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HOW THE GAS PROGRAM WORKS WITH A NOTE ON SIMULATING TURTLES WITH TOUCH SENSORS

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How the GAS Program Works

The GAS program is a display simulation of a 2-dimensional ideal gas. Barriers, or walls, are line segments, and molecules, alias particles or balls, are circles. Collisions occur between balls and other balls as well as between balls and walls. All collisions are elastic. Global gravitational, electric, and magnetic fields can be imposed to act on the particles. The following is a discription of some of the inner workings of the program.

The scope is divided into 2000 (=40*40) 40x40 squares.* 204 extra 40x40 squares are imagined to be placed around the scope perimeter. There is a 42x42 table each of whose entries is a list of objects in the corresponding 40x40 square on or around the scope. This includes all walls going through the square, all wall ends in the square, and any balls whose centers lie in the square.

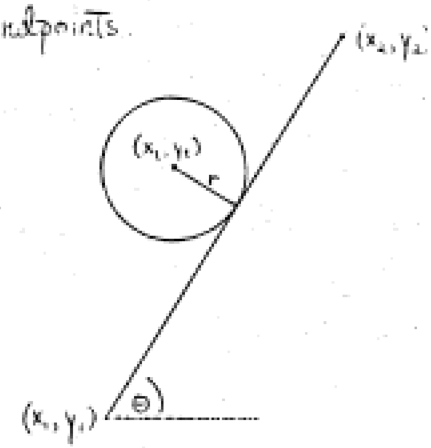
To perform a single time step (on the order of 1/30 second), all ball entries are removed from the table. Then each ball in turn has its velocity added to its position, and is then entered in the table by appending it to the list of objects in its square (the square in which its center lies). Just before the ball is actually appended to the list, a check is made to see if it is about to collide with something already there. After it is appended, a check is made for collisions with objects in the lists for the 10 neighboring squares.

A collision check is performed as follows: For a ball, a simple check is made to see that its relative velocity with respect to the current ball constitutes a possible collision course. If so, a fairly complicated calculation is done to find out if a collision actually occurs, and if so, at what time. For a wall which has not already been considered as a collision candidate for the current ball, a simple calculation finds the time at which the ball hits the wall (were it infinitely extended). If this time is in the desired range, another simple calculation is made to check that the ball hits the part of this infinite wall which really exists. An additional calculation is performed for wall endpoints, first checking that the ball hits the point, and then if it does so in the desired time range.

The first collision in the desired time range is then made to happen (a long calculation), and a recalculation is done to find all (possibly changed) later collisions. If there are no collisions to be performed, ball velocities are updated and the next time step is taken.

* The size (and therefore number) of squares is actually an assembly parameter. The squares should be big enough so that their diameter is greater than the sum of speeds and radii for any two balls.

Wall Collisions



Let (x_1, y_1) and (x_2, y_2) be the coordinates of the wall endpoints.
Let $\ell = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ be the length of the wall.
Let (x_0, y_0) be the position of the center of the ball at the beginning of the time step.
Let (u, v) be the velocity of the ball.
Let r be the radius of the ball.

Then, the time at which a ball-wall collision would occur is given by

$$t = \frac{\pm \ell r - x_0(y_2-y_1) + y_0(x_2-x_1) - (x_2 y_1 - x_1 y_2)}{u(y_2-y_1) - v(x_2-x_1)}$$

where the $\pm$ is to be taken opposite the sign of the denominator.

The collision actually occurs iff

$$\left| x_1^2 + y_1^2 - x_2^2 - y_2^2 + 2\left[x_t(x_2-x_1) + y_t(y_2-y_1)\right] \right| \leq \ell^2$$

where $(x_t, y_t) = (x_0 + ut, y_0 + vt)$ is the position of the center of the ball at the collision time t.

After the collision, the new velocity of the ball is

$$(u', v') = (u \cos 2\theta + v \sin 2\theta,\ u \sin 2\theta - v \cos 2\theta)$$

where θ is the angle the wall makes with the horizontal, so

$$\sin 2\theta = 2(x_2-x_1)(y_2-y_1) / \ell^2 \quad \text{and}$$
$$\cos 2\theta = \left[(x_2-x_1)^2 - (y_2-y_1)^2\right] / \ell^2 .$$

The time at which a ball-wall-end$_i$ collision would occur is given by

$$t = -\frac{(x_0-x_i)u + (y_0-y_i)v + \sqrt{\left\{ r^2(u^2+v^2) - \left[(x_0-x_i)v - (y_0-y_i)u\right]^2 \right\}}}{u^2+v^2} \qquad [i=1,2]$$

where the quantity in braces must be positive for a real collision.

The velocity of the ball after the collision is given by

$$(u', v') = (-u \cos 2\theta - v \sin 2\theta,\ v \cos 2\theta - u \sin 2\theta)$$

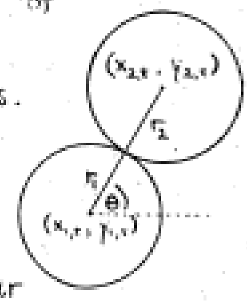
where θ is the angle from the horizontal of the ball radius joining the center of the ball to the wall end at the collision time t, so

$$\sin 2\theta = 2(x_t - x_i)(y_t - y_i) / r^2 \quad \text{and}$$
$$\cos 2\theta = \left[(x_t-x_i)^2 - (y_t-y_i)^2\right] / r^2 ,$$

where $(x_t, y_t) = (x_0 + ut, y_0 + vt)$ is the position of the ball center at time t.

Ball Collisions

Let (x_1, y_1) and (x_2, y_2) be the positions of the centers of the two balls at the beginning of the time step.
Let (u_1, v_1) and (u_2, v_2) be the velocities of the two balls.
Let r_1 and r_2 be the radii of the two balls.
Let m_1 and m_2 be the masses of the two balls.



Then, the time at which a ball-ball collision would occur is given by

$$t = -\frac{(x_1-x_2)(u_1-u_2) + (y_1-y_2)(v_1-v_2) + \sqrt{\left\{(r_1+r_2)^2\left[(u_1-u_2)^2+(v_1-v_2)^2\right] - \left[(x_1-x_2)(v_1-v_2)-(y_1-y_2)(u_1-u_2)\right]^2\right\}}}{(u_1-u_2)^2+(v_1-v_2)^2}$$

where the quantity in braces must be positive for a real collision.

The velocities after collision are given by

$$(u_i', v_i') = \big(-(u_i-u_c)\cos 2\theta - (v_i-v_c)\sin 2\theta + u_c\,,\; +(v_i-v_c)\cos 2\theta - (u_i-u_c)\sin 2\theta + v_c\big) \qquad [i=1,2]$$

where $(u_c, v_c) = \frac{m_1(u_1,v_1) + m_2(u_2,v_2)}{m_1+m_2}$ is the velocity of the center of mass and θ is the angle from the horizontal of the line joining the two ball centers at the time of collision, so

$$\sin 2\theta = 2(x_{2,t} - x_{1,t})(y_{2,t} - y_{1,t}) / (r_1+r_2)^2 \quad \text{and}$$
$$\cos 2\theta = \left[(x_{2,t}-x_{1,t})^2 - (y_{2,t}-y_{1,t})^2\right] / (r_1+r_2)^2\,, \quad \text{where}$$

$(x_{i,t}, y_{i,t}) = (x_i, y_i) + (u_i, v_i)\,t$ is the position of the center of ball i at the collision time t.

How to do it with Vectors

[It's simpler, and it works in 3-space]

Wall Collisions:

Let $\hat{n}$ be a unit normal to the wall. [In 2-space, $\hat{n} = (-\sin\theta, \cos\theta) = (-\frac{y_2 - y_1}{\ell}, \frac{x_2 - x_1}{\ell})$]

Let $\vec{p}$ be a point on the wall. [In 2-space, $\vec{p}\cdot\hat{n} = \frac{x_2 y_1 - x_1 y_2}{\ell}$]

Let $\vec{x}$ be the position of the center of the ball at the beginning of the time step.

Let $\vec{v}$ be the velocity of the ball.

Let r be the radius of the ball.

Then, the time at which a ball-wall collision would occur is given by

$$t = \frac{\pm r + \vec{p}\cdot\hat{n} - \vec{x}\cdot\hat{n}}{\vec{v}\cdot\hat{n}}$$

where the $\pm$ is to be taken opposite the sign of the denominator $(\vec{v}\cdot\hat{n})$.

The new velocity of the ball is given by

$\vec{v}\,' = \vec{v} - 2\,\vec{v}\cdot\hat{n}\,\hat{n}$ where the 2 may be replaced by $1+\epsilon$ for inelastic collisions

Wall-end Collisions:

Let $\vec{x}$ be the position of the center of the ball with respect to the wall endpoint.

Let $\vec{v}$ be the velocity of the ball.

Let r be the radius of the ball.

Then, the time at which a ball-wall-end collision would occur is given by

$$t = -\frac{\vec{x}\cdot\vec{v} + \sqrt{r^2|\vec{v}|^2 - |\vec{x}\times\vec{v}|^2}}{|\vec{v}|^2} \qquad [\text{Note } |\vec{x}\times\vec{v}|^2 + (\vec{x}\cdot\vec{v})^2 = |\vec{x}|^2|\vec{v}|^2.]$$

where the quantity under the radical must be positive for a real collision.

The new velocity of the ball is given by

$\vec{v}\,' = \vec{v} - \frac{2\,\vec{v}\cdot\vec{r}}{|\vec{r}|^2}\vec{r}$ where $\vec{r} = \vec{x} + \vec{v}t$, and so $|\vec{r}| = r$; for inelastic collisions the coefficient 2 may be replaced by $1+\epsilon$.

Ball Collisions:

Let $\vec{x}_i$ be the initial position of the center of ball i. Let $\vec{x} = \vec{x}_2 - \vec{x}_1$.

Let $\vec{v}_i$ be the velocity of ball i. Let $\vec{v} = \vec{v}_2 - \vec{v}_1$.

Let m_i be the mass of ball i. Let $M = m_1 + m_2$.

Let r_i be the radius of ball i. Let $r = r_1 + r_2$.

Then, the time at which a ball-ball collision would occur is given by

$$t = -\frac{\vec{x}\cdot\vec{v} + \sqrt{r^2|\vec{v}|^2 - |\vec{x}\times\vec{v}|^2}}{|\vec{v}|^2}$$

where the quantity under the radical must be positive for a real collision.

The new velocities are given by

$v_i' = v_i - \frac{2\frac{m_{3-i}}{M}(\vec{v}_i - \vec{v}_{3-i})\cdot\vec{r}}{r^2}\vec{r}$ where $\vec{r} = \vec{x} + \vec{v}t$, and so $|\vec{r}| = r$; for

On Simulating Turtles with Touch Sensors

We wish to create a display simulation of turtles with touch sensors. Such a simulation is similar to that done by the "GAS" program, in particular, turtles are like slow balls, and walls are walls. The turtle simulator need not be nearly as fast or as accurate as the GAS program, and turtle-turtle collisions may be excluded (in the single turtle case). Furthermore, the result of a collision is not an elastic bounce as in the GAS program, but just a dead stop (or perhaps some hairy inelastic collision would be a better model, depending on what the real turtles plus software do), and speeds are not important (except perhaps for multiturtle simulation). Multiple turtles seem to be a very interesting idea, however, with different, concurrently running, programs for each turtle. Perhaps games could be played such as blindfold tag. An interesting "physics" demonstration would be produced by two mirror-image-programmed turtles acting as if there were a wall between them.

The simulation may proceed in a way similar to the GAS program. We divide the (rectangular) area to be simulated into an s1 x s2 array of squares*, and consider an extra row of squares outside the perimeter, making an s1+2 x s2+2 array. Corresponding to this array of squares, we have an array each element of which is a list containing an entry for each wall going through and for each wall-end in the corresponding square. For each time step (perhaps corresponding to a turtle step) each turtle is in turn examined: its position is updated (its program may have to be run in order to do this) and it is checked for collision with other objects (see description of GAS program for further details). The first collision in the desired time step is made to happen (at which point touch-sensor interrupts may be generated, to be stored for use at the next time step) and the procedure repeated for later collisions during the time step. If there are no collisions during a pass through the turtles, the next time step is taken.

The actual simulation is only one part of the simulator. There is also input-output. In order to input walls, it seems that the easiest thing to do is to have a display turtle draw the walls (Greenblatt suggested this). Of course the other items of input, i.e. turtle movements as a function of time, would be directly under program control. As far as output is concerned, it would be desirable to be able to display either a fixed window or a window moving with a given turtle (or many of these simultaneously), with variable window size and variable location with respect to the simulation area or a turtle. All display parameters and options would be under (real-time) program control.

Additional ideas for extra hair are walls invisible (in the sense of not really there) to some turtles, invisible turtles, and time-dependent (position, length, orientation, invisibility) walls.

* The squares should be big enough so that their diameter is greater than the sums of speeds and radii for any two turtles.