conjunction with each other to abstract further the printed output. Two special selection lists, LISTI and LISTU can also be used in this connection. LISTI identifies the set of all infeasible equations (rows) and LISTU identifies the set of all unbounded variables (columns) at the time of the call for OUTPUT.
4. Whether to display the original or current coefficients: referring to the simplex tableau, the original coefficients are the Coefficients, Right-Hand-Side Coefficients, Slack Coefficients, and objective Function coefficients for the initial tableau (all slack basis). Contrasted with this, the "current" coefficients are
those for the simplex tableau corresponding to the current basis.

Output Medium: the report prepared by OUTPUT is directed to the standard printing device.

Table 13. Parameters for OUTPUT

| Parameter | Output Device (PRINTER) | Function of Parameter |
| :---: | :---: | :---: |
| CURRENT | Optional | The requested elements of the matrix are premultiplied by the inverse to bring them up to date with the current basis. |
| CODED | Optional | Provides a condensed, coded picture of matrix tableau. |
| BYROWS | Optional | The nonzero elements of the row along with the names of the column in which they reside are displayed. (Matrix displayed row by row.) |
| BYCOLS | Optional | The nonzero elements of the column along with the names of the rows in which they reside are displayed. (Matrix displayed column by column.) |
| COUNTS |  | The name, type, and element count of each row, column, and RHS is printed according to the following codes. |
|  |  | The type for a row is printed: |
|  |  | Row Type Meaning |
|  |  | $N \quad$ Nonrestraining |
|  |  | E Equality |
|  |  | G Greater than |
|  |  | GR Greater than with a range |
|  |  | L Less than |
|  |  | LR Less than with a range |
|  |  | The type for a column or RHS is printed: |
|  |  | Row Type Meaning |
|  |  | FX Fixed |
|  |  | FR Free |
|  |  | LO Lower bounded |
|  |  | UP Upper bounded |
|  |  | LU Lower and upper bounded |
| MATRIX |  | Outputs the matrix in card image form on the card punch or to a CARD communication file if the FILE, 'filename' parameters are specified. The contents of CR variable ADATA will be placed in columns 15 to 22 of the generated NAME card. |
| ROWS | Optional | Indicates that row selection or exception lists are to be used. |
| COLS | Optional | Indicates that column selection or exception lists are to be used. |
| EXCEPT | Optional | Indicates that the following parameter is a list reference and items in list are to be excepted from output. |
| LISTR | Optional | Used in connection with ROWS parameter to specify that LISTR contains the row selection or exception list. |
| LISTC | Optional | Used in conmection with COLS parameter to specify that LISTC contains the column selection or exception list. |

Table 13. Parameters for OUTPUT (cont.)

| Parameter | Output Device <br> (PRINTER) | Function of Parameter |
| :--- | :--- | :--- |
| LISTI | Optional | Optional <br> Used in connection with ROWS parameter to specify that the row <br> selection list is composed of all infeasible rows. <br> Used in connection with COLS parameter to specify that the col- <br> umn selection list is composed of unbounded columns. <br> Indicates that requested output be written on internal communi- <br> cation file (as well as printed). <br> Used in connection with FILE parameter to specify 'filename' of <br> internal communication file. |

## Notes:

Either BYROWS or BYCOLS must be specified, but not both.
Element values displayed are the original ones as loaded by INPUT unless the parameter CURRENT is specified.
Unless BYROWS or BYCOLS is specified, the matrix is displayed in tableau format.
Parameter ROWS, if specified, must always be part of one of the following parameter sequences:

$$
\begin{aligned}
& \text { ROWS, LISTR } \\
& \text { ROWS, LISTI } \\
& \text { ROWS, EXCEPT, LISTR } \\
& \text { ROWS, EXCEPT, LISTI }
\end{aligned}
$$

This parameter specifies that only those elements in the rows specified in LISTR or LISTI are to be output or to be excluded from output.

Parameter COLS, if specified, must always be part of one of the following parameter sequences:
COLS, LISTC
COLS, LISTU
COLS, EXCEPT, LISTC
COLS, EXCEPT, LISTU
This parameter specifies that elements in the columns specified in LISTC or LISTU are to be output or excluded from output.

The following control program statements are useful in determining the cause of infeasibility or unboundedness if it occurs during CRASH, OPTIMIZE, PARAOBJ, or PARARHS:

```
C INITIALIZE UNBOUNDEDNESS INTERRUPT
        CELL TO TRANSFER TO 500
        ASSIGN 500 TO KUBS
    C INITIALIZE INFEASIBILITY INTERRUPT CELL
        CELL TO TRANSFER TO }51
        ASSIGN 510 TO KNFS
    C ENTRY FOR UNBOUNDED PROBLEM INTERRUPT
        500 CALL OUTPUT (BYCOLS,COLS,LISTU)
        503 CALL SOLUTION
            STOP
C ENTRY FOR INFEASIBLE PROBLEM INTERRUPT
        510 CALL OUTPUT (BYROWS,ROWS, LISTI)
            GO TO 505
```

        In case of unboundedness, the matrix columns for the un-
        bounded variables are output.
        In case of infeasibility, the matrix rows for the infeasible
        constraints are output.
    The following example illustrates the use of OUTPUT to display the original form of the elements in the rows specified in LISTR but not in the columns specified in LISTC.

> CALL OUTPUT (BYROWS, ROWS, LISTR, COLS, EXCEPT, LISTC)

The following interrupts may occur within OUTPUT

$$
\text { Interrupt } \quad \underline{\text { Causes }}
$$

KMAJER 1. No matrix has been processed by INPUT.
2. There is no file with the name 'filename'.

1. Null selection list.
2. Invalid parameters.
3. Illogical combination of parameters
Irrecoverable input/output error.

SOLUTION The OPTIMIZE procedure does not automatically print the solution values when an optimal solution is reached. Its only purpose is to produce the optimal basis. Calling for the SOLUTION procedure allows the user to output the actual solution report.

The same mode of operation applies for parametric programming on the Right-Hand-Side and Costrow. Parametric procedures PARARHS and PARAOBJ create the basis for various values of the parameter FTHETAR and FTHETAC but do not print the solutions, this requires a call to SOLUTION.

Keeping the solution output function separate from the optimization or parametric procedures allows greater flexibility in the use of these procedures. Also, since the solution is called from the control program, tests may be programmed in the control program, using the IF statement to print the solution only under certain conditions. Additionally, several solution reports may be created for a given problem using different selection lists.

SOLUTION may also be used after a call to RESTORE, thereby printing the solution for a problem previously saved on a RESTART file, or after the sequence CALL BASISIN, CALL INVERT to output the solution pertaining to a user-specified basis.

The normal mode of SOLUTION is to print the solution on the standard printing device. If the optional parameter FILE is specified, the specified information is also placed on communication file 'filename'. In this case, the RCHAPTER and/or CCHAPTER parameters must be used to specify the columns of output to be filed.

SOLUTION output is prepared in two chapters, ROWS and COLUMNS. The ROWS chapter contains information on the selected rows in the matrix. The report contains nine columns of information. Table 14 describes each of the nine columns for the ROWS chapter. The COLUMNS chapter contains information on the selected columns in the matrix. The columns report contains eight columns which are described in Table 15.

If the FILE option is used, it is possible to file the data columns selectively in each chapter as well as select which rows and columns to output. Each data column has been assigned a number. Tables 14 and 15 list the numbers as well as the headings in each chapter.

The data columns are selected for filing by using the keyword parameters RCHAPTER and CCHAPTER, each followed by the numbers of the data columns to be filed.

Table 14. ROWS Chapter Column Description

| Column | Heading | Description of Information in Column |
| :---: | :---: | :---: |
| 1 | NUMBER | The internal serial number associated with the row. |
| 2 | ROW | The name of the row (slack). |
| 3 | AT | A two-character code indicating status of row. |
|  |  | Code $\quad$ Meaning |
|  |  | BS Slack variable in basis and feasible. |
|  |  | ** Slack variable in basis and infeasible. |
|  |  | EQ Artificial slack variable, nonbasic. |
|  |  | UL Row at upper limit. |
|  |  | LL Row at lower limit. |
| 4 | ACTIVITY | Activity of row, that is, the original right-hand-side minus the activity of the slack. |
| 5 | SLACK ACTIVITY | Activity of slack variable. |
| 6 | LOWER LIMIT | Lowest activity that row may have. |
| 7 | UPPER LIMIT | Highest activity that row may have. |
| 8 | DUAL ACTIVITY | Otherwise known as simplex multiplier, or PI value for row. |
| 9 | SLACK PRICE | Slack price if specified during input. If slack is priced, reduced cost of slack is equal to the DUAL ACTIVITY + or - the SLACK PRICE, where + or - refers to minimizing or maximizing, respectively. |

Table 15. COLUMNS Chapter Column Description

| Column | Heading | Description of Information in Column |
| :---: | :---: | :---: |
| 1 | NUMBER | The internal serial number associated with column. |
| 2 | COLUMN | The name of the column. |
| 3 | AT | A two-character code indicating status of column. |
|  |  | Code $\quad$ Meaning |
|  |  | BS Column in basis and feasible. |
|  |  | ** Column in basis and infeasible. |
|  |  | FR Column basic and free. |
|  |  | EQ Column nonbasic and fixed. |
|  |  | UL Column nonbasic at upper limit. |
|  |  | LL Column nonbasic at lower limit. |
| 4 | ACTIVITY | The value of the column in the solution. |
| 5 | INPUT COST | The objective function coefficient of column. |
| 6 | LOWER LIMIT | Lowest activity column may have. |
| 7 | UPPER LIMIT | Highest activity column may have. |
| 8 | REDUCED COST | The DJ of the column. The rate of change in the objective value per unit change of the column. Note that the reduced cost of an upper-bounded variable at upper bound will be negative. It may also be negative on a fixed variable. |

Chapter 2 describes the means of accessing the filed solution and the structure of each record.

The example shown below illustrates some uses of SOLUTION.

> CALL SOLUTION (ROWS, LISTR, COLS, LISTC, FILE, 'SOLFILE', RCHAPTER, $2,5,8$, CCHAPTER, $2,4,8)$

In the example, SOLUTION is used to perform the following tasks.

1. File the output on communication file 'SOLFILE' as well as on the printer.
2. File only the rows specified in row selection list LISTR.
3. File only the columns specified in column selection list LISTC.
4. File only the row name, slack activity, and dual activity columns of the ROWS chapter. (All columns appear on the printer report.)
5. File only the column name, activity, and reduced cost columns of the COLUMNS chapter. (All columns appear on the printer report.)

The optional parameters available to SOLUTION are given below.

| Parameter | Explanation |
| :--- | :--- |
| ROWS | Indicates that row selection or <br> exception list follows. |


| Parameter | Explanation <br> COLS |
| :--- | :--- |
| EXCEPT | Indicates that column selection or <br> exception list follows. |
| LISTR | Indicates that following list reference <br> is exception list. <br> Used in connection with ROWS to <br> specify row selection or exception <br> list. |
| LISTC | Used in connection with COLS to <br> specify column selection or exception <br> list. |
| FILE | Indicates that requested output be <br> written on internal communication <br> file 'filename'. |
| 'filename' | Used in connection with FILE to specify <br> 'filename'. |
| RCHAPTER | Indicates ROWS chapter data column <br> selection numbers follow. |
| CCHAPTER | Indicates COLUMNS chapter data <br> column selection numbers follow. |

The following interrupts may occur within SOLUTION.

## Interrupt Causes

KMAJER 1. No matrix defined.
2. There is no file with name 'filename'.

KIOER

## Causes

3. Data column selection indicated but specifications missing.
4. Invalid parameter.
5. Illogical combination of parameters.

Irrecoverable input/output error.

ERRORS The ERRORS procedure substitutes the current primal and dual solutions into the original primal and dual problems and computes and outputs any rounding error that exists to the standard printing device. Any error less than the tolerance FABSZT is considered zero, and no line of print will occur.

The output is prepared in two sections. The first section contains the dual errors and consists of the following information.

1. Name of the basis variable.
2. Magnitude of error.

The second section contains the primal errors and consists of the following information.

1. Name of the row.
2. Right-hand-side value of row.
3. Magnitude of error.

The following interrupts may occur in ERRORS.

| $\frac{\text { Interrupt }}{\text { KMAJER }}$ | $\frac{\text { Causes }}{\text { No matrix defined. }}$ |
| :--- | :--- |
| KIOER | Irrecoverable input/output error. |

CONDITION The CONDITION procedure outputs to the standard printing device the following information:

1. Contents of communication region.
2. Current status of all active files.

GET The GET procedure allows the user to retrieve information about a row or column, and to alter his strategy in the control language. All or any part of the following items may be accessed on a call for GET.

| Code |  | Meaning |
| :--- | :--- | :--- |
| UB |  | Upper bound |
| LB |  | Lower bound |
| CJ |  | Objective function coefficient |
| BI |  | Activity level |
| DJ | Reduced cost |  |
| ZJ | PI value |  |

The general form of a call for GET is

where
NAME is the name of a row or column.
op is one of the codes listed above.
FWxx is a user working cell.
In addition to placing requested information in the specified working cells, GET also prints information on the standard printing device. The following example illustrates the use of GET to obtain the activity in FWO1, the input cost in FW02, and the upper bound in column RUNCRUDE in FW03.

CALL GET ('RUNCRUDE', BI, FW01, CJ, FW02,UB,FW03)

## PRESERVATION/RESTORATION PHASE

The preservation/restoration phase contains four procedures, BASISOUT, SAVE, BASISIN, and RESTORE. An outline of each is given in Table 16 below.

Table 16. Preservation/Restoration Procedures

| Procedure | Purpose |
| :--- | :--- |
| BASISOUT | Preserves the basis structure. <br> SAVE <br> Breserves the contents of data <br> areas and files. |
| RESTORE | Restores a basis structure. <br> Restores the contents of data <br> areas and files. |

BASISOUT The BASISOUT procedures punches or files
(FILE parameter) the current basis structure and bounds status. The punched or filed data deck is preceded by a NAME card which contains (in columns 15 to 22) the contents of CR cell ADATA. In addition, the data deck is followed by an ENDATA card.

The data deck produced by BASISOUT is in the correct format to be used as input data to the BASISIN procedure.

Chapter 5 describes the format of data cards produced by BASISOUT and required as input by BASISIN.

Optional parameters for BASISOUT are:

| Parameter | $\frac{\text { Explanation }}{\text { Indicates that the output is to be }}$ |
| :--- | :--- |
| written on communication file |  |
| 'filename'. If FILE is not speci- |  |
| fied, the output will be produced |  |
| on the standard punch device. |  |

The following interrupts may occur within BASISOUT:

| $\underline{\text { Interrupt }}$ | $\underline{\text { Causes }}$ |
| :--- | :--- |
| KMAJER |  |
|  | 1. No matrix defined. |
|  | 2. 'filename' undefined. |
|  | 3. Invalid parameter. |
| KIOER |  |
|  |  |

SAVE The SAVE procedure saves the contents of the communication region, the various internal work areas, and all internal files (MATRIX, INVERSE, etc.) on the tape file RESTART. Only one problem may be saved on the RESTART tape. Any number of SAVEs may be made to the same restart tape, but the last one overlays previous ones. If several SAVE files are desired, the tape unit for RESTART may be changed in the control program by a new ATTACH statement preceding the SAVE. Note that user working-storage and communication files are not saved.

The following interrupts may occur within SAVE.

| $\frac{\text { Interrupt }}{\text { KMAJER }}$ |  |
| :--- | :--- |
|  |  |
|  | 1. REauses |
| KIOER | 2. RESTART file not on a tape unit. |
|  | Irrecoverable input/output error. |

BASISIN The BASISIN procedure either inputs a new basis, or modifies the existing basis. Provision is made to allow both the specification of variables to be entered into the basis and the removal of variables at upper or lower bound. In addition, the user may specify which nonbasic variables are to be placed at upper or lower bound.

If the MODIFY parameter is used, the current basis will be used to process the input. Chapter 5 describes the format
of the input cards. If the MODIFY parameter is not used, an all-slack basis will be used to process the input, and all variables will initially be set at lower bound.

A call for the INVERT procedure must be made following the BASISIN procedure.

The optional parameters for BASISIN are given below.

| Parameter | Explanation <br> Indicates that the input data is to be <br> processed against the current basis <br> structure (instead of the slack basis). |
| :--- | :--- |
| FILE | Indicates that the input is on file <br> 'filename' instead of the normal card <br> reading device. |
|  | The symbolic name of the input file. |

The following interrupts may occur within BASISIN.

| $\underline{\text { Interrupt }}$ | $\underline{\text { Causes }}$ |
| :--- | :--- |
| KMAJER | 1. Invalid parameter. |
|  | 2. 'filename' undefined. |
| KIOER | Irrecoverable input/output error. |

RESTORE The RESTORE procedure restores the data areas and internal files saved by SAVE from file RESTART. Note that any internal file restored by RESTORE must be defined prior to the call for RESTORE.

The following interrupts may occur within RESTORE.

| Interrupt | $\underline{\text { Causes }}$ |
| :--- | :--- |
| KMAJER | 1. RESTART file undefined. |

2. Internal file undefined.
3. RESTART file not on a tape unit.
4. Insufficient core available for restoring data areas.

KIOER Irrecoverable input/output error.

## 7. SEPARABLE PROGRAMMING OPERATING MODE

Use and operation of procedures in the separable programming (SEP) operating mode will be described in this chapter. A general description of this operating mode is provided followed by descriptions of specific procedures. The procedures are presented in four logical phases.
i. Input
2. Optimization
3. Output
4. Preservation and Restoration

## GENERAL DESCRIPTION OF SEP MODE

Separable programming provides the FMPS user with the capability of handling certain types of nonlinear functions.

The nonlinearities must comply with the following important restrictions:

1. A nonlinear function in n variables must be "separable" into the sum of $n$ functions, each in terms of only one of these variables, as in

$$
y=f\left(x_{n}\right)=f_{1}\left(x_{1}\right)+f_{2}\left(x_{2}\right)+\ldots+f_{n}\left(x_{n}\right)
$$

2. Each of the $n$ functions must be representable by a piece-wise linear approximation of that function. The graph of the function in Figure 7 is shown in solid lines, a piece-wise linear approximation of the function is shown in broken line.


Figure 7. Piece-Wise Linear Approximation to a Separable Function

## SEP ALGORITHM

A full description of the delta-method algorithm, together with a discussion of methods available to ensure that the problem complies with the above conditions, is found in Non-Linear and Dynamic Programming by G. Hadley. ${ }^{\dagger}$ Some details of this algorithm are outlined below.

1. Each variable $x$ participating in a nonlinear function $f(x)$ has associated with it a set of special variables. These special variables depict the piece-wise linear approximation to $f(x)$; each special variable represents the distance progressed along some particular section of the piece-wise linear approximation. That is, $d x_{k}$ is the $k$ th of $r$ special variables used to approximate $\mathfrak{f}(x)$. It may be written as

$$
d x_{k}=\frac{x-x_{k-1}}{x_{k}-x_{k-1}}
$$

where $x_{k-1}$ and $x_{k}$ are successive intercepts on the $x$ coordinate (see Figure 7).
2. Each of the special variables has a lower bound of zero and an upper bound of 1 . Their order specifies a direction along the $\times$ coordinate.
3. A special variable may become basic only if one of the adjacent variables is basic or the preceding variable is at upper bound. A bound shift is allowed only if the preceding variable is at upper bound. No two special variables in the same set may be basic at a given iteration.
4. The activity of the variable approximated is given by a grid equation of the form

$$
x=x_{0}+\operatorname{sum}_{k=1}^{r} \Delta x_{k} \cdot d x_{k}
$$

(See "Applicability of the SEP Algorithm" below.)
5. Any subset of the objective function and the problem constraints may be separable functions. A variable $x$ may appear linearly in some functions and as a set of special variables approximating it in other functions. The user must only observe the requirements for establishing interrelationship.

## PIECE-WISE LINEAR APPROXIMATION

Figure 7 shows a piece-wise linear approximation to some function $f(x)$. This function is to be included in a set of equations for optimization. The function may be part of

[^0]the objective or of some constraint. Note that the function is defined only over certain limits of $x$, that is,
$$
x_{0} \leq x \leq x_{r}
$$

Special variables $\mathrm{dx}_{1}, \mathrm{~d} x_{2}, \ldots \mathrm{~d} x_{r}$ are now defined. These variables collectively form the set of special variables required to approximate $f(x)$. The special variable $d x_{1} d e-$ fines the interval between the two $x$ intercepts $x_{0}$ and $x_{1}$; $d x_{2}$, the next interval between $x_{1}$ and $x_{2}$, and so on. The relationship is given by

$$
\begin{aligned}
x= & x_{0}+d x_{1}\left(x_{1}-x_{0}\right)+d x_{2}\left(x_{1}-x_{2}\right) \\
& +\ldots+d x_{r}\left(x_{r}-x_{r-1}\right)
\end{aligned}
$$

or, simply,

$$
x=x_{0}+\operatorname{sum}_{k=1}^{r} \Delta x_{k} \cdot d x_{k}
$$

where

$$
\begin{aligned}
& 0 \leq d x_{k} \leq 1 \\
& \Delta x_{k} \text { are user-defined intervals along the } x \text { axis. }
\end{aligned}
$$

The $\Delta x_{k}$ may be as small or as large as required, and may vary as necessary to obtain the user-required degree of approximation to any section of $f(x)$.
The value of $f(x)$ at $x_{0}$ is $f\left(x_{0}\right)$, at $x_{1}$ it is $f\left(x_{1}\right)$, and so on to $f\left(x_{r}\right)$ at $x_{r}$. Defining

$$
\Delta f\left(x_{k}\right)=f\left(x_{k}\right)-f\left(x_{k-1}\right)
$$

the relationship for $f(x)$ along the first interval of the piecewise linear approximation is obtained by equating

$$
f(x)=f\left(x_{0}\right)+\Delta f\left(x_{1}\right) \cdot d x_{1}
$$

where

$$
\begin{aligned}
& 0 \leq d x_{1} \leq 1 \\
& d x_{2}=d x_{3}=\ldots=d x_{r}=0
\end{aligned}
$$

This relationship can be extended to any point on the approximation, as in

$$
f(x)=f\left(x_{0}\right)+\underset{k=1}{r} \Delta f\left(x_{k}\right) \cdot d x_{k}
$$

This is a linear relationship in $d x_{k}$. If the $d x_{k}$ are variables in the linear program, then this function may be included in the linear program as long as the following restriction is observed:
for

$$
0 \leq d x_{k} \leq 1
$$

$$
\begin{aligned}
& d x_{0}=d x_{1}=\ldots d x_{k-1}=1 \\
& d x_{k+1}=\ldots=d x_{r}=0
\end{aligned}
$$

The variable $d x$ is the only variable in the set that may be basic. All other variables in the set are at upper or lower bound.

## APPLICABILITY OF THE SEP ALGORITHM

There are two points, $A$ and $B$, on the piece-wise linear approximation (Figure 7) from which the value of $f(x)$ decreases irrespective of the direction along the $x$ coordinate. Assuming that $f(x)$ is an objective to be maximized, it is apparent that starting from $[x, f(x)]$, the point $A$ would be reached and the optimum would be indicated. However, A represents only a local optimum. The global optimum is point $B$. By starting at $x_{r}$, and proceeding in the opposite direction, point $B$ is attained. The use of the SETBOUND procedure can assist in finding the global optimum in such cases, but there is no guarantee that an optimum attained using separable programming is the global optimum unless all functions have the appropriate properties of convexity and concavity.

The problem of local optima is also raised by separable nonconvex constraints. If the objective for the problem for which Figure 7 represents a constraint was $z=x$, then, depending on the direction in which $x$ is moving, the algorithm may decide that $A$ or $B$ is the optimum.

## EXAMPLES USING SEPARABLE PROGRAMMING

The following two problems illustrate the use of separable programming to model nonlinearities in the objective function and in a constraint.

## NONLINEAR OBJECT FUNCTION

Volume-related discounts on a certain petrochemical feedstock are to be applied to the objective function according to the following table:

The total cost of feed, which is the amount by which the objective function should be decremented, varies with volume according to the following polygonal curve.


The pseudo costs associated with the four special variables entered into the problem are the difference in total cost found on this curve divided by the range of volume associated with the special variable. Those differences are $\$ 237.5, \$ 637.5, \$ 1125.0$ and $\$ 1500$ respectively. The matrix tableau would appear as follows.

|  | Purchase Feedstock |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | SPVAR1 |  |  |  |
|  | -237.5 |  |  |  | | SPVAR2 |
| :--- |
| -637.5 | SPVAR3 | SPVAR4 |
| :--- |
| -1125.0 |
| -1500.0 |
| Feedstock <br> Material <br> Balance $\mathrm{-50}$ |

Note that the scaling of the special variables must be done manually and will affect all coefficients of the feed vector.

## NONLINEAR CONSTRAINT

This example illustrates the use of separate programming to model a nonconvex specification row. Two products, A and $B$, are to be blended to meet a maximum pourpoint specification of $20^{\circ} \mathrm{F}$.

The Pour Point versus Mix Curve is illustrated below. To prepare the curve for modeling, an arbitrary choice of ranges is made for the separable segments. In this case, ranges are $0-20 \%, 20-60 \%, 60-100 \%$ of Component B.


It is assumed that we wish to make 10 Mbbls of the blend. One vector is used to represent $100 \%$ A, and three "delta" vectors are used to represent the addition of Component $B$, as shown in the following tableau.

| Separable Set <br> (Unscaled) |  |  |  |  |  |
| :--- | ---: | ---: | :--- | :--- | :--- |
| Upper <br> Bound Row | 100 A <br> OB | 80 A <br> 20 B | 40 A <br> 60 B | 0 A <br> 100 B | RHS |
| Material <br> Balance on A | +1.0 | -0.2 | -0.4 | -0.4 | 10 |

Since the input requires the separable set to be scaled to have upper bound of 1 , multiply each vector by 10 . This results in the final tableau below as entered in the problem.

| Separable Set <br> (Scaled) |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Upper <br> Bound Row | 1 | 1 | 1 | 1 |  |
| Material <br> Balance on A | +10.0 | -2.0 | -4.0 | -4.0 |  |
| Material <br> Balance on B |  | 40 A <br> 60 B | 0 A <br> 100 B | RHS |  |
| Pour Point <br> Maximum <br> Specification | $400^{\circ}$ | $-220^{\circ}$ | $+100^{\circ}$ | $+180^{\circ}$ | $\leq 200^{\circ}$ |

The separable programming operating mode requires different internal treatments of the work matrix than the linear programming operating mode. There, it is necessary to set the mode of operation at the beginning of a run by means of the ENTER procedure.

The procedures in the separable programming operating mode are presented in four logical phases.

1. Input
2. Optimization
3. Output

## 4. Preservation and Restoration

Each phase will be explained in detail. Note that many procedures in the separable programming operating mode are identical to corresponding procedures in the linear programming operating mode. Descriptions of these procedures are repeated in this section for user convenience. A note at the beginning of each procedure indicates whether or not the procedure is identical to the corresponding linear programming procedure.

## INPUT PHASE

The input phase consists of two procedures, INPUT, and REVISE. An outline of each is given in Table 17 below.

Table 17. SEP Input Procedures

| Procedure | Purpose |
| :--- | :--- |
| INPUT | Accepts the initial statement <br> of the SEP problem |
| REVISE | Makes revisions to the SEP <br> problem |

INPUT Except for the restrictions and conditions described in the following paragraphs, the INPUT procedure for the separable programming operating mode is the same as the INPUT procedure for the linear programming operating mode.

The INPUT procedure specifies a separable programming problem to FMPS. INPUT processes input data (in standard data card format only) and converts it into a compact internal representation on internal file MATRIX. The following internal files (see Table 7) must be defined before the call to INPUT.

1. MATRIX
2. INVERSE
3. UTILI
4. UTIL2

Also, if INPUT's data are on file, the user's communication file must also be defined.

The data deck setup for the input procedure is shown in Chapter 5.

The special variables may appear in any row in the problem. They are identified as such in the COLUMNS chapter, and this identification is the only difference between separable and linear programming data. The 'MARKER' parameters are used to bracket each set of special variables. (The singlequotation marks are included in the keywords.) There are two types of 'MARKER' cards distinguished by the keywords 'SEPORG' or 'SEPEND' in columns 40 to 47 of the 'MARKER' data card. The format of a 'MARKER' data card is shown below.

| Columns |  | Description |
| :--- | :--- | :--- |
| 1-4 |  | Blank. |
| $5-12$ |  | Unique column name. |
| $13-14$ |  | Blank. |
| 15-22 |  | 'MARKER' |
| $23-39$ |  | Blank. |
| $40-47$ |  | 'SEPORG' or 'SEPEND' |
| $48-72$ |  | Blank. |

All of the special variables in a set must be contained between two 'MARKER' cards. A set may be embedded anywhere within the body of the matrix columns. The beginning of a new set is recognized when a 'SEPORG' type of 'MARKER' card is read. The name of the set is the name in columns 5 to 12 of the 'SEPORG' card which precedes the set. The end of a set is recognized when either a 'SEPEND' or 'SEPORG' type of 'MARKER' card with a unique name in columns 5 to 12 is processed. Contiguous
sets do not require a 'SEPEND' type of 'MARKER' card as a separator.

Data cards describing the special vectors in a set have the same format as normal linear variables. The order of appearance of the variables in a set defines the required sequence $d x_{1}, \ldots, d x_{r}$.

Each of the separable special variables must have an upper bound of 1 . This bound is automatically assigned to each of the special variables. The user may, if he so desires, include these bounds in the BOUNDS chapter. However, if any other bound besides this preempted bound is assigned, it will be registered as a minor error.

The following CR variables must be initialized before the call for INPUT.

| CR Variable | Explanation |
| :--- | :--- |
| ADATA | Contains the name of the data deck <br> for data reading procedures such as <br> INPUT, REVISE, etc. Also used by <br> data outputting procedures such as <br> BASISOUT to name output data deck. |
| APBNAME $\quad$The name to be assigned to the SEP <br> problem. |  |

Optional parameters for INPUT are

Parameter $\quad$| Explanation |
| :--- |
| FILE |
| found on file 'filename'. If the pa- |
| rameter is not used, INPUT data is |
| assumed to be on the standard card |
| input device. |

'filename' The symbolic name of the communication file on which the input data resides.

The following interrupts may occur with INPUT.

$$
\text { Interrupt } \quad \text { Causes }
$$

KMAJER

1. Invalid parameter.
2. Input data not found.
3. Minimum required input not found (ROWS, COLUMNS, and RHS).
4. Undefined files.
5. Rows chapter exceeds available memory.
6. FILE 'filiename' undefined.
7. Invalid 'MARKER' card.

KMINER

KIOER An irrecoverable input/output error has occurred.

REVISE This procedure is identical to the corresponding procedure in the linear programming mode.

The REVISE procedure modifies a matrix according to the input data from the standard card input device or from an internal communication file. Any element of the matrix can be modified, deleted, or inserted. REVISE requires that the matrix to be revised be currently input and that all of the standard FMPS internal files be defined. Communication region variable ADATA contains the name of the REVISE data deck or identification record name if data are on file. New sets of special variables must be bracketed by the required 'MARKER' cards.

It is mandatory (unless a slack starting basis is desired) that a BASISIN procedure and an INVERT procedure follow REVISE to resume from an advanced base.

The data card format is the same as for INPUT. Refer to Chapter 5 for information about data deck setup.

Optional parameters for REVISE are given below.

| Parameter | Explanation <br> Indicates that the input data for <br> REVISE is on the file 'filename'. |
| :--- | :--- |
| 'filename' | The symbolic name of the communica- <br> tion file on which the input data <br> resides. |

The following interrupts may occur within REVISE.
Interrupt Causes
KMAJER 1. Invalid parameter.
2. Input data not found.
3. Undefined files.

Interrupt Causes
KMAJER (cont.) 4. ROWS chapter exceeds available memory.
5. No matrix exists to REVISE.
6. Invalid 'MARKER' card.

KMINER

1. Duplicate columns. The duplicate column is ignored.
2. Duplicate element. The duplicate element is ignored.
3. Invalid indicator in ROWS or BOUNDS chapter.
4. Invalid combination of indicators in BOUNDS chapter.
5. Columns out of sort in BOUNDS chapter.
6. Illegal bound for a special variable. The illegal bound is ignored.

KIOER
An irrecoverable input/output error has occurred.

## SEP OPTIMIZATION PHASE

The optimization phase contains three procedures in the separable programming operating mode, OPTIMIZE, INVERT, and SETBOUND. An outline of each is given in Table 18 below.

Table 18. SEP Optimization Procedures

| Procedure | Purpose |
| :--- | :--- |
| OPTIMIZE | Attempts to find optimal, feasible solu- <br> tion to the existing matrix while ensur- <br> ing that the special variables comply <br> with their basic entry rules. |
| INVERT | Restates the product form of the inverse <br> in terms of the minimum number of <br> transformations required to state the <br> basis. |
| SETBOUND | Tries different solution paths by setting <br> the special variables in specified sets <br> to bound. |

OPTIMIZE OPTIMIZE is similar to the LP OPTIMIZE, except that in the SEP operating mode, the CR variable INCAND is not available for user setting.
The OPTIMIZE procedure attempts to find a feasible optimal solution to the separable programming matrix using the

SEP algorithm. If the matrix has no solution, or if the solution is unbounded, OPTIMIZE will cause the KNFS or KUBS interrupts to occur.

While the model is infeasible, OPTIMIZE uses a composite pricing (PI) vector. The function of the composite PI vector is either to maintain or to move toward optimality while achieving feasibility. Communication region cell FCMPDJ is the compositing factor which determines the balance between the drive for optimality and/or feasibility. As an example, a value of 0.5 for FCMPDJ implies a balanced driving force between optimality and feasibility while a value of 0.0 implies total disregard for optimality. When a balanced driving force is requested, OPTIMIZE systematically reduces FCMPDJ by 0.125 if the drive for feasibility is insufficient. FCMPDJ will be reduced if only one candidate from the selected subset is chosen to enter the basis, and the sum of infeasibilities is not decreasing.

Communication region variable IIWGHT is used to weight individual infeasibilities. The standard setting for IIWGHT is 0 , which implies all infeasibilities are given equal weight. If IIWGHT is set to -1 , individual infeasibilities are weighted by the amount by which they are infeasible. If IIWGHT is set to +1 , individual infeasibilities are weighted by the reciprocal of the amount by which they are infeasible.

The communication region variables utilized by OPTIMIZE are listed below. Of all the cells in the list, only ARHS, $A O B J$, and FOBJWT must be initialized by the user prior to calling OPTIMIZE.
CR Variable

Explanation

## ARHS

AOBJ
FOBJWT

FCMPDJ
Factor used in determining effective
DJ when infeasible, as in

$$
\begin{aligned}
\text { DJE } & =\text { FCMPDJ * DJ }+(1.0-\text { FCMPDJ }) \\
& * \text { DJI }
\end{aligned}
$$

where
DJE is the effective DJ of the column.

DJ is the true DJ of the column.

DJI is the DJ based on infeasibility removal qualities of the column.

IIWGHT
Infeasibility weighting switch, according to codes shown below.

| CR Variable | Explanation |  |
| :---: | :---: | :---: |
| IIWGHT (cont.) | Code | Meaning |
|  | -1 | Weight by amount of infeasibility. |
|  | 0 | All infeasibilities given equal weight. |
|  | $+1$ | Weight by reciprocal of amount of infeasibility. |

FDJZT DJ zero tolerance. If the absolute value of the reduced cost (DJ) is less than FDJZT, it is considered zero.

FINFZT Infeasibility zero tolerance. If the absolute value of the amount of infeasibility is less than FINFZT, the variable is considered feasible.

FMPIVT Minimum pivot tolerance. During any optimization procedure (here, INVERT is not considered an optimization procedure), an element is not considered as potentially pivotal unless its absolute value is greater than FMPIVT.

ILOGP Iteration logging frequency for console printer.

ILOGSS On/Off switch for printing column selection messages during pricing of matrix.

IFREQI Iteration frequency interrupt for inversion. The KINV interrupt will occur every IFREQI iterations (IFREQI >0).

IFREQA Iteration frequency interrupt. If IFREQA is 0 , no interrupt will occur. Otherwise, the KFREQA interrupt will occur every IFREQA iterations.

The length of time, in minutes, before the KTIME interrupt will occur. The KTIME interrupt does not occur if ITIME is set to zero. Whenever the KTIME interrupt occurs, ITIME is set to zero. Time for KTIME is measured from the time of the last initialization of ITIME.

The following interrupts may occur within OPTIMIZE.


KMAJER

1. AOBJ or ARHS undefined.
2. No matrix to optimize.

| Interrupt | Causes |
| :---: | :---: |
| KIOER | 1. Irrecoverable input/output error. |
|  | 2. File capacity exceeded. |
| KNFS | No feasible solution. |
| KUBS | Unbounded solution. |
| KINV | 1. Inversion frequency (IFREQI) satisfied. |
|  | 2. Correcting numerical errors. |
|  | 3. Inverse exceeding file storage. |
|  | Corrective action requires calling the INVERT procedure. |
| KFREQA | User iteration frequency (IFREQA) satisfied. |
| KTIME | User-specified time increment reached. |

INVERT This procedure is identical to the corresponding procedure in the linear programming mode.

The INVERT procedure establishes the product-form inverse for the currently specified basis. To minimize the number of elements in the inverse and, therefore, reduce numerical rounding error and computation time, INVERT uses the most modern techniques in triangularization and subtriangularization. INVERT may be either called explicitly by the user or called as the result of the KINV interrupt.

Periodic calls to INVERT from OPTIMIZE help preserve numerical accuracy and reduce total optimization time. Such calls are automatically executed at suitable time intervals. Setting CR variable INVTIME to a negative value inhibits these automatic calls.

CR variable IFREQI, if set to a positive nonzero value, controls the maximum number of iterations that can occur between occurrences of the KINV interrupt. Exceptional conditions such as the INVERSE procedure exceeding file storage, or loss of accuracy during OPTIMIZE, PARARHS, or PARAOBJ procedures may also cause the KINV interrupt to occur.

In general, operating with $\operatorname{INVTIME}=0$ and IFREQI $=0$ gives the best speed and accuracy. CR region variable FMINVT is used by INVERT as the minimum pivot tolerance. Elements are not considered pivotal if their value is smaller than FMINVT. FMINVT should be initialized to a value smaller than the value used for FMPIVT, the minimum pivot tolerance for OPTIMIZE.

The following interrupts may occur within INVERT.

$$
\text { Interrupt } \quad \text { Causes }
$$

KMAJER

KIOER

1. No matrix defined.
2. No basis to invert to.

Irrecoverable input/output error.

SETBOUND The SETBOUND procedure may be called at any stage of problem solution, provided that a matrix exists on the file MATRIX. Due to the possibility of obtaining a local optimum to a problem (depending on the solution path taken), it is of interest to examine the solutions obtained by proceeding along different parhs. SETBOUND provides this capability.

Independent of problem status, SETBOUND will set all the special variables in the sets specified to upper bound.

The two possible calls to SETBOUND are
CALL SETBOUND
and
CALL SETBOUND (LISTC)
The first of these calls will result in all the special variables in all the sets being set to upper bound.

The second call will result in all the special variables in those sets listed in a previously loaded column selection list (see LOADLIST) being set to upper bound. The sets required are specified by including the column name given on the 'SEPORG' type of 'MARKER' card in the list of names in the column selection list.

For example, if a set of special variables is preceded in the INPUT data by a card with the format outlined below,

| Columns |  | Description |
| :--- | :--- | :--- |
| 5-12 |  | FIRSTSET |
| 13-14 |  | Blank. |
| $15-22$ |  | 'MARKER' |
| $23-39$ |  | Blank. |
| $40-47$ |  | 'SEPORG' |

and the name FIRSTSET is included in the LOADLIST data, then the call

## CALL SETBOUND (LISTC)

will set all the special variables in the set bracketed by the above and the next 'MARKER' card to upper bound. All other special variables will remain at their previous bound setting.

Note that if LISTC is specified and no list is set up, then all special variables will be set to bound.

Optional parameters for SETBOUND are given below.

| Parameter | Explanation <br> Indicates that a previously estab- <br> lished column selection list should be <br> searched for the set names of the var- <br> iables to change bound. |
| :--- | :--- |

The following interrupts may occur within SETBOUND.

| Interrupt | Causes |  |
| :--- | :--- | :--- |
| KMAJER |  | No matrix setup. |
| KMINER | No selection list setup and optionai <br> parameter specified. |  |
| KIOER | An irrecoverable input/output error <br> has occurred. |  |

## OUTPUT PHASE

The output phase contains four procedures, OUTPUT, SOLUTION, ERRORS, and CONDITION.

An outline of each is given in Table 19 below.
Table 19. SEP Output Procedures

| Procedure | Purpose |
| :--- | :--- |
| OUTPUT | Displays the matrix in various <br> forms. |
| SOLUTION | Reports the solution values. <br> ERRORS |
| EXamines the errors in the solution. |  |
| CONDITION | Displays the condition of various <br> FMPS regions and files. |

Note that, except where explicitly noted, the 'MARKER's are not included in any of the output generated by the following procedures.

OUTPUT This procedure is identical to the corresponding procedure in the linear programming operating mode.
The OUTPUT procedure displays the entire matrix of a selected subset on the standard printing device, or files on the internal communications device. OUTPUT displays the entire original matrix in tabular form on the standard printing device.

Parameters for OUTPUT make it possible to:

1. Display updated elements.
2. Select specific rows and/or columns.
3. Output nonzero elements only.
4. File results.

Table 13 in Chapter 6 contains a complete list of parameters for OUTPUT.

The filed output consists of two logical records. The first, the identification record, is labeled OUTPUT and is followed by the second record containing the selected data. Chapter 2 describes the basic means of accessing the filed records in FORTRAN and lists the detailed structure of each record.

The following interrupts may occur within OUTPUT.

| Interrupt | Causes |
| :---: | :---: |
| KMAJER | 1. No matrix has been processed by INPUT. |
|  | 2. There is no file with the name 'filename'. |
| KMINER | 1. Null selection list. |
|  | 2. Invalid parameter(s). |
|  | 3. Illogical combination of parameters. |
| KIOER | Irrecoverable input/output error. |

The following example illustrates the use of OUTPUT to display the original form of the elements in the rows specified in LISTR but not in the columns specified in LISTC.

```
CALL OUTPUT (BYROWS,ROWS,LISTR,COLS,
    EXCEPT,LISTC)
```

SOLUTION SOLUTION output for the separable programming operating mode is prepared in three sections: the IDENTIFIER section, the ROWS section, and the COLUMNS section. The IDENTIFIER section is for display of problem status and indicates the operating mode. The ROWS and the COLUMNS sections are the same as for the linear programming operating mode with one addition in the COLUMNS section. The column names of the 'MARKER' cards will be included in the column name list in the position they had in the INPUT data column order. These names mark off each set of special variables, and have no entries against them. If the user requires the activity of the variable $\times$ approximated by the $d x_{1} \ldots, d x_{r}$, he must include the grid equation (see "SEP Algorithm", above) in the problem.

The SOLUTION procedure prepares the current solution of the separable programming matrix for display. The normal mode of SOLUTION is to print the solution on the standard printing device. if the optional parameter FilE is used, the specified information is placed on internal communication file 'filename'.

SOLUTION output is prepared in three chapters for the separable programming operating mode. The first, the IDENTIFIER chapter, is for display of problem status. The second, the ROWS chapter, contains information on the selected rows in the matrix. The report contains nine columns of information. The COLUMNS chapter contains information on the selected columns in the matrix. The COLUMNS report contains eight columns.

If the FILE option is used, it is possible to file the data columns selectively in each chapter, as well as to select which rows and columns to output. Each data column has been assigned a number.

Table 14 in Chapter 6 describes the nine columns of the row report. Table 15 in the same chapter describes the 8 columns of the columns report. These tables also indicate the number and the heading assigned to each data column.

The data columns are selected for filing by using the keyword parameters RCHAPTER and CCHAPTER, each followed by the numbers of the data columns to be filed.

Chapter 2 describes the means of accessing the filed solution and the structure of each record.

The example shown below illustrates some uses of SOLUTION.
1 CALL SOLUTION (ROWS, LISTR, COLS, LISTC, FILE, 'SOLFILE', RCHAPTER,2,5,8,CCHAPTER, $2,4,8$ )

In the example, SOLUTION is used to perform the following tasks:

1. File the output on communication file 'SOLFILE' as well as on the printer.
2. File only the rows specified in row selection list LISTR.
3. File only the columns specified in column selection list LISTC.
4. File only the row name, slack activity, and dual activity columns of the ROWS chapter. All columns appear on the printed report.
5. File only the column name, activity, and reduced cost columns of the columns chapter. All columns appear on the printed report.

The optional parameters available to SOLUTION are given below.

| Parameter | Explanation <br> ROWS |
| :--- | :--- |
| Indicates that row selection or excep- <br> tion list follows. |  |
| COLS | Indicates that column selection or <br> exception list follows. |
| EXCEPT | Indicates that following list reference <br> is exception list. |


| Parameter | Explanation <br> LISTR |
| :--- | :--- |
| Used in connection with ROWS to specify <br> row selection or exception list. |  |
| FISTC | Used in connection with COLS to specify <br> column selection or exception list. |
| 'filename' | Indicates that requested output be written in connection with FILE to specify <br> on internal communication file 'filename'. <br> 'filename'. |
| RCHAPTER | Indicates ROWS chapter data column <br> selection numbers follow. |
| CCHAPTER | Indicates COLUMNS chapter data column <br> selection numbers follow. |

The following interrupts may occur within SOLUTION.

## Interrupt Causes

KMAJER 1. No matrix defined.
2. There is no file with name 'filename'.
3. Data column selection indicated but specifications missing.

KMINER 1. Invalid parameter.
2. Illogical combination of parameters.

KIOER Irrecoverable input/output error.

ERRORS This procedure is identical to the corresponding procedure in the linear programming operating mode.

The ERRORS procedure substitutes the current primal and dual solutions into the original primal and dual problems and computes and outputs any rounding error that exists to the standard printing device. Any error less than the tolerance FABSZT is considered zero, and no line of print will occur.

The output is prepared in two sections. The first section contains the dual errors and consists of the following information.

1. Name of the basis variable.
2. Magnitude of error.

The second section contains the primal errors and consists of the following information.

1. Name of the row.
2. Right-hand-side value of row.
3. Magnitude of error.

The following interrupts may occur in ERRORS.

| Interrupt | Causes |
| :--- | :--- |
| KMAJER | No matrix defined. |
| KIOER | Irrecoverable input/output error. |

CONDITION This procedure is identical to the corresponding procedure in the linear programming operating mode.

The CONDITION procedure outputs to the standard printing device the following information.

1. Contents of communication region.
2. Current status of all active files.
3. Current status of all assigned input/output devices.
4. Amount of storage (words) in use by each file.
5. Maximum amount of storage used in the run by each file.

## SEP PRESERVATION/RESTORATION PHASE

The preservation/restoration phase contains four procedures, BASISOUT, SAVE, BASISIN, and RESTORE. An outline of each is given in Table 20 below.

Table 20. SEP Preservation/Restoration Procedures

| Procedure | Purpose |
| :--- | :--- |
| BASISOUT | Preserves the basis structure. |
| SAVE | Preserves the contents of data <br> areas and files. |
| BASISIN | Restores a basis structure. <br> RESTORERestores the contents of data areas <br> and files. |

These procedures are identical to the corresponding procedures in the linear programming operating mode.

BASISOUT The BASISOUT procedure punches or files (FILE parameter) the current basis structure and bounds status. The punched or filed data deck is preceded by a NAME card which contains (in columns 15 to 20) the contents of CR cell ADATA. In addition, the data deck is followed by an ENDATA card.

The data deck produced by BASISOUT is in the correct format to be used as input data to the BASISIN procedure.

Chapter 5 describes the format of data cards produced by BASISOUT and required as input by BASISIN.

The optional parameters for BASISOUT are

| Parameter | Explanation <br> FILE |
| :--- | :--- |
| Indicates that the output is to be written <br> on communication file 'filename'. If |  |
| FILE is not specified, the output will be <br> produced on the standard punch device. |  |
| 'filename'The symbolic name of a communication <br> file. |  |

The following interrupts may occur within BASISOUT.

| $\underline{\text { Interrupt }}$ |  |
| :--- | :--- |
| KMAJER | 1. No matrix defined. |
|  | 2. 'filename' undefined. |
|  | 3. Invalid parameter. |
| KIOER | Irrecoverable input/output error. |

SAVE The SAVE procedure saves the contents of the communication region, the various internal work areas, and all internal files (MATRIX, INVERSE, etc.) on the tape file RESTART. Note that user working-storage, and communication files are not saved.

The following interrupts may occur within SAVE.

| Interrupt | Causes |
| :--- | :--- |
| KMAJER | 1. RESTART file undefined. |
|  | 2. RESTART file not on a tape unit |
| KIOER | Irrecoverable input/output error. |

BASISIN The BASISIN procedure either inputs a new basis or modifies the existing basis. Provision is made to allow both the specification of variables to be entered into the basis and the removal of variables at upper or lower bound. In addition, the user may specify which nonbasic variables are to be placed at upper or lower bound.

If the MODIFY parameter is used, the current basis will be used to process the input. Chapter 5 contains the format of the input cards. If the MODIFY parameter is not used, an al!-slack basis will be used to process the input and al! variables wili initiaily be set at lower bound. A cail for the INVERT procedure must be made following the BASISIN procedure.

The optional parameters for BASISIN are given below.

| Parameter | Explanation <br> Indicates that the input data is to be <br> processed against the current basis <br> structure (instead of the slack basis). |
| :--- | :--- |
| FILE | Indicates that the input is on file <br> 'filename ${ }^{1}$ instead of the normal card <br> reading device. |
| 'filename' $\quad$ The symbolic name of the input file. |  |

The following interrupts may occur within BASISIN.
Interrupt Causes
KMAJER 1. Invalid parameter.
2. 'filename' undefined.

KIOER Irrecoverable input/output error.

Note that basis specifications which conflict with the rules for basic and upper bounded variable (see "SEP Algorithm", above) selection will be resolved by ignoring invalid specifications.

RESTORE The RESTORE procedure restores the data areas and internal files saved by SAVE from file RESTART. Note that any internal file restored by RESTORE must be defined prior to the call for RESTORE.

The following interrupts may occur within RESTORE.
Interrupt Causes
KMAJER

KIOER

1. RESTART file undefined.
2. Internal file undefined.
3. RESTART file not on a tape unit.
4. Insufficient core available for restoring data areas.

Irrecoverable input/output error.

## 8. OPERATING PROCEDURES

This chapter includes a description of the BPM control cards necessary for FMPS runs, and the relationship between BPM !ASSIGN control cards and FMPS control language CALL DEVICE statements. Also included are guidelines for the efficient use of FMPS. The user should reference the SIGMA 5/7 Batch Processing Monitor Reference Manual for complete discussion of BPM control cards. Error messages and error types are given in Appendix B.

## BPM CONTROL COMMANDS USED IN FMPS RUNS

Figure 8 illustrates the general deck sequence for an FMPS run. The run deck always starts with a set of BPM control cards. Following the !DATA control card are the user's FMPS control language program terminated by an END statement and input data decks. Each input data deck is preceded by a NAME card and followed by an ENDATA card.

## ASSIGN AND CALL DEVICE INTERACTION

The interrelationships between !ASSIGN control card parameters and the arguments in the CALL DEVICE control language statement are shown in the following examples.

In the command

## CALL DEVICE('EXAMPLE',TAPE,'E')

the keyword TAPE dictates an !ASSIGN control card which establishes a RAD file, labeled or unlabeled tape, and specifies that file or tape organization be consecutive-sequential (see Table 21).

In the command

## CALL DEVICE('EXAMPLE2', DISC, 'C')

the keyword DISC dictates an !ASSIGN control card which establishes a RAD file, and specifies that file organization be keyed direct-access (see Table 22).

The user should note that the compiled FMPS control language statements are written to a file or tape using the $\mathrm{F}: 1 \mathrm{DCB}$. A BPM !ASSIGN control card must be in each run deck for $F: 1$, and the organization must be consecutivesequential. The control language compiler within FMPS simulates the following pair of control language statements.

```
CALL DEVICE('PREPDEVI', TAPE, 'A')
CALL ATTACH ('PREPOUT', 'PREPDEVI')
```

The (INOUT) clause should be included in !ASSIGN control cards for all FMPS internal files and user communication files to assure the ability to read and write the file.

Should the user wish to save the RESTART tape after using the CALL SAVE procedure in an FMPS run, the (SAVE) clause should be included on the ! ASSIGN control card associated with the tape.

Note that all FMPS internal files and user FORTRAN communication files are binary files; the!ASSIGN control card should have the (BIN) clause included.

Table 21. Consecutive-Sequential File Assignments

| FMPS Control Language Statement |  |
| :--- | :--- |
| CALL DEVICE('EXAMPLE', TAPE, 'E') |  |
| Acceptable BPM !ASSIGN Control Cards |  |
| RAD File | !ASSIGN F:5, (FILE, EXAMP), |
|  | (CONSEC), (SEQUEN). . |
| Labeled Tape | !ASSIGN F:5, (LABEL, EXAMP), |
|  | (CONSEC), (SEQUEN) . . . |
| Unlabeled Tape | !ASSIGN F:5, (DEVICE, 9T), |
|  | (CONSEC), (SEQUEN). . |

Table 22. Direct-Access File Assignments

| FMPS Control Language Statement |  |
| :--- | :--- |
| CALL DEVICE('EXAMPLE2', DISC, 'C') |  |

## EFFICIENT USE OF FMPS

## ORGANIZING THE CONTROL PROGRAM

For simplicity and in order to avoid sequence errors, it is recommended that the control program always start with the following statement order:

```
CALL ENTER
ASSIGN statements for KMAJER and KMINER
CALL DEVICE
CALL ATTACH
```

If standard tolerance settings are to be used, the user need only be concerned with the following initializations:

| CR Variable | Explanation |
| :--- | :--- |
| ADATA | Initialize prior to the call for <br> any procedure requiring input <br> data, or producing output on |



Figure 8. General FMPS Deck Structure

Card types and their uses are explained below.

| Card Type | Parameter | Purpose |
| :---: | :---: | :---: |
| 1. | ! JOB | Identifies the account number and the user for the job. |
| 2. | !LIMIT | Sets the maximum execution time, number of printer pages and number of temporary RAD granules to be in effect for the run. This card is required only when the user expects the job to exceed the default BPM limits defined during BPM system generation. |
| 3. | !ASSIGN | Mandatory for the five standard FMPS files and also for any additional files or tapes the job will require (for example RESTART). If the CALL BASISOUT procedure is to be used, the assign card for $\mathrm{F}: 106$ must be included. Note that all the standard FMPS files may be assigned to either RAD or tape; however, for improved execution speed they should be assigned to RAD as keyed direct-access files. The control language file ( $\mathrm{F}: 1$ ) should always be a RAD file and must have consecutive-sequential organization. |
| 4. | !RUN | Causes BPM to load FMPS into core and commence execution. |
| 5. | ! DATA | Signals BPM that following cards are user data decks to be read by FMPS. |


| CR Variable | Explanation <br> ADATA (cont.) <br> AOBJ, ARHS <br> tape or cards, except for SAVE, <br> RESTART, and INPUT when SHARE <br> is specified. |
| :--- | :--- |
| FOBJWT | Initialize these two cells early in <br> the control program since they are <br> used by many procedures. |
|  | Initialize at -1.0 for maximization, <br> or 1.0 for minimization. |

It is always necessary to initialize the KINV interrupt cell and to program a sequence of action for that interrupt because the KINV interrupt may occur for reasons beyond the user's control (such as the occurrence of excessive numerical errors). Also, the KINV interrupt may be activated by the timing routine built into the OPTIMIZE procedure, whenever more frequent calls for INVERT would help reduce the time per iteration within the OPTIMIZE procedure.

The SAVE procedure can be used for two purposes:

1. To preserve the problem status on tape in order to be able to restart from an advanced basis if it is necessary to discontinue the run, or if hardware errors occur.
2. To create a working copy of a problem in a compact format on magnetic tape; for instance, calling the SAVE procedure after reading a large matrix from cards allows use of the RESTART tape rather than the cards at a later time.

Execution of the SAVE procedure several times during one run causes the latest status to be preserved on tape.

Whenever a call for SAVE is executed, any information written on tape by previous calls for SAVE is overlaid by the new information being written. When restarting a run by means of the RESTART procedure, care must be used in the sequence of control program statements. Any statements that modify the communication region (CR) must appear after the call for RESTART, since execution of the RESTART procedure initializes the $C R$ to the status at the time the problem was saved. For this reason, it is recommended that the CALL RESTART statement be placed immediately after the calls for DEVICE and for ATTACH.

## MULTIPLE ATTACHMENTS OF RESTART TAPE

It is sometimes desired to use different tapes for RESTART and SAVE. In this case, it is permissible to ATTACH the RESTART file several times as in the following sequence.

```
CALL DEVICE('MATRIXIN',TAPE,'F')
CALL DEVICE('MATRXOUT',TAPE, 'G')
CALL ATTACH(RESTART, 'MATRIXIN')
CALL RESTART
    \vdots
CALL ATTACH(RESTART, 'MATRXOUT')
    CALL SAVE
```

In the above sequence, the problem is restarted using RESTART tape F; following the call for RESTART, tape $G$ is attached to the RESTART file, so that any information saved during subsequent calculations is written on that tape, rather than on tape $F$.

## APPENDIX A. PARAMETRIC PROGRAMMING

This appendix describes three posf-optimal procedures, RANGE, PARAOBJ, and PARARHS, that are available as options to FMPS. An outline of each is given in Table 23 below. Note that post-optimal procedures are available only in the linear programming operating mode.

Table 23. Parametric Programming Procedures

| Procedure | Purpose |
| :--- | :--- |
| RANGE | Generates and outputs an <br> analysis of the current LP <br> solution. |
| PARAOBJ | Performs parametric pro- <br> gramming on the objective <br> row after optimality. | | Performs parametric pro- |
| :--- |
| gramming on the RHS after |
| primal and dual optimality. |.

After an optimal solution has been obtained, the procedures RANGE, PARAOBJ, and PARARHS may be used to determine the sensitivity of the optimal solution in regard to RHS and objective function values. The RHS range computes how far the activity level of a given nonbasic variable can be changed in either direction, while holding all other nonbasic variables at the current activity level, before the optimal basis for the current RHS will change. The COST range computes how far the cost coefficient of a given basic variable can be changed in either direction, while holding the cost coefficients of all other variables constant, before the optimal basis for the current cost coefficients will change. Parametric programming is an extension of RANGES, and is used to determine how the optimal basis will change when more than one coefficient moves over a special range of values. Before performing parametric procedures, a change row or column must have been defined. Depending upon which parametric procedure is requested, a matrix cost row or RHS is changed continuously until the specified maximum change has been obtained. The cost row or RHS is called a composite because it consists of the original elements plus a given amount of a change element. The function of parametric procedures is to retain optimality and feasibility as the problem continues to change.

RANGE The RANGE procedure generates and outputs an analysis of the current LP solution.

RANGE will produce two different types of reports depending upon the optional parameters. The first parameter, BASIC, generates a report of 11 columns for the variables currently basic or at intermediate levels. The
second parameter, NONBASIC, creates another report of 12 columns for the variables currently nonbasic or at limit levels. Tables 24 and 25 list column numbers as well as headings in each level. If neither BASIC nor NONBASIC is specified, both outputs will be given.

The optional parameters available to RANGE are given below.

| Parameter | Explanation <br> Indicates that output is to in- <br> clude only those columns cur- <br> rently in the basis. |
| :--- | :--- |
| NONBASIC | Indicates that output is to in- <br> clude only those constraint <br> rows whose slack variables are <br> currently nonbasic and those <br> columns currently nonbasic. |
| ROWS | Indicates that row selection <br> or exception list parameter <br> follows. |
| COLS | Indicates that column selection <br> or exception list parameter <br> follows |
| EXCEPT | Indicates that following list <br> reference is for exception list. |
| LISTC | Used in connection with ROWS <br> to specify row selection or ex- <br> ception list. |
| Used in connection with COLS <br> to specify column selection or <br> exception list. |  |

The following interrupts may occur within RANGE.

| Interrupt | Causes |
| :--- | :--- |
| KMAJER | No matrix defined. |
| KMINER | 1. Invalid parameter. <br> 2. Illogical combination of <br> parameters. |
| KINV | 1. Solution is primal or dual <br> feasible. Typical response <br> to this interrupt would be: <br> CALL INVERT <br> CALL OPTIMIZE <br> RETURN |
| KIOER | Irrecoverable input/output error. |

Table 24. Output for Basic Variables

| Column | Heading | Description of Information in Column |
| :---: | :---: | :---: |
| 1 | NUMBER | The internal number associated with the BASIC variable. |
| 2 | NAME | Name of the basic variable. |
| 3 | AT | A two-character code indicating the status of the BASIC variable. |
|  |  | Code Meaning |
|  |  | BS Basic variable |
|  |  | ** Separator used to distinguish slack from nonslack |
| 4 | ACTIVITY | Activity of the basic variable. |
| 5 | INPUT COST | Input cost specified by the user. |
| 6 | LOWER PROCESS | The name of the variable that would change its status (enter the basis) if the cost coefficient of the basic variable in column 2 was decreased by more than the amount in column 7. |
| 7 | LOWER <br> INCREMENT | The maximum amount of cost coefficient decrease for the basic variable in column 2 which would not change the status of any variable. If the cost coefficient is changed beyond this amount, the variable in column 6 would change its status. |
| 8 | LOWER AT | The current status (at upper limit [UL] or at lower limit [LL]) associated with the process specified in column 6. |
| 9 | UPPER PROCESS | The name of the variable that would change its status (enter the basis) if the cost coefficient of the basic variable in column 2 was increased by more than the amount in column 10. |
| 10 | UPPER INCREMENT | The maximum amount of the cost coefficient increase for the basic variable which would not change the status of any variable. If the cost coefficient was changed beyond this amount, the status of the variable in 9 would be changed. |
| 11 | UPPER AT | The current status (at upper limit [UL] or at lower limit [LL]) associated with the variable in column 9. |

Table 25. Output for Nonbasic Variables

| Column | Heading | Description of Information in Column |
| :---: | :---: | :---: |
| 1 | NUMBER | The internal number associated with the NONBASIC variable. |
| 2 | NAME | Name of the nonbasic variable. |
| 3 | AT | A two-character code indicating the status of the NONBASIC variable. |
|  |  | Code Meaning |
|  |  | EQ Artificial variable. |
|  |  | UL Row at upper limit for slack variable, or column at upper limit for nonslack variable. |
|  |  | LL Row at lower limit for slack variable, or column at lower limit for nonslack variable. |
|  |  | Separator to distinguish slack variables from nonslack variables. |
| 4 | LOWER LIMIT | The lower bound on row activity for slack variables. The lower bound on column activity for nonslack variables. |
| 5 | UPPER LIMIT | The upper bound on row activity for slack variables. The upper bound on column activity for nonslack variables. |
| 6 | $\begin{aligned} & \text { REDUCED } \\ & \text { COST } \end{aligned}$ | The DJ of the variable in column 2. |

Table 25. Output for Nonbasic Variables (cont.)

| Column | Heading | Description of Information in Column |
| :---: | :---: | :---: |
| 7 | LOWER PROCESS | The name of the basic variable that would leave the basis if the original activity level of the variable in column 2 was decreased beyond the amount in column 8. |
| 8 | LOWER INCREMENT | The maximum amount of original activity decrease of the variable in column 2 which would not change the status of any variable. If the activity level decreased beyond this amount, the basic variable in column 7 would leave the basis. (The lower limit of the variable is ignored.) |
| 9 | LOWER AT | A two-character code indicating the status at which the BASIC variable in column 7 would leave the basis. |
| 10 | UPPER PROCESS | The name of the basic variable that would leave the basis if the original activity level of the variable in column 2 decreased beyond the amount in column 11. |
| 11 | UPPER <br> INCREMENT | The maximum amount of original activity increase of the variable in column 2 which would not change the status of any variable. If the activity level was increased beyond this amount, the basic variable in column 10 would leave the basis. (The upper limit of the variable is ignored.) |
| 12 | UPPER AT | A two-character code indicating the status at which the BASIC variable in column 10 would leave the basis. |
|  |  | Code <br> Meaning |
|  |  | UL Variable leaves basis at upper limit. |
|  |  | LL Variable leaves basis at lower limit. |

PARAOBJ The PARAOBJ procedure is used to perform parametric programming on the objective row after an LP problem has reached optimally. From any LP program a series of related problems can be defined by replacing the objective row with the original row plus a multiple of a change objective row. This multiple, FTHETAC, is the parameter commonly known as THETA. In PARAOBJ, each value of FTHETAC defines a different problem with different cost coefficients. The function of this procedure is to trace the whole series of solutions, varying FTHETAC from zero up to a maximum parameter of FTHETACM defined by the user. FTHETAC is gradually increased while the solution is kept primal and dual feasible by changing the basis when necessary. Solution printout may be obtained optionally at a basis change or at a chosen interval of FTHETAC.
PARAOBJ produces an iteration lob at each basis change which is identical to that of OPTIMIZE with the exception of the THETA column which represents the current value of the parameter.
The following parameters must be defined, in addition to those parameters requested by OPTIMIZE procedure, before PARAOBJ procedure is called.

| $\frac{\text { Parameter }}{\text { APOBJ }}$ | $\frac{\text { Explanation }}{\text { Contains name of objective func- }}$tion row. |
| :--- | :--- |
| FTHETAC | Initial value of THETA for PARAOBJ. |


| Parameter <br> FTHETACM | Explanation <br> Maximum value of THETA for <br> PARAOBJ. |
| :--- | :--- |
| FTHETACP | The incremental value for THETA <br> during PARAOBJ for which the <br> KSOLTN interrupt will occur. |

PARAOBJ will terminate at one of the following three conditions.

1. The parameter is at its maximum value of FTHETACM. The message
```
'MAXIMUM OF PARAMETER OF THETA AT
. XXXXXX'
```

is printed and FTHETAC is set to FTHETACM.
2. The problem becomes unbounded at the current value of the parameter and no further basis change will occur. The message

```
'PREMATURE MAXIMUM OF THETA AT
.XXXXXX'
```

is printed and FTHETAC retains the current value.
3. The parameter has reached a value beyond which it can be increased indefinitely without any basis change to maintain optimality. The message

```
'NO MAXIMUM FOR PARAMETER OF THETA AT
. XXXXXX'
```

is printed and FTHETAC is set to FTHETACM.
The following interrupts may occur within PARAOBJ.

| $\frac{\text { Interrupt }}{\text { KMAJER }}$ | $\frac{\text { Causes }}{\text { 1. AOBJ, ARHS or APOBJ }}$undefined. |
| :--- | :--- |
| KINV | 2. No matrix to parameteriz |
|  | 1. Problem is initially prim |
| or dual infeasible. |  |

KSOLTN $\begin{aligned} & \text { Solution printing is requested. A } \\ & \text { typical response to this interrupt }\end{aligned}$ would be:

## CALL SOLUTION RETURN

| KIOER | 1. Irrecoverable input/output <br> error. |
| :--- | :--- |
| KFREQA | 2. File capacity exceeded. |
| KTIME | User iteration frequency (IFREQA) <br> satisfied. |
| User-specified time increment <br> reached. |  |

PARARHS The PARARHS procedure is used to perform parametric programming on the RHS after a problem has reached primal and dual optimality. From any LP problem a series of related problems can be defined by replacing the RHS with the original RHS plus a multiple of a change RHS. This multiple, FTHETAR, is the parameter commonly known as THETA. In PARARHS each value of FTHETAR defines a different LP problem with a different RHS. The function of this procedure is to trace the whole series of solutions by varying FTHETAR from zero up to a maximum parameter of FTHETAM defined by the user. FTHETAR is gradually increased while the solution is kept primal and dual feasible by changing the basis when necessary. Solution printouts may be obtained optionally at basis changes or at a chosen interval of FTHETAR.

PARARHS produces an iteration log at each basis change which is identical to that of OPTIMIZE with the exception of the THETA column representing the current value of FTHETAR.

The following parameters must be defined before PARARHS is called.

| Parameter | Explanation <br> APRHS |
| :--- | :--- |
| FTHETAR | Initial value of THETA for <br> PARARHS. |
| FTHETARM | Maximum value of THETA for <br> PARARHS. |
| FTHETARP | The incremental value for THETA <br> during PARARHS for which the |
|  | KSOLTN interrupt will occur. |

PARARHS will terminate for one of the following three conditions.

1. The parameter is at its maximum value of FTHETARM. The message

$$
\begin{aligned}
& \text { 'MAXIMUM OF PARAMETER OF THETA AT } \\
& \text {. XXXXXX' }
\end{aligned}
$$

is printed and FTHETAR is set to FTHETARM.
2. The problem becomes infeasible at the current value of parameter and no further basis change can occur. The message.

## 'PREMATURE MAXIMUM OF THETA AT . XXXXXX'

is printed and FTHETAR retains the current value.
3. The parameter has reached a value beyond which it can be increased indefinitely without any basis change to maintain feasibility. The message
'NO MAXIMUM FOR PARAMETER OF THETA AT . XXXXXX '
is printed and FTHETAR is set to FTHETARM.
The following interrupts may occur within PARARHS.

| Interrupt | Causes <br> KMAJER |
| :--- | :--- |
|  | 1. AOBJ, ARHS or APRHS <br> undefined. |
| KINV | 2. No matrix to parameterize. |
|  | 2. Problem initially primal or <br> dual infeasible. <br> dual feasibility due to num- <br> erical error. |

KSOLTN

## Causes

3. Inversion frequency satisfied.

Normal interrupt response for KINV would be:

CALL INVERT
CALL OPTIMIZE
RETURN
Solution printing is requested.
A typical response to this in-
Solution printing is requested.
A typical response to this interrupt would be:

CALL SOLUTION
RETURN

Interrupt

KIOER

## Causes

1. Irrecoverable input/output error.
2. File capacity exceeded.

User iteration frequency (IFREQA) satisfied.

User-specified time increment reached.

## APPENDIX B. FMPS ERROR MESSAGES

## CONTROL LANGUAGE COMPILER DIAGNOSTICS

The following list specifies the error messages that can be produced by the control language compiler at compile time. Any error during compilation precludes execution of the control program. Note that all error lines are prefixed with

ERROR*****.
Computer diagnostics are listed below. Note that in the INVALID PARAMETER message, aaaaaaaa contains the name, in from one to eight characters, of the incorrect parameter.

ILLEGAL STATEMENT
STATEMENT NUMBER MUST BE NUMERIC
ASSIGN STATEMENT MUST REFER TO INTERRUPT CELL REQUIRED FIELD MISSING

THE STATEMENT NUMBER OF A GO TO STATEMENT MUST BE NUMERIC OR KTYPE

ARGUMENT ON LEFT OF EQUAL SIGN MUST BE EITHER USER OR COMMON STORAGE VARIABLE

EQUAL SIGN MISSING
INVALID PARAMETER aaaaaaaa
MISSING LEFT PARENTHESIS
LOGICAL OPERATOR MUST BE ENCLOSED IN PERIODS

ILLEGAL LOGICAL OPERATOR
MISSING RIGHT PARENTHESIS
INVALID PROCEDURE NAME
UNDEFINED STATEMENT NUMBER

## DUPLICATE STATEMENT NUMBER

NOT ENOUGH CORE AVAILABLETO PROCESS THIS MANY STATEMENTS

MISSING TERMINAL QUOTE

## INPUT/OUTPUT ERROR TYPES

The following table describes the input/output error messages that can occur during an FMPS run.

Table 26. Input/Output Error Types
\(\left.$$
\begin{array}{|l|l|}\hline \text { Error Type } & \begin{array}{l}\text { Description }\end{array} \\
\hline \text { 1. } & \begin{array}{l}\text { A file is referenced but no } \\
\text { ATTACH was made. } \\
\text { No DEVICE is attached to } \\
\text { a file. } \\
\text { Device read error. }\end{array} \\
\text { 3. } & \begin{array}{l}\text { Device write error. } \\
\text { Volume of storage for de- } \\
\text { vice exceeded during a } \\
\text { write operation. }\end{array} \\
\text { 5. } & \begin{array}{l}\text { Attempt to write on a file } \\
\text { in read or closed status. }\end{array} \\
\text { 7. } & \begin{array}{l}\text { Attempt to read on file in } \\
\text { write or closed status. }\end{array} \\
\text { 8. } & \begin{array}{l}\text { Attempt to read beyond } \\
\text { written information. }\end{array} \\
\text { 9. } & \begin{array}{l}\text { Dynamic core pointer for } \\
\text { a file buffer points to an } \\
\text { illegal core area. } \\
\text { Undefined type of device, }\end{array}
$$ <br>

i.e., device not DISC or\end{array}\right\}\)| TAPE. |
| :--- |
| Insufficient core available |
| to create even one file |
| buffer. |

# APPENDIX C. FMPS SAMPLE RUNS 

```
J0B 326.SUMD
LIMIT (TIME,90), (LE,1000),(UE,1000),(0,,1000)
ASS!ĞN F:106,(OEVICE,CPAO&)
ASSIGN F:I,(FILE,CLANG),(BIN), (WRITE,ALL) )(CONSEC) , (SEQUEN),
(QUTIN);(RECL,30000) (NEAD,ALL)
ASSIGN F!2,(FILE,UTILI):(BIN):(WRITE,ALL),(KEYED),(EIRECT)O|
OUTIN):(RECL:30000),(READ;ALL)
ASSIGN F:3,(FILE,UTILZ):(BIN):(WRITE,ALL)&(KEYED):(OIRECT)&)
(OUTIN):(RECL,30000),(READ,ALL)
```



```
(OUTIN):(RECL,30000),(READ;ALL)
ASSIGN F:5,(FILE,IVSE) (BINS,(WRITE,ALL),(DIRECT),(KEYED),;
(QUTIN), (RECL,30000):(READ,ALL)
ISSIGN F{G,(OEV{CE,9T):(INOUT),(INSN,026),(BIN),(WRITE,ALL),(SAVE)
ASSIGN FiG,IDEV
DATA
```

THIS IS A COMMENT PPUNCHED C IN CQL II
DEFINE HEADING AND ENTER LDP. MEDE
TITLE SDS SIGMA 5/7 - SAMPLE FMPS L•P. RUN
THIS BENCHMARK HAS BEEN PURPOSELY MADE QUITE COMPGEX TO CEMONSTRATE
MANY OF THE OPTIONS AVA!LABLE IN FMPS. USUALLY, CONTROL PRBGRAMS
ARE MUCH SIMPLER AND THE STANDAKD EPTIDNS ARE USED.
CALL ENTER(LP)
INITIALIZE MAJUR ERRER INTERRUPT VARIABLE
ASSION 300 TO KMAJER
INITIALIZE MINER ERRER INTERRUPT VARIAELE
ASEIGN 300 TO KMINER
SET TIME LIMIT OF 5 MINUTES FREM EXECUTIEN AF THIS STATEMENT
ITIME 5
INITIALIZE TIME=OUT INTERRUPT VARIABLE
ASIGN 45 TO KTIME
SPECIFY FEUR SYMEOLIC UNITS (WORKINT, FILES) ON RAO
CALL DEVICE('FILEI',DISC,'B')
CALL DEVICE('FILEC',DISC, 'C')
CALL DEVICE('FILE3',DISC,ID')
CALL DEVICE('FILE4'ODISCSIEV)
SPECIFY A SYMBOLIC UNIT ON TAPE (LOGICAL NUMEER A)
CALG DEVICE (ITAPEA',TAPE, IF')
ATYACH THE FOUR STANDARC L\&P. EMPS FILES TO THE
PREVIOUSLY DEFINED FOUR SYMBOLIC UNITS (RAO)
CALL ATTACH(MATRIX, 'FILEI')
CALL ATTACH(INVERSE,'FtLEZ')
CALL ATTACH(UTILIEIFILE3')
CALL ATTACHIUTIL2, 'FILE4')
$C$
$C$
$C$
ATTACH THE RESTART FILE TO LOGICAL TAPE A PREVIBUSLY DEFINED
CALL ATTACH\&RESTART, 'TAPEA')
nonnanの
NETE FQR THE ABOVEOMATRIX, INVERSE OUTILIPUTILZ, AND RESTART
ARE INTERNAL FILES WHICH MUST ALWAYS BE ATTACHEC
EXCEPT RESTART IF NO SAVING OR RESTARTING IS PREGRAMMED
SELECT DESIRED INFUT OATA RECQRO ANE SPECIFY PRESLEM NA:E
ADATA " ALLBYS'
APBNAME 'FUSIEN"
$\stackrel{C}{C}$
LOAD INPUT MATRIX FROM CARDS, USING RECORD 'ALLOYS:
$c$
CALL Input
C Call input(fileafilename) would result in searehing inplt file
CALL INPUT(FILEAFILENAME) WOULD RESULT IN SEARCHING INPLT FILE
CALLED FILENAME FBR RECORD ALLOYS AND LOAOING IT AS INPGT MATRIX
IA THIS CASE ONE SHOULD FIRST OEFINE THE FILE AND ATYACH IT
IA THIS CASE ONE SHOULD FIRST OEFINE
BY MEANS OF DEVICE AND ATTACH CALLS.
IDENTIFY RIOHT=MAND-SIDE COLUMN AND COST ROW TO BE USED
ARHS : 'Aloys'
A日BJ : 'Valuél
nomanoma
VARIOUSOPYIONSTODISPLAYMATRIX
dISplay original matrix in standard farmat

CAlL Eupput
DIsplay oricinal matrix in ceoed form
CALL OUTPUT(CODED)
DISPLAY ORIGINAL MATRIX IN ROW ORDER
CALL EUTPUTEBYREWS:
OISPLAY ORIGINAL MATRIX IN COLUMN ERDER
CALL EUTPUT\&BYCOLS)

EXAMPLEOFSOLUT10ヘ

Varlous Initializations for solution (optimize)
SET TO INVERT NG LESS FREQUENTLY THAN AT INTERVALS EF ITERATIONS
IFREOI *
ASSIGN WEIGHT OF $1 \cdot 0$ TO OBJECTIVF ROW
(1.O RESULTS IN MINIMIZATION: -IVC IN MAXIMIZATION)

FOBJWT : 1.0
SET TO PRINT ITLRATION LOG EACH ITERATION (PRINTER BUTPLTI


CALL SOLUTION
FTHETARM - 10.0

```
C SET TO PRINT SOLUTION AT THETA INTERVALS OF 1.O
FTHETARP 1:O
C IDENPIFY RHS PARAMETRIC GLUMN (THE ONE TE EE MULTIPLIED BY THETA)
C EXRHS EOELPRODC'
C EXECUTE PARAMETRIC RHS RUN
    CALL PARARHS
    CALL SOLUTION
C
    STEP
C
    PKESERVE PROBLEM STATUS ON RESTART TAPE
    4S CALL SAVE
        TERMINATE RUN
        STOP
        ENTER HERE WHEN INVERSIEN INTERRUPT OCCLURS
    OO CALL INVERT
    RETURN TO PRBCEDURE THAT CAUSED THE KINV INTERRUPT
    RETURN
    ENTER HERE IN CASE UF MAJER OR MINER ERRORS
    OISPLAY COMMUNICATION REGION VARIABLES AND FILE STATUS
    OO CALL CONDITION
    TERMINATE FMPS EXLCLITION
    STOP
    ENTER MERE FOR MINOR ERROR INTERRUPT DURING OPTIMIZE PNASE
    DISPLAY FMPS STATLS
    CALL CONDITION
    OO SAME AS IF TIMLOUT ECCURED
    G0 10 45
    ENTER MERE WHEN SOLLTIBN PRINT-BUT IS REQUESTEC IBASIS CHANGF
    0R SOLUTION PRINTCOUT INTERVAL GF THETA SATISFIED)
    PRINT SOLUTIGN
    600 CALL SOLUTION
    PRINT VALUE OF ITIRATION COUNT
    PRINY VALUE
    WRITE ITCNT 
    RETURN
```

C 700 ENTER HERE IF NUMERICAL ACCURACY CAUSFS INFEASIRILITY DLRING WARAMETRICS
CALL OPTIMIZE
RETURN
C ENO OF CONTROL PROGFAM
END

| $\begin{aligned} & \text { NAME } \\ & \text { REWS } \end{aligned}$ | Alleys |  |
| :---: | :---: | :---: |
| A value |  |  |
| E YIELO |  |  |
| L FE |  |  |
| $L$ MN |  |  |
| 1 CU |  |  |
| $L$ MG |  |  |
| $G$ AL |  |  |
| 1 Sl |  |  |
| N DELEST |  |  |
| COLUMNS |  |  |
| OIN1 | Value | 0.03000 |
| B\N1 | YJELD | 1.00000 |
| 8tN1 | FE | 0.15000 |
| OIN1 | Cu | 0.03000 |
| BIN1 | MN | 0.02000 |
| EINI | Mg | 0.02000 |
| BIN2 | ${ }^{\text {A }}$ | 0.70000 |
| B\N1 | S! | 0.02000 |
| BIN1 | DELCST | -10.0 |
| BIN2 | value | 0.08000 |
| BIN2 | YIELD | 1.00000 |
| B!N2 | FE | 0.04000 |
| BIN2 | Cu | 0.05000 |
| BIN2 | HN | 0.04000 |
| -1N2 | Mg | 0.03000 |
| 61N2 | ${ }_{6}$ | 0.75000 |
| BIN2 | Sl | 0.06000 |
| B[n] | value | 0.17000 |
| B [ N3 | YIELO | 1.00000 |
| BIN3 | FE | 0.02000 |
| BlN3 | Cu | 0.08000 |
| Bin3 | MN | 0.01000 |
| BIN3 | ${ }^{\text {A }}$ | 0.80000 |
| OIN3 | SI | 0.08000 |
| BIN4 | Value | 0.12000 |
| BIN4 | YIELO | 1.00000 |
| BIN4 | FE | 0.04000 |
| BIN4 | Cu | O.0ざర00 |
| BIN4 | Mn | 0.02000 |
| BIN4 | ${ }^{4}$ | 0.75000 |
| BiN4 | S1 | 0.12000 |
| 81N5 | value | 0.15000 |
| 8 IN5 | YIELO | 1.00000 |
| 8IN5 | FE | 0.02000 |
| B!N5 | Cu | 0.06000 |
| BIN5 | MN | 0.02000 |
| BIN5 | MG | 0.01000 |


| BIN5 | AL | 0.80000 |
| :--- | :--- | :--- |
| BINS | SI | 0.02000 |
| ALUM | VALUE | 0.21000 |
| ALUM | YIELD | 1.00000 |


| alum | FE | 0.01000 |
| :---: | :---: | :---: |
| ALUM | Cu | 0.01000 |
| Alum | AL | 0.97000 |
| Allim | S! | 0.01000 |
| SILCON | VALUE | 0.38000 |
| SILCON | YIELD | 1.00000 |
| SILCON | FE | 0.03000 |
| silcen | S: | 0.97000 |
| RWS |  |  |
| ALOYg | YIELD | 2000.00000 |
| Aloys |  | 60.00000 |
| ALOYg | Cu | 100:00000 |
| AL̇OY1 | MN | \$0,00000 |
| ALOYI | Mg | 300.00000 |
| ALeri | Al | 1500:00000 |
| ACByd | SI | 300.00000 |
| OELPRODC | YIEGD | 20000.0 |
| Ranges |  |  |
| ALI | S】 | 50.00000 |
| BOUNDS |  |  |
| UP PRODI | Bind | 200.00000 |
| UP PRODI | BIN2 | 2500.00000 |
| L6 Prodi | BiN3 | 400.00000 |
| UP PREDI | BiN3 | 800.00000 |
| Lo Prodi | Bin4 | 100.00000 |
| UP PRODI | BiN4 | 700.00000 |
| UP PRODI | Bins | 1500.00000 |
| Endata |  |  |

```
11:34%FES 12,169 1D=0000
J0B 3̈26,SOMO
LMIT (TIME,90),(L0,1000),(U#,1000)%(00,1000)
ASSION F:106, (DEVICE,CPAO4)
ASSIGN FII,(FILE,CLANG)P(BIN),(WRITE,A(L),(CONSEC),(SEOUEN),:
(OUTIN),(RECL)30000), (READ;ALL)
ASSION FIZ;(FILE,UTILI)P(BIN):(WRITE,ALL)P(KEYED);(DIRECT).)
(OUTIN),(RECL,30000),(READ,ALL)
ASSIGN F:3&(FILE,ÜTILZ),(BIN)&(WRITE,ALL),(KEYED),(CIRECT),)
(OUTIN),(RECL. 30000),(READ, ALL)
ASSIGNF:4,(FILE,MTRX)&(BIN),(WRITE,ALL),(DIRECT),(KEYEO),,)
(OUTIN),(RECL,30000),(READJALL)
ASSIGN F:5:(FILE,IVSE)&(EIN),(WRITE,ALL),CDIRECT),(KEYED):,
(OUTIN), (RECL,30000), (READ,ALL)
ASSION FIGSIOEVICE,GT)=(INEUT),(INSN,O26),(BINI)(WRITE,ALL),(SAVE)
RUN (LMNDFMPS)
```

| 12FEb69 |  |
| :---: | :---: |
| INTERNAL |  |
|  | THIS is a COMMENT (PUNCHED C in COL 1) |
|  | Define heading and enter lapg mede |
| $1 *$ | TITLE SOS SIGMa 5/7 - Sample frps lop. Run |
|  | this benchmark has been purposely made cuite complex to demonstrate many bf thl uptigns ayallable in fmps. usually, control programs are much simpler and the standard options are used. |
| $2 . *$ | CALL ENTER(LP) |
|  |  |
|  | INITIALIZE MAjer errer interrupt variable |
| $3 *$ | ASSIGN 300 TO KMAJER <br> INITIALIZE MINOR ERRGR INTERRUPT VARIABLE |
|  | ASSIGN 300 TO KMINER |
| 5 * | SET TIME LIMIT OF 5 MINUTES FR日M EXECLTION OF THIS STATEMENT |
|  | Initialize timeabut interrlipt variable |
| $6 *$ | ASSIGN 45 TO KTIME |
|  | SPEEIfy feur symbulic units fherking filesi on rau |
| $7 * *$ | CALL DEVICE!'FILE!!,OISC, 'b') |
| $8 * *$ | CALL DEVICE('FILE2', DISC, 'C') |
|  | CALL DEVICE (FILE3',DISC, 'C') |
| 10** | CALL DEVICE(?FILE4: DISC, 'E') |
|  |  |
|  | specify a symbolic unit on tape ilogical numgen a) CALL UEVICE('TAPEA'stAPE,'F') |
| 11 ** | attach the four standard lop. fmps files to the PREVIQUSLY DEFINED FBUR SYMBOLIC UNITS ;RAD: |
|  |  |
| $12 * *$ | CALL attachematrix, 'FILEI') |
| 13 * | CALL attach(inverse, IFILES') |
| 1400 | CALL ATPACHPUT!LIE'F!LE3') |
| 15 $=$ |  |
|  | attach the restart file to legical tape a previ.jusly defined |
| $16 * *$ | call attach(restart, 'tapea') |

C NOTE FER THE ALIOVE-MATRIX,INVERSESUTILI,UTILE, ANO RESTART
ARE INTERNAL FILES WHICH MUST ALGAYS BE ATTACMED
ARE INTERNAL FILES WHICH MUST ALHAYS BE ATTACRED
EXCEPT RESTART IF NO SAVINC OR RESTARTINO IS PIRGRAMMEU
SELECT DLSIRED INPUT DAIA RECARD AND SPECIFY PROBLEM NAME


|  | ASSIGN WLIGHT GF 1.0 TO OBJECTIVE REW (1.0 RESLLIS IN MINIMIZATIUN, -1.0 in MAXIMIZATION) |
| :---: | :---: |
| 27 ** | FORJWT : 2.0 |
|  | SET TO PRINT ITERATIEN LEG EACH [TERATIBN (PRINTER OUTPUT) |
| 28 ** | $\begin{aligned} & \text { ILOGP : I } \\ & \text { SET PRICING TO BE MADE FROM GROUPS OF TWE PRGFITABLE VARIABLES } \end{aligned}$ |
| 29 * | INCAND 2 |
|  | SET INVERSIEN INTERRUPT CELL PO TRANSFER TE STATEMENT 200 |
| 30 * | ASSIGM 200 TO KINV |
| 31 ** | now set miner error interrupt to cause creation of restart tape IF IT WERE TO OCCUR DURING THE OPTIMIZE PHASE ASSIGN 400 TO KMINER |
| $32 *$ | SET OPTIMIZE TO DISREGARD BPTIMALITY DURING PHASE GNE FCMPDJ 0.0 |
|  | SOLVE LOP. MATRIX |
| $33 *$ | CALL OPTIMILE |
|  | PRESERVE BASIS OF GPTIMAL SOLUTION |
| 34. | CALL SAVE |
|  | PRINT SOLUTION VALUES (COLUMNS AND REwS) |
| $35 *$ | CALL SOLUTION |
|  | PRINT PRIMAL AND DUAL ERRORS |



| INTERNAL | STATEMENT | NLMBEF | 1 | TIME | - 11:35 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| INTERNAL | STATEMENT | NUMBER | 2 | TIME | - 11135 |
| INTERNAL | STATEMEN ${ }^{\text {S }}$ | NUMBER | 3 | TIME | - 11:35 |
| INTERNAL | STATEMEN ${ }^{\text {P }}$ | NUMBER | 4 | TIME | - 11:35 |
| INTERNAL | STATEMEN ${ }^{\text {P }}$ | NUMBER | 5 | TIME | - 11:35 |
| INTERNAL | STATEMENT | NUMBER | 6 | TIME | - 12135 |
| INTERNAL | STATEMEN ${ }^{\text {P }}$ | NUMBER | 7 | TIME | - 11:35 |
| INTERNAL | STATEMENT | NUMBER | 8 | TIME | - 11135 |
| INTERNAL | STATEMENT | NUMEER | 9 | TIME | - 11135 |
| INTERNAL | STATEMENT | NUMBER | 10 | TIME | - 11835 |
| INTERNAL | STATEMENT | NUMAER | 12 | IIME | -11835 |
| INTERNAL | STATEMENT | NUMEER | 12 | TIME | -11135 |
| INTERNAL | STATEMENT | NUMBER | 13 | TIME | - 11:35 |
| INTERNAL | STATEMENT | NUMEER | 14 | TIME | - 11835 |
| INTERNAL | STATEMENT | NUMBER | 15 | IIME | - 11:35 |
| INTERNAL | STATEMENT | NUMBER | 16 | TIME | -11835 |
| INTERNAL | STATEMENT | NUMBER | 17 | TIME | - 11:35 |
| INTERNAL | STATEMENT | NUMBER | 18 | IIME | - 11:35 |
| INTERNAL | STATEMENT | NUMBER | 19 | TIME | -11:35 |

GUFFER SIZES (BYTES) ARE O MATRIX = 648 INVERSE 10240







$12 F E 669$ SDS SIGMA $5 / 7$ - SAMPLE FMPS LOP. RUN $\quad$ 0. 13.2.

| 1 - ROWS PrImal-DUAL OUTPUT |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| numbern | -0LABEL. | AT | . . .ACTIVITY... | Slack activity | - -lower limit. | - - Upper limit. | -DUAL | ACtivity | - 0 INPUT CEST.* | -REDUCED COST. |
| 1 | value | FR | 296.216553 | -296.え16797 | NONE | NENE |  | 1.000000 | 1.000000 | 0.000000 |
| 2 | YIELD | EO | 2000:000000 | 0.000000 | 2000.000000 | 2000.000000 |  | 0.013596 | 0.000000 | 0.083696 |
| 3 | FE | $U$ | 60.0000000 | 0.0000000 | NONE | ${ }^{30} 0.000000$ |  | E.56̂Ez3i | B. ${ }^{\text {cesogai }}$ | 20 \%extai |
| 4 | MN | UL | 400000000 | 0.000000 | NONE | 40.000000 |  | 0.544404 | 0.000000 | 0.644404 |
| 5 | cu | BS | 83.967499 |  | NanE | 1000.0000000 |  | 0.000000 | 0.000000 | 0.000000 |
| 6 | MG | 85 | 19,9602 ${ }^{\text {1 }}$ | 10.039711 | 1500.00 NENE | 30.000000 |  | 0.000060 | 0.00000 | 0.000000 |
| 7 | AL | 4 | 1500:000000 | 0,000000 | 1500.000000 | -rehe |  | -0.251986 | 0.000000 | 0.251986 |



IN NON-COMPLETELY TRIAHGULARIZED PART, OF THESE 3 WHERE NOT TRIANGULARIZED ANO 0 WERE REJECTED FAR TOO SMALL A PIVOT: mathix po be Inverted had 9 cols and 38 elements. Inverse has 10 cols and 39 elements.

GTER MS FOR INVERT
MEGAIIVE OS COUNT - 0 SELECTED 0 VARIABLES EEST DS - $0.0000000+00$
OPTIMAL SELUTION. OBJECTIVE VALUE 0 0.289751360*03


| ITER! | BUM OF INF | NINF | -8JEC | VALUE | $\underline{V}$-IN | Move | Reduced cest | ACPIVITY | veout | meve | Pivet | THETA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 13 | 0,00000000D+00 | 0 | 0.2574814 | 80+03 |  | L-8 | $0.104619570 \times 60$ | 0.133981070402 | 12 | $8=1$ | 0.244849340402 | 0.625185190-01 |
| 14 | 0,000000000+00 | 0 | 0.2457446 | S80+03 |  | $6=8$ | 0.370370370-01 | 0.876808510402 |  | $\mathrm{B}=\mathrm{U}$ | -0.494348570400 | $0.700106380 \cdot 01$ |
| Ne mãx | IMUM PARAMETER | AT THET | TAE 0.700 | 1060 0 |  |  |  |  |  |  |  |  |
| INTERN | L STATEMENT NU | MBEP | I5 TIME | - 1 |  |  |  |  |  |  |  |  |

$12 F E B 69$ SOS SIGMA S/7 - SAMPLE FMPS L.P. RUN $\quad$ o. 20. 1.

JOENTIFIER SECTION


|  | 12FE869 | SOS SIGMA 5/7. SAMPLE FMPS LOPP RLN | 0. 20. |
| :---: | :---: | :---: | :---: |


| SECTION | $\underline{1}$ - ROws |  |  | PRIMAL-dUAL | autput |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| numaer | - LABEL. | ${ }^{\text {AT }}$ | - . -ACTIVITY... | SLACK ACTIVITY | OPLOHER LIMIT. | : SUPPER LIMIT: $^{\text {L }}$ | - DUAL ACTIVITY | :-INPUT Cest.: | -REDUCED CAST: |
| 1 | VALUE | FR | 385,765869 | -385.766113 | NONE | - NONĖ | 1.000000 | 1.000000 | 0,000000 |
| 2 | YIELO | EO | 2000\%000000 | 0.000000 | 2000.000000 | 2000.000000 | -0.270000 | 0.000000 | -0.270000 |
| 3 | FE | UL | 60:000000 | 0.000000 | NONE | 60.000000 | 6.308510 | 0.000000 | 6.308510 |
| 4 | MN | 88 | 12-170213 | 27. ${ }^{\text {a }}$ 29773 | NONE | 40.000000 | 0.000000 | 0.000000 | 0,000000 |
| 5 | CU | BS | 56:468079 | 43.531906 | NENE | 100.000000 | 0.000000 | 0.000000 | 0.000000 |
| 6 | Mo | BS | 5:035106 | 24.314886 | NONE | 30.000\% 0 \% | 0.000000 | ס.0̄00000 | 0.008006 |
| 7 | ${ }^{\text {AL }}$ | 8 S | \$587-680634 | 87.680847 | 1500.000000 | - NGNE | 0.000000 | 0.000000 | 0.000000 |
| , | \$1 | 4 | 250,000000 | 50,000000 | 250.000000 | 300.000000 | -0.3085II | 0.000000 | -0.305515 |
| 9 | delcst | FR | -2000:000000 | 1999.999736 | NENE | NONT | 10.000000 | \$0¢. 0 ర00000 | 0.000000 |

S2FEB69 SOS SIGMA 5/7 = SAMPLE FMPS L•P. RUN


```
NEGATIVE DJ COUNT = O SELECTED OQ VARIABLES gEST DJ = 0.0000000*00
```

OPTIMAL SOLUTION, OBJECTIVE VALUE O O:317018770.03
INTEFNAL STATEMENT NUMEER 67 TIME : $11: 36$



IDENTIFIER SECTION


| SECTIEN | 1 - Rens |  |  | PRIMAL-DUAL | sutput |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| mumber | - olabel. | AT | ...oACTIVITY... | Slack activity | -.LOWER LIMIT. | - -UPPER LIMIT* | - DUAL | activity | -.InPUT COST.- | -REDUCED COST: |
|  | value | FR | 490.441406 | -890.442650 | NenE | -- noñ |  | 1.000000 | 1.000000 | 0.000000 |
| 8 | YJELD | EO | 2000.000000 | 0.000000 | 2000,000000 | 2000.000000 |  | -0.270000 | 0.000000 | -0.270000 |
| 3 | FE | U | 60:000000 | 0.000000 | NONE | 60.000000 |  | 6.308510 | 0.000000 | 6.308510 |
| 4 | MN | 88 | 17:521271 | 22.\$78714 | NONE | \$0.000000 |  | 0.000000 | 0.000000 | 0.000000 |
| 5 | Cu | 85 | 100,000000 | 0.000000 | NONE | 100.000000 |  | 0.000000 | 0.000000 | 0.000000 |
| 6 | H6 | BS | 5:730638 | 24. 239354 | NONE | 30.000000 |  | 0.000000 | 0.000000 | 0.000000 |
| 7 | AL | BS | 3905:1594Ė4 | 2405.159424 | 1500.000000 | - $\mathrm{NONE}^{\text {N }}$ |  | 0.000000 | 0.000000 | 0.000000 |
| 8 | SI | 4 | 250,000000 | 50.000000 | 250,000000 | 300.000000 |  | -0.3085! | O.000000 | -0.3083II |
| 5 | DELCST | FR | 00:0000000 | $0.0 \overline{0} \overline{0} 0 \overline{0} 0$ | NONE | Ṅ̄̇NE |  | 0.000000 | $\overline{0} \cdot \overline{0} \overline{0} 00000$ | 0.0̄0̄0000̄ |

12FEB69 SDS SIGMA 5/7 - SAMPLE FMPS L.P. RLUN $C$. 22. 3.

12FEG69 SOS SIGMA S/7 - SAMPLE FMPS LOP. RUN 23. 1.

INTERNAL STATEMENT NUMBER 53 TIME $11: 36$ EXIT*

| TETAL JOB TIME | 1.30 |
| :---: | :---: |
| PROCESSOR EXECUTION TIME |  |
| PRACESSOR I/O TIME | .77 |
| PROCESSER OVERHEAD TIME | .07 |
| USER EXECUTION TIME | . 48 |
| USER 1/8 TIME | . 52 |
| USER GVERHEAD TIME | .75 |
| - OF Cards reao | 316 |
| - OF CARDS PUNCHED |  |
| - EF PROCESSER PAGES OUT | 2 |
| - of user pages uut | 35 |
| - Of diagnostić pages out | 0 |
| - OF SCRATCH TAPES USED |  |
| - of Save tapes used | - |
| * OF DISK READS AND WRITES | 1594 |
| - OF DIEC READS AND WRITES | 2957 |
| TEMPORARY DISC SPACE LSED | 17 |
| PERMANENT DISC SPACE LSED |  |

[^1]```
ASSIGN F:106, (DEVICE,CPAO4)
ASSIGN F:IO(FILE,CLANG),(BIN):(WHITE,ALL),(CONSEC), (SEQUEN), )
(BUTIN),(KECL,30000)/(READ,ALL)
ASSIGNF:R,(FILE,UTILI)P(BIN)P(WRITE,ALL),(KEYEDI,(DIRECT),&
(OUTIN):(RECL; 30000), (READ,ALL)
ASSIGN F:30(FILE,UTILZIP(BIN)=(WRITE,ALL),(KEYED),(OIRECT),)
(OUTIN):(RECL,30000):(READ,ALL)
ASSIGN FI4&(FILE,MTRX)O(UIN),(WRITE,ALL),(DIRECT),(KEYEO),)
(OLTIN), (RECL,30000), (READ.ALL)
ASSIGN F:S,(FILE,IVSE),(EIN),(WRITE;ALL):(OIRECT),(KEVED),I
(OUTIN),(RECL,30000), (READ,ALL)
(GUTIN),(RECL:300
CATA
```

```
Ć define mEADing and enter seperable prggaamming mode
    TITLE NON-LINEAR PRUBLEM NO 6
    CALL ENTER(SEP)
C
    CALL DEVICE('FILEI',DISC,'8'
    CALL DEVICE('FILE2I,DISC,IC')
    CALL DEVICE(FFILE3',DISCPIO
    CALL DEVICE''FILEHI,DISC,'E')
nanon
    ATtACH the fOUR STANDARD fupg FILES to the four
    PREYIZUSLY DEFINED SYM゙BELIC UNITSIRADI.
    CALL ATTACH(MATRIX.!FILE!')
    CALL ATTACH(INYERSE; FILEZI)
    CALL ATPACH(UTIGIOIFILEJ!)
    CALL ATTACH(UTILZa'FILE4';
C
    ASSIGN 100 t0 kmAjER
    ASIGN 200 t0 KIOEP
    ASION 300
    ABSIGN 400 to KUBS
    ASSION 500 TO KINV
c
C
call imput
C
    ADATA : 'NLPSTDOI'
OENTIFY RIGHT-HAND-SIDE COLUMN AND COST ROW TO BE USED
AOBJ :O8JT :
C SET TO INYERT NO LESS FREOUENTLY THAN AI INTERVALSS OF
    5O ITERATIONSINOTL: AUTOMATIC INVERT ON-TIME IS BY DEFAULT
    IN OPERATIONO).
    -2*****
    NGTE: TO TURN OFF THE AUTOMATIC INVERT ON TIME, THE FOLLOAING
        STATEMENT SWBULD BE USED.
        STATEMEN
    -****
        IFREQI - 50
C INITIALIZE ITERATION LOGGING FREQLENCY TO PRINT EVERY ITERATION
```

ILEGP : 1
C SPECIFY MINIMIZATION
F08, 1.
$C$ SOLVE SEPERABLE MATRIX
CALL OPTIMIZE
$C$
$C$
$C$
DISPLAY Problem selution
CALL SOLUTIEN
STOP
C ENTER HERE FOR MAJOR ERROR CONDIIIONS
100 CALL CONOITION
STOP
C ENTER HERE FOR I/O ERROR CONOITION
200 CALL CONDITION
STEP
$C$
$C$
$C$
ENTER MERE FER NO FEASIBLE SOLUTION CONOITION
300 CALL CONDITION
C ENTER HERE FOR UNBOLNDED SOLUTION CENOITION






| 6526 | Rows | 2026499 |  |
| :---: | :---: | :---: | :---: |
| 6527 | Rew1 | $3.0{ }^{-}$ |  |
| 6527 | Rews | 2.30245 |  |
| 6528 | Rew 1 | 3.0 |  |
| 6528 | Rew5 | 2.18651 |  |
| 6529 | Rewi | 3.2 |  |
| 6 ¢S29 | news | 2.21698 |  |
| 6530 | Rewi | 3.2 |  |
| 6530 | Row 5 | 2̀.10928 |  |
| 6\$31 | Rewi |  |  |
| 6531 | Rew5 | 2.07295 |  |
| 6 6332 | Rowt | 3.0 ¢ |  |
| 6 632 | Row5 | 1.91984 |  |
| 6533 | Row 1 | 3.0532 |  |
| 6533 | Rews | 1.75590 |  |
| s7bound | 'marker' |  | 'SEPGRG' |
| 751 | Rowe | 1.9973 |  |
| 751 | Row6 | 1.97803 |  |
| 752 | Rews | $2.7{ }^{\circ}$ |  |
| 75 | Row6 | 2.62592 |  |
| 75 | Rewa | $2 \cdot 6$ |  |
| 753 | Row6 | 2.52883 |  |
| 754 | Rew2 |  |  |
| 75* | Rew6 | $2 \cdot 38965$ |  |
|  | Rewe | 2.8 |  |
| 755 | Rew6 | 2.2゙6499 |  |
|  | Rowz | 3.0 |  |
| 756 | Rew6 | 2.30245 |  |
| 757 | Rewr | $3.0{ }^{\circ}$ |  |
|  | Row 6 | 2.15 3651 |  |
| 758 758 | Rown Rewt | 302 ${ }^{\text {a }}$ 2 21698 |  |
| 759 | Rowa | 3. ${ }^{\text {c }}$ |  |
| 759 | Rew6 | E.10928 |  |
| 7510 | Row2 | 3.3 |  |
| 7510 | Raw6 | 2007295 |  |
| 7511 | Rews | 3.2 |  |
| 7511 | ROW6 | 1.91984 |  |
| 7512 | Rowz | 3.0532 |  |
| 7512 | Rew6 | 1.75590 |  |
| 7513 | Rew? | 2.8308 |  |
| 7513 | Rew6 | 1.56692 |  |
| 7514 | Rew? | 3.116 |  |
| 7514 7515 | Row6 | 1066182 |  |
| 7515 | Rew6 | 3.54204 |  |
| 7516 | Rew2 | 3.0 |  |
| 7516 | Rews | 1.48925 |  |
| 7517 | Row2 | 3.0 |  |
| 7517 | Rew6 | 1.43978 |  |
| 7518 | Row2 | 3.0 |  |
| 7518 | Row6 | 1.39358 |  |
| 7519 | Rowz | 3.0 |  |
| 7519 | Row6 | 1.35025 |  |
| 7520 | Rew |  |  |
| 7520 | Rew6 | 1.30953 |  |
| 7 7S2 | Rewa | 3.0 |  |
| $7 \$ 21$ | Rew6 | 1.25719 |  |
| 7522 | Row2 | 300 |  |
| 7522 | Row6 | 1.23505 |  |
| 7523 | Rowz | 3.0 |  |
| $7{ }^{7} \mathbf{2} 2$ | Rew6 | 1.20099 |  |
| 7524 | Rewz | 3.0 |  |
| 7524 | Rew6 | 1.16857 |  |
| 7 7S3 | Rewa | $3.0{ }^{\circ}$ |  |
| 7525 | Row6 | 1.13796 |  |
| 7526 | Row | 3.00 |  |
| 7526 | Rew6 | $\frac{1}{3} .108080$ |  |
| 7827 | Rew6 | i.00128 |  |
| 7528 | Rowz | ${ }_{3}{ }^{\circ} 0^{-}$ |  |
| 7528 | Rew6 | 1.05502 |  |
| 7529 | Rew? | 3.0 |  |
| 7529 | Rew6 | $1.03000$ |  |
| 7530 | Rewt | 3.05 |  |
| 7530 | Row6 | \$.00613 |  |
| 7531 | Rewz | $\underline{1} .688$ |  |
| 7531 | Row6 | . 55393 |  |
| S8isouno | 'marker' |  | 'SEPARG' |
|  | Row3 ReW7 | 1.06936 $i .00549$ |  |
| 85 | Rew 3 | 3.0532 |  |
| 85 | Rew 7 | 1.75590 |  |
| 853 | ROW3 | 2.8308 |  |
| $8{ }^{8} 3$ | Rew 7 | 1.56692 |  |
| ${ }_{6}{ }^{\text {S }}$ | Rew3 | 3.116 |  |
|  | Rew Row | 1.66182 3.6 |  |
| 85 85 85 | ROW3 Rew | 3.0 1.54204 |  |
| \% 56 | Rew3 | 3.0 ${ }^{-1}$ |  |
| \%s 6 | Row 7 | 1.48915 |  |
| 85 | Rew3 | $3.0{ }^{-1}$ |  |
| 857 | Row 7 | 1.43978 |  |
| 858 | Row 3 | $3.0{ }^{-}$ |  |
| 858 | Row 7 | 4.30358 |  |
| \%s 9 | Rew3 | 300 |  |
| 8510 88 | Rew 7 Rew3 |  |  |
| 8510 | ROW 7 | j. 30953 |  |
| ssil | Row3 | $3.0{ }^{\circ}$ |  |



| 1RHS | Row5 | . 0052 |
| :---: | :---: | :---: |
| 1RHS | Rew6 | 31.60104 |
| 1RHS | Row 7 | 8.46602 |
| 1RhS | Rews | +13771 |
| 1RHS | Rows | . 001 |
| Bounds |  |  |
| UP BṄD | U1 | 1. |
| UP BND | UP | 1. |
| UP 8NO | 43 | 1. |
| UP END | 44 | 1. |
| UP BÑD | 45 | 1. |
| UP BNO | 35 1 | 1. |
| UP ENO | 352 | 1. |
| UF BND | 3 s 3 | 6. |
| UP 日ṄD | $33^{4}$ | 1. |
| UP añ | 3s 5 | 1. |
| UP BND | 3s 6 | 1. |
| UP END | 3s 7 | 1. |
| UP EṄD | 3 s 8 | 1. |
| UP ENO | 3 S 9 | 1. |
| UP ENO | 3510 | . |
| UP BNO | 3s11 | 1. |
| UP END | 3 S 12 | 1. |
| UP bino | 3s13 | 1. |
| LP BND | ${ }^{3} 514$ | 1. |
| UP BNO | 3515 | 1. |
| UP BNO | 3516 | 1. |
| UP END | 451 | 1. |
| UP END | $4{ }^{4} 2$ | 1. |
| UP BND | 453 | 1. |
| UP BNO | 454 | i. |
| UP BNO | 45 | 1. |
| UP BNO | 456 | 1. |
| UP BNO | 457 | 1. |
| UP BND | 458 | 1. |
| UP ENO | 459 | I. |
| UP BNO | 4 Sio | 1. |
| UP OND | 4511 | 1. |
| up Bnd | 4512 | 1. |
| UP 8NO | 4513 4 4 | $1:$ |
| UP OND | 4515 | 1. |
| LP bnd | $4{ }^{4} 16$ | 1. |
| UP BND | $5 s 1$ | 1. |
| LP BND | 5 S 2 | 1. |
| UP BNO | 553 | I. |
| 4 P 8ND | 5 S 4 | 1. |
| UP BNO | 5s 5 | 1. |
| UP BNO | 5 S 6 | 1 , |
| UP anic | 5s 7 | 1. |
| up bio | 5 s 8 | 1. |
| UP BND | 5 S 9 | 1. |
| UP EȦD | 5510 | 1. |
| UP BRD | $5 \mathrm{Sl1}$ | 1. |
| LP BṄ | 5512 | 1. |
| UP BNO | 5513 | 1. |
| LP END | 5514 | 1. |
| UP gNo | $5 S 15$ | 1. |
| up bĩo | 5516 | 1. |
| UP BND | SS17 | 1. |
| UP BND | 5 Sl 3 | i. |
| UP BNC | 5519 | 1. |
| UP bic | 5 S 20 | 1. |
| LP BND | 5s2ı | 1. |


| UP BND | 5522 | 1. |
| :---: | :---: | :---: |
| UP BNO | 5 S 23 | 1. |
| UP BND | 5 S 24 | 1. |
| UP 8NO | 5525 | 1 , |
| LP Bind | 5526 | 1. |
| UP BNO | $5 \mathrm{S2} 7$ | 1. |
| UP BṄO | 5528 | , |
| LP BND | 5 s 29 | - |
| UP BND | SS30 | $1 \cdot$ |
| UP BNO | 5S31 | - |
| UP BNO | $5 \mathrm{S32}$ | 1. |
| UP BND | 5 S 33 | - |
| LP BND | 5534 | - |
| LP BṄO | 5535 | , |
| UP BND | 5 S 36 | ${ }^{\circ}$ |
| UP BND | $5 \mathrm{S37}$ | 1. |
| LP BNO | 5 S 38 | 1. |
| UP BND | 5539 | . |
| UP END | 5540 | . |
| UP BND | $5 \mathrm{S4}$ 1 | - |
| UP BNO | 5 S 42 | 1. |
| UP BNO | 5343 | . |
| UP END | 5544 | - |
| UP BND | 651 | 1. |
| UP END | 652 | 1. |
| UP BND | 653 | - |
| UP BNO | 6S | 1. |
| LS BND | 655 | 1. |
| UP BND | 656 | . |
| UP END | 657 | ${ }^{\circ}$ |
| UP BND | 658 | . |
| UP BND | 6s 9 | . |
| up bino | 6510 | . |
| UP BNO | 6511 | 1. |


| up bido | 6512 | 1. |
| :---: | :---: | :---: |
| UP ANC | 6513 | 1. |
| UP BNO | 6514 | 1. |
| le ano | 6515 | 1. |
| UP BND | 6516 | 1. |
| UP BNO | 6517 | I. |
| UP OND | 6518 | 1. |
| UP GND | 6519 | 1. |
| LP BND | 6520 | 1. |
| UP BNO | 6S21 | 1. |
| UP ENO | 6522 | 1. |
| up ano | 6523 | 1. |
| ue aido | 6524 | 1. |
| UP BNO | 6S25 | 1. |
| UP BNO | 6S26 | 1. |
| UP BEND | 6527 | 1. |
| UP BNO | 6528 | 1. |
| UP BṄO | 6529 | 1. |
| up aid | 6530 | 1. |
| UP END | 6531 | $1 \cdot$ |
| UP AND | 6532 | 1. |
| UP GND | 6533 | 1. |
| UP ONC | 7s 1 | 1. |
| UP END | 7s 2 | 1. |
| LP ${ }^{\text {PND }}$ | 753 | i. |
| UP OND | 754 | 10 |
| LP ANC | 7s 5 | 1. |
| UP BNO | 756 | 1. |
| UP BNO UP BND | 757 | i. |
| $\begin{aligned} & \text { UP END } \\ & \text { UP BND } \end{aligned}$ | 758 759 | i. |
| UP BND | 7510 | 1. |
| UP BND | 7s11 | 1. |
| UP BNO | 7512 | 1. |
| UP aNO | 7513 | 1. |
| UP OND | 7S14 | 1. |
| UP BNO | 7515 | 1. |
| UP OND | 7516 | $1{ }^{\circ}$ |
| UP BND | 7517 | \% |
| UP 8NO | 7518 | 1. |
| UP anio | 7 s 20 | 1. |
| UP BṄO | 7521 | 1. |
| UP BNO | 7822 | 1. |
| LP BNC | 7523 | 1. |
| UP OND | 7524 | 1. |
| UP 8ND | 7525 |  |
| UP And | 7526 | 1. |
| UP END | 7527 7528 | 10 |
| UP BND | 7 729 | 1. |
| up and | 7530 | 1. |
| UP BNO | 7 s 31 | I. |
| UP bio | 851 | 1. |
| UP 8NO | 85 | 1. |
| UP END | $8{ }^{85} 3$ | i. |
| UP 8io | 854 | 1. |
| UP END | 855 | 1. |
| Lip and |  | 1. |
| UP BNO | 857 | 1. |
| UP BND | $8 \mathrm{8S} 8$ | 1. |
| UP BND UP BND |  | 10 |
| UP BND | 8s1! | 1. |
| LP BND | 3512 | $1 \cdot$ |
| UP BNC | 8S! 3 | 1. |
| UP BND | $8 \mathrm{SI4}$ | 1. |
| UP BND | 8515 | 1. |
| UP BND | 8516 8517 | $1:$ |
| UP gio | 8518 | 1. |
| UP END | 8519 | 1. |
| UP BNO | 8 szo | 1. |
| UP BND | 8521 | $1{ }^{1}$ |
| UP 8MD | 951 | 1. |
| up ond | 952 | 1. |
| UP giNo | 953 | 1. |
| UP BNC | 954 | 1. |
| UP 8ND |  | 10 |
| UP SNO | 9s 7 | 1. |
| Lip gno | 958 | 1. |
| UP OND | 959 | 1. |
| UP ENO | 9510 | 1. |
| UP ANO | 9511 | 1. |
| UP 日NO | 9512 | 1. |
| UP Bro | 9513 | 1. |
| UP 8ido | 9514 | 1. |
| UP akio | 9515 | 1. |
| UP and | 5516 | 1. |
| UP OND | 9 S 17 | 1. |
| UP ONO | 9518 | 1. |
| UP OND | 9519 | 1. |
| up onio | 151 | 1. |
| UP AND | 152 | 1. |
| UP OND UP ANO | 153 154 | 1. |
| UP ENO |  |  |


| UP END | 156 | 1. |
| :---: | :---: | :---: |
| UP BNO | 157 | 1. |
| UP BND | 158 | 1. |
| UP BNO | 159 | 1. |
| UP END | 1510 | 1. |
| UP BṄC | 1511 | 1. |
| UP 8ad | 1512 | 1. |
| UP BNO | 1513 | 5. |
| UP BNO | 1514 | 1. |
| LP BND | 1515 | 1. |
| Endata |  |  |


12FEB69 0. 0. 1.
INTERNAL STATEMENT NUMBEK O TIME 11:37
DEFINE HEACING AND ENTER SEPERABLE PROGRAMMING MODE TITLE NON-LINEAR PROBLEM NO 6

        CALL ENTER(SEP)
    
        SPECIFY FOUR SYMBOLIC UNITS(WERKING FILES) ON RAD
    
        CALL DEVICE\{'FILEI',DISC,'B'\}
    
        CALL OLVICE TFILESI,DISC,IC',
    
        CALL DEVICE YFILES',DISC,IDI,
    
        CALL DEVICE (YFILE', DISC,'E')
    
        ATTACH THE FOUR STANDARD FMPS FILES TO THE FOUR
    
        PREVIOUSGY DEFINED 8YMBOLIC UNITS(RAD)•
    
        CALL ATTACH(MATRIX, 'FILEI')
    
        CALL ATTACH(INyERSE; 'FILEC')
    
        CALL ATTACM\{UTILSO'FILE3!
    
        CALL ATTACH(UTILE, TFILE4:)
    
        INITIALIZE JNTERRUPT VARIABLES
    
        ASSIGN 100 TO KMAJER
    
        ASSIGN 200 TO KIOER
    
        ASSIGN 300 TO KNFS
    
        ASSIEN 400 TO KUBS
    
        ADATA ' 'NLPSSTCO1'
    
        LOAD INPUT MATRIX FROM CARDS, USING RECERD INLPSTOOI:
    
        call input
    
        IDENTIFY RIGHT=HAND=SIDE COLUMN AND COST ROW TO BE USED
    
        AOGJ \(=188,1\)
    ARHS $=1$ IRHS

    C SET TO INVERT NO LESS FREQUENTLY THAN AT JNTERVALS OF
    
        SO ITERATIONSINOTE: ALTUMATIC INVERT OR TIME IS BY DEFAULT
    
        IN OPERATION*).
    
    NOTE: TO TLRN OFF THL AUTOMATIC INVERT ON TIME, THE FOLLOWING
    
        STATLMENT SHOLLD BE USED.
    
        !NYTIME - -
    
    *****
    12FE869

| $20 \text { * }$ | Preaj - 50 |
| :---: | :---: |
|  | initialize iteration logging freguency to print every iteration |
| 21 ** | ILOGP - 1 |
| c | specify minimization |
| 22. | Febjut - 1, |
| c | selve seperable matrix |



| 12FEB69 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| JNTERNAL | Statement | NUMBER | 1 | time | 11:37 |
| INTERNAL | STAPEMENT | number | 2 | TIME | 11:37 |
| INTERNAL | Statement | number | 3 | TIME | - 11:37 |
| INTERNAL | STATEMENT | number | 4 | TIME | - 11:37 |
| INTERNAL | STATEMENT | nUMBER | 5 | TIME | - 11:37 |
| INIERNAL | STATEMENT | NUMBER | 6 | TIME | - 11:37 |
| INTERNAL | STATEMENt | NLMBER | 7 | TIME | - 11:37 |
| INTERNAL | gTATEMENT | NUMBER | 8 | IIME | - 11:37 |
| INTERNAL | STATEMENT | NUMEER | 9 | TIME | - 11137 |
| INTERNAL | STATEMENT | NUMBER | 10 | TIME | - 11:37 |
| INTERNAL | Statement | NUMBER | 11 | TIME | - 11:37 |
| INTERNAL | STATEMENT | NUMBER | 12 | IIME | - 11:37 |
| INTERNAL | STAPEMENT | NUMBER | 13 | TIME | - 11:37 |
| jnternal | 8TATEMENT | NUMBER | 14 | TIME | - 11:37 |
| INPERNAL | STATEMENT | NUMBER | 15 | TIME | - 11:37 |
| TNTERNAL | STAPEMENT | number | 16 | IIME | - 11:33 |
| JniERNAL | gTatement | nlmaER | 17 | TIME | 11:37 |

OUFFER SLZES (BYTES) ARE.. MAPRIX . 7160 INVERSE $=10240$


d2FEB69 NON-LINEAR PROBLEM N日 6

$\begin{array}{ll}11 & 0.398175600+02 \\ 12 \\ 10.775887300+02\end{array}$
$0.775887300+02$
$0.727796300+02$
$0.712375900+02$
$6-0.147048510+02$
$6-0.147048510+02$
$6=0.14704851 D+02$
$6=0.14700000 D+02$
$135 \mathrm{~L}-\mathrm{U}$
166 L
$188 \mathrm{~L}=\mathrm{U}$
$0.000000000+00$ $.000000000+00$ $0.000000000+00$ $0.116183270+00$


NEGATIVE DJ CQUNT 5 SELECTED 5 VARIABLES BEST DJ. $0.210928 D+01$



600 MS FOR INVERT
INTERNAL STATEMENT NUMEER 34 TIME : 11838
INTERNAL STATEMENT NUMEEF 23 TIME :11838
NEGATIVE DJ CQUNT - 1 SELECTEO 1 VAFIABLES BEST DJ : 0.3000000001

12FEB69 NONELINEAR PROELEM NE 6 0.

8 NBN-BASIC SLACKS. COMPLETELY TRIANGLLARIZED I ROWS AND 9 COLS.


NEGATIVE DJ CGUNT - I SELECTED 1 VARIABLES BEST DJ. - 0.3000000 001
ITER SUM OF INF NIAF OBJECT VALUE V INN MOVE REDUCED COSF ACTIVITY V-OUT MOVE PIVOT

NEGATIVE OJ COUNT $\quad$ SELECTED 4 VARIABLES BEST DJ. -0.1235230 + O
ITEN: SUM OF INF NINF BBJECTVALUE V-IN MOVE REDUCED COST ACTIVITY V-OUT MOVE PIVET

$\begin{array}{ll}\text { SEP VAR: } & 172 \text { REJECTED } \\ \text { SEP VAR: } & 174 \text { REJECTED }\end{array}$



```
NEGATIVE DJ COUNT & SELECTED S VARIABLES BEST DJ - O.7503300-01
ITER: SUM OF INF NINF OBJECT VALUE VOINMGVE REOUCED CEST ACTIVITY VGOUT MEVE PIVET
    RER;
SEP YAR. 56 REJECTEO
SEP VAR. 54 REJECTED
NEGATIVE DJ CQUNT U SELECTED O VARIARLES BEST DJ - 0.0000000%$00
INTERNAL, STATEMENT NUMBER 33 TIME - 11:38
```

12FEB69 NBN-LINEAR PROBLEM NO 6
0. 1. 8.

9 NON-BASIC SLACKS. COMPLETELY TRIANGULARIZED 1 REWS AND 7 COLS.
2 IN NON-CGMPLETELY TRIANGULARIZED PART. OF THESE I WHERE NOT TRIANGULARIZED AND O WERE REJECTED FBR TED SMALL A PIVET.


1200 MS FOR INVERT
$\begin{array}{llll}\text { INTERNAL STATEMENT NUMBER } & 34 & \text { TIME - } 11: 38 \\ \text { INTERNAL STATEMENY NUMBER } & 23 & \text { TIME } \\ \text { I1:38 }\end{array}$

12FEB69 NON•LINEAK PRBELEM NO 0 2. 1.

IOENTIFIER SECTIEN

1EFEB69 NON-LINEAR PFOSLEM NO 6 2. 2.

| SECTION | 1 - R8ws |  |  |  | Imaledual | gupput |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MUMEER | - LABEL. | AT | -. ACYIVITY.0. | SLACK | ACIIVITY | - LOWER LIMIT. | - UPPPER WIMIT* | - DUAL activity | - INPUT CESP:* | - Reduced costo |
| 1 | OBJT | PR | -9.020030 |  | 9.0̄20029 | NONE | NONE | 1.000000 | 1.000000 | 0.000000 |
| 2 | ROW\% | EO | 30.016886 |  | 0.000000 | 30.016586 | 30.016886 | 0.300000 | 0.000000 | 0.300000 |
| 3 | Rowz | EQ | 459959442 |  | 0.000000 | 44.959442 | 44.9534 ${ }^{\text {a }}$ 2 | 0.280000 | 0.000000 | 0.230000 |
| 4 | Rew3 | EO | 27-414490 |  | 0.000000 | 27.414490 | 27.414490 | 0.003934 | 0.000000 | 0.063354 |
| 5 | Row 4 | E0 | 99.836899 |  | 0.000000 | 99.836899 | 99.836899 | 0.000000 | 0.000000 | 0.000000 |
| 6 | Rows | EO | 0.005200 |  | 0.000000 | 0.005200 | 0.0005200 | -0.061052 | -0.0̇ō00000 | -0.06105 |
| 7 | Rew6 | E0 | 31:601028 |  | 0.000000 | 31.601028 | 31.601028 | -0.641452 | 0.000000 | -0.641452 |
| \% | Rew 7 | E0 | 8.466020 |  | 0.000000 | 8.466020 | - 8.466020 | -0.009013 | 0.060000 | -0.009013 |
| 9 | Rews | E0 | -137710 |  | 0.000000 | 4.137710 | 4.137710 | 0.000000 | 0.000000 | 0.000000 |
| 10 | Rows | E0 | 0.001000 |  | 0.000000 | 0.002000 | 0.001009 | 0.583422 | 0.000000 | 0.513422 |

12FEB69 NONOLINEAR PFBBLEM NG :

| SECTION | $2-\mathrm{colu}$ |  |  | PRIMAL = UUAL | QUTPUT |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Numsen | - 1 LABEb. | AT | -*ACTIVITY** | - INPUT CAST. | - - Lewer limit. | - - upper limit. | -REDUCEO Cest. |
| 11 | $\times 5$ | ES | 0.514710 | 0.000000 | 0.000000 | NONE | 0.000000 |
| 12 | USOUNDS | $E$ | $0: 000000$ | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 13 | U2 | BS | 0.949472 | -9.000000 | 0.000000 | 1.000000 | 0.000000 |
| 14 | U2 | 46 | 0.000000 | -3.100000 | 0,000000 | 1.0000000 | -0.100000 |
| 15 | USOUNDE | E0 | $0: 000000$ | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 16 | U3 | BS | 0:169566 | -2.800000 | 0.000000 | 1.000000 | 0.000000 |
| 17 | 4 | 6 | 0.000000 | -2.900001 | 0.000000 | 1.000000 | -0.100000 |
| 58 | 45 | LL | 0:000000 | -24.000000 | 0.000000 | 1.000000 | -1.600000 |
| 19 | S3BEUND | E0 | 00000000 | 0.0000000 | 0.000000 | 0.00 ÓŌō | 0.000000 |
| 20 | 3 s 1 | 46 | 0.060000 | 0.0000000 | 0.000000 | 1.0000000 | 0.003676 |
| 2 | 3s 2 | 46 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | 0.002288 |
| 22 | 353 | 6 | $0: 000000$ | 0.000000 | 0.000000 | 1.000000 | 0.000837 |
| ¢3 | 3 S 4 | Ll | $0: 000000$ | 0.00000000 | 0.000000 | $1.0000 \overline{0} 0 \overline{0}^{1}$ | -0.000600 |
| $2{ }_{4}$ | 3s 5 | 46 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.002019 |
| 25 | 356 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.004496 |
| 26 | 357 | 46 | ¢.000000 | 0.000000 | 0.000000 | 1.0600000 | -0.006622 |
| 27 | 358 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.008797 |
| 28 | 359 | 6 | 0.000000 | 0.000000 | 0.000000 | 1.0000000 | -0.010907 |


| 29 | 3510 | LL | 00000000 | 0.000000 | 0.000000 | 1.000000 | -0.015836 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 30 | 3511 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.018794 |
| 31 | 3 s 12 | 46 | -0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.021711 |
| 32 | 3513 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.024588 |
| 33 | $3{ }^{3} 14$ | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.027429 |
| 34 | 3515 | 46 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.025002 |
| 35 | 3516 | 46 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.026929 |
| 36 | SidBeund | E0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 37 | 451 | UL | 1:000000 | -. 000000000 | 0.000000 | 1.000000 | 0.014691 |
| 38 | 482 | UL | 1.000000 | 0.000000 | 0.000000 | 1.000000 | 0.013432 |
| 39 | 453 | UL | i, 0000000 | 0.000000 | 0.000000 | 1.00000000 | 0.012110 |
| 40 | 454 | UL | 1:000000 | 0.000000 | 0.000000 | 1.000000 | 0.010755 |
| 51 | 455 | UL | 1.000000 | 0.000000 | 0.000000 | 1.000000 | 0.009511 |
| 52 | $4{ }^{4} 6$ | U6 | 1.000000 | 0.000000 | 0.000000 | 1.000000 | 0.010098 |
| 43 | 457 | UL | 1.000000 | 0.000000 | 0.000000 | 1.000000 | 0.008171 |
| 44 | 458 | UL | 1:000000 | 0.000000 | 0.000000 | 1.000000000 | 0.006200 |
| 45 |  | UL | $1: 000000$ | 0.0000000 | 0.000000 | 1.000000 | 0.004291 |
| 46 | 4510 | U6 | 1.000000 | 0.000000 | 0.000000 | 1.000000 | 0.002667 |
| 57 | 4511 | Bs | 0.000042 | 0.000000 | 0.000000 | 1.000000 | 0.000000 |
| 48 | 4512 | 4 | 0.000000 | O.0000000 | 0.000000 | 1.00̄̄ōōōo | -0.002627 |
| 59 | 4513 | 46 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.005215 |
| 50 | 4584 | 46 | 0.0000000 | 0.000000 | 0.000000 | 1.0000000 | -0.007766 |
| 51 | 4515 | LL | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.008395 |
| b2 | 4516 | LL | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.010121 |
| 53 | S5Bound | Ea | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 54 | 5 S 1 | UL | 1.000000 | 0.000000 | 0.000000 | 1.000000 | 0.007376 |
| 55 | 5 S 2 | 85 | 0.160604 | 0.000000 | 0.000000 | 1.00000000 | 0.000000 |
| 56 | 5s 3 | 6 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.010786 |

L2FEB69 NON-LINEAR PROBLEM NO 6
0.2 .4.

SECTION 2 - COLUMNS PRIMAL-DUAL OUTPUT

| NUMEEA | - ${ }_{\text {chabel. }}$ | ${ }^{\text {at }}$ | - . -ACTIVITY... | - - input cost | - -LOWER LImit. | - ${ }^{\text {cupper limit- }}$ | - REDUCED COST. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5s ${ }^{\text {a }}$ | L | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.020508 |
| 58 | 555 | 4 | $0: 000000$ | 0.000000 | 0.000000 | 1,000000 | -0.029915 |
| 59 | 556 | LL | 0:000000 | 0.000000 | 0.000000 | 1,000000 | -0.037532 |
| 60 | 557 | 46 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.044928 |
| 61 |  | L6 | 0,000000 | 0.000000 | 0.000000 | 1.000000 | -0.051767 |
| 62 | 559 | 46 | 0.000000 | 0.000000 | 0.000000 | 1,000000 | -0.058iIt |
| 63 | 5510 | 6 | 6:000000 | 0.000000 | 0.000000 | 1,000000 | -0.064042 |
| 64 | 5 S 11 | 4 | 0.000000 | 0.000000 | 0.000000 | \$.000000 | -0.070844 |
| 65 | 5 S 12 | 6 | 6:000000 | 0.000000 | 0.000000 | 1.0000000 | -0.074910 |
| 66 | 5513 | 6 | 6,000000 | 0.000000 | 0.000000 | i. 000000 | -0.079837 |
| 67 | $5 \mathrm{SI4}$ | L6 | 0.000000 | 0.000000 | 0.000000 | 1,000000 | -0.084508 |
| 68 | 5815 | 46 | 00000000 | 0.0000000 | 0.000000 | 1.0000000 | -0.088946 |
| 69 | 5S16 | L6 | 0:000000 | 0.000000 | 0.000000 | 1,000000 | -0.093176 |
| 70 | 5517 | 66 | 6:000000 | 0.000000 | 0:000000 | 1.000000 | -0.098929 |
| 71 | 5518 | $L$ | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.153149 |
| 72 | 5819 | 6 | 0:000000 | 0.000000 | 0.000000 | 1.000000 | -0.161372 |
| 73 | 5s20 | 4 | 0:000000 | 0.000000 | 0.000000 | 1.000000 | -0.169165 |
| 74 | 5s21 | 6 | 0,000000 | 0.000000 | 0.000000 | 1.0000000 | -0.118959 |
| 75 | $5 s 22$ | 4 | 0:000000 | 0.000000 | 0.000000 | 1.000000 | -0.181422 |
| 76 | 5823 | 4 | 6.000000 | 0.000000 | 0,000000 | 1.006000 | -0.188320 |
| 77 | $5 \$ 24$ | LL | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.194958 |
| 78 | 5825 | 46 | 0:000000 | 0.000000 | 0.000000 | 1,000000 | -0. 203716 |
| 79 | 5526 | 4 | 0:000000 | 0.000000 | 0,000000 | 1.000000 | -0.278209 |
| 80 | 5527 | 46 | 0:000000 | 0.000000 | 0.000000 | 1.000000 | -0.288844 |
| 81 | 5528 | $6 L$ | 0.000000 | 0,000000 | 0,000000 | 1.000000 | -0.301764 |
| -2 | 5529 | 4 | 0.000000 | 0.000000 | 0.000000 | i 0000000 | -0.309239 |
| 83 | 5830 | 4. | O.000000 | 0.0000000 | 0.000000 | 1.0000000 | -0.319003 |
| 8 | 5S31 | 4 | 0:000000 | 0.000000 | 0,000000 | 1.000000 | -0.331446 |
| 85 | 5 S 32 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.424036 |
| 86 | 5 S 33 | 4 | 0.000000 | 0.000000 | 0.000000 | 1,000000 | -0.530995 |
| 87 | 5534 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.593545 |
| 8 | $5 s 35$ | 46 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.657459 |
| 89 | 5 S36 | 46 | 0.000000 | 0.000000 | 0.000000 | 1,000000 | -0.684285 |
| 90 | 5837 | 4 | 00000000 | 0.000000 | 0.000000 | $1,00 \overline{0} \overline{0} \overline{0} 0$ | -0.710959 |
| 91 | 5538 | 4 | $0: 000000$ | 0.000000 | 0,000000 | 1.000000 | -0.737911 |
| 92 | ${ }^{5} 533$ | 61 | 00000000 | 0.000000 | 0.000000 | 1,000000 | -0.765172 |
| 93 | 5S40 | LL | 0:000000 | 0.000000 | 0,000000 | 1,000000 | -0.792864 |
|  | 5S41 | 4 | 6:000000 | 0.006000 | 0.000000 | 1,000000 | -1.242437 |
| 95 | 5 S 42 | 46 | 0:000000 | 0.000000 | 0.000000 | 1.000000 | -1.308294 |
| 96 | 5 S 43 | 6 | \%:000000 | 0.000000 | 0,000000 | 1.000600 | -1.377036 |
| 97 | $5{ }^{5} 44$ | 4 | 0.000000 | 0.000000 | 0,000000 | 1.000000 | -1.391363 |
| 98 | Sobeund | E0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 99 | 651 | UL | 1.000000 | 0.000000 | 0.00000 | 1.000000 | -0.02491 |
| 100 | $65^{2}$ | UL | 1.000000 | 0.000000 | 0,000000 | 1.000000 | -0.003702 |
| 101 | $65^{6}$ | 6 | 0.000000 | 0.0000000 | 0.000000 | 1.000000 O] | 0.019853 |
| 102 | 65 | 4 | 0.000000 | 0.000000 | 0,000000 | 1.000000 | 0.041505 |

LEFEB69 NON-LINEAR PROELEM NO 6
0. 2. 5.

SECTION 2 - COLUMNS PRIMAL=DUAL OUTPUT

| NUMPER | -CLABEL* | ${ }^{\text {AT }}$ | -**ACPIVITY-** | - InPUT Cest.0 | - GLUER bIMIT. | - upper limito | - REDUCED Cest. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 103 | 6S 5 | 4 | 0.000000 | 0.000000 | 0,000000 | 1.000000 | 0.067268 |
| 104 | 686 | 4 | $0: 000000$ | 0.000000 | 0.000000 | 1,000000 | 0.095586 |
| 105 | 657 | 46 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | 0.114636 |
| \$06 | 658 | 4 | 6,000000 | 0.000000 | 0.000000 | 1.000000 | 0.156885 |
| S07 | 659 | L | -5000000 | 0.000000 | 0.000000 | 1.000000 | 0.189295 |
| 108 | 6310 | 6 | 0,000000 | 0.000000 | 0.000000 | 1.000000 | 0.223843 |
| 103 | 6511 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | 0.257760 |
| 110 | 6512 | 46 | 0,000000 | 0.000000 | 0.000000 | 1.000000 | 0.292652 |
| 111 | 6513 | 6 | 0.000000 | 0.000000 | 0.000000 | 1.0000000 | 0.327772 |



SECTION 2 - COLUMNS PRIMAL-DUAL OUTPUT

| NUMBER 195 |  | AT | - QACTIVITY.0. | - INPUT COST0.0 | - LANER LIMIT. | - ©UPPER LIMIT. 1.000000 | -REOUCED CEST. 0.000000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 196 | 9S 9 | $L$ | -000000 | 0.000000 | 0.000000 | 1.000000 | 0.000000 0.000000 |
| 197 | 9S11 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | 0.000000 |
| 198 | 9512 | $L$ | 0.000000 | 0.000000 | 0.000000 | 1,000000 | 0.000000 |
| 199 | 9513 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | 0.000000 |
| 200 | 9514 | 4 | 0:000000 | 0.000000 | 0.000000 | 1.000000 | 0.000000 |
| 201 | 9S15 | 6 | 0:000000 | 0.000000 | 0.000000 | 1.000000 | 0.000000 |
| 802 | 9516 | 46 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | 0.000000 |
| 203 | 9517 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | 0.000000 |
| 804 | 9838 | 6 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | 0.000060 |
| 205 | 9519 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | 0.000000 |
| 206 | S1080UnO | EO | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 207 | 151 | 4 | $1: 000000$ | 0.000000 | 0.000000 | 1.000000 | 0.019473 |
| 208 | 15 | UL | \$0000000 | 0.000000 | 0.000000 | 1.000000 | 0.097063 |
| 203 | is 3 | UL | 12000000 | 0.000000 | 0.000000 | 1.000000 | 0.059661 |
| 210 | Is 4 | U | 10000000 | 0.000000 | 0.000000 | 1,000000 | 0.028804 |
| 211 | Is 5 | BS | 0:374868 | 0.000000 | 0.000000 | 1.000000 | 0.000000 |
| 212 | 156 | 4 | 0.000000 | 0.000000 | 0.000000 | \$.000000 | -0.026954 |
| 213 | 1s 7 | 6 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.052234 |
| 214 | 158 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.075991 |
| 215 | 159 | 66 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.098359 |
| 216 | 1510 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.119444 |
| 217 | 1511 | 66 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.139374 |
| 2i8 | 1512 | 4 | Ǒ.00́0000 | 0.0000000 | 0.000000 | 1.000000 | -0.igse30 |
| 219 | 1513 | 6 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.176089 |
| 220 | 1514 | 4 | 0.000000 | 0.000000 | 0.000000 | 1.000000 | -0.193043 |
| 2ट̇1 | 1515 | 6 | 00.000000 | 0.00000000 | 0.000000 | 1.000000 | -0.146168 |
| 222 | SEPEND | E0 | \%-000000 | 0.800000 | 0.000000 | 0.000000 | 0.000000 |

12FEB69 NONOLINEAR PROGLEM NO 6

INTERNAL STATEMENT NUMBER 25 TIME 11:39
-EXIT*

| tetal job time | 2.03 |
| :---: | :---: |
| Processor execution time | .00 |
| PROCESSER $1 / 0$ TIME | . 07 |
| PRecesser overhead time | . 08 |
| USER EXECUTION TIME | . 56 |
| USER 1/0 TIME | . 60 |
| USER OVERHEAD TIME | \% |
| - of caros read | 994 |
| - Of Caros punched | 0 |
| - of processer pages out | 2 |
| - of user pages out | 18 |
| - of diagnestic pages out | 0 |
| - OF SCRatch tapes used | 0 |
| - of Save tapes used | 0 |
| - of disk reads and writes | 1436 |
| - OF disc reads and wrifes | 2814 |
| TEMPGRARY DISC SPACE USED | 34 |
| PERMANENT DISC SPACE USED | $\bigcirc$ |
| ACCUM P PERM. DISC SPACE USED | 0 |


[^0]:    ${ }^{\dagger}$ G. Hadley, Non-Linear and Dynamic Programming. Reading, Massachusetts: Addison-Wesley Publishing Company, 1964, Chapter 4.

[^1]:    JOE 326,3UMD
    LIMIT (TIME,90), (L0,1000), (U0,1000), (10., 1000)

