General Information Manual

PERT . . . a dynamic project planning & control method

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NOTE

Most of the illustrations and much of the information appearing in this manual were made available to the IBM Corporation by the U. S. Navy Special Projects Office in Washington, D. C.

Other material resulted from meetings with personnel from U. S. Naval Air Development Center, Johnsville, Pennsylvania; Headquarters Air Force Systems Command, Washington, D. C.; Air Force Systems Command, Dayton, Ohio; Aero Jet-General, Sacramento, California; Douglas Aircraft Company, Inc., Santa Monica, California; General Electric, Utica, New York; Lockheed Missile System Division, Sunnyvale, California; North American Aviation, Inc., Los Angeles, California.

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INTRODUCTION

Program Evaluation and Review Technique (PERT) is a method of planning, replanning and progress evaluation in order to better control a major research and development program. The PERT technique was developed during 1958 at the Navy Special Projects Office by a project team which studied the application of statistical and mathematical methods to the planning, evaluation, and control of research and development (R&D) effort. The project team consisted of personnel from the U.S. Navy Special Projects Office, Booz-Allen and Hamilton, and Lockheed Missile Systems Division. The Navy first applied PERT to the development of the Polaris submarine. By 1961 the technique had found application in certain areas of the Air Force, Army, and special agencies of government as well as private industry.

PERT is used to define what must be done in order to accomplish program objectives on time. Through its use, areas of a project that require remedial decisions can be detected and the effect of trade-offs among the three basic factors — time, resources, and technical performance — can be determined. One of the major advantages of PERT is that it provides a method for the diagramming (establishing a network) of a program. Each event is depicted and its relationship to the others expressed. PERT uses time as the common denominator to reflect planned resource application and performance specifications.

PERT also provides:

- Aids in planning and scheduling a program.
- Better communications.
- Continuous, timely progress reports, identifying potential problem areas where action may be required.
- A simulation of the effects of alternate decisions under consideration and an opportunity to study their effect upon the program deadlines, prior to implementation.
- Probability of successfully meeting deadlines.

The nature of PERT is such that the larger and more complex the project, the greater will be the benefits. Its implementation does not require a change in management organization or policy. The technique makes it possible to organize existing data into a more meaningful form for immediate use by management. PERT has been and is presently under continued development; modifications could incorporate quality, cost and resources as additional variables.

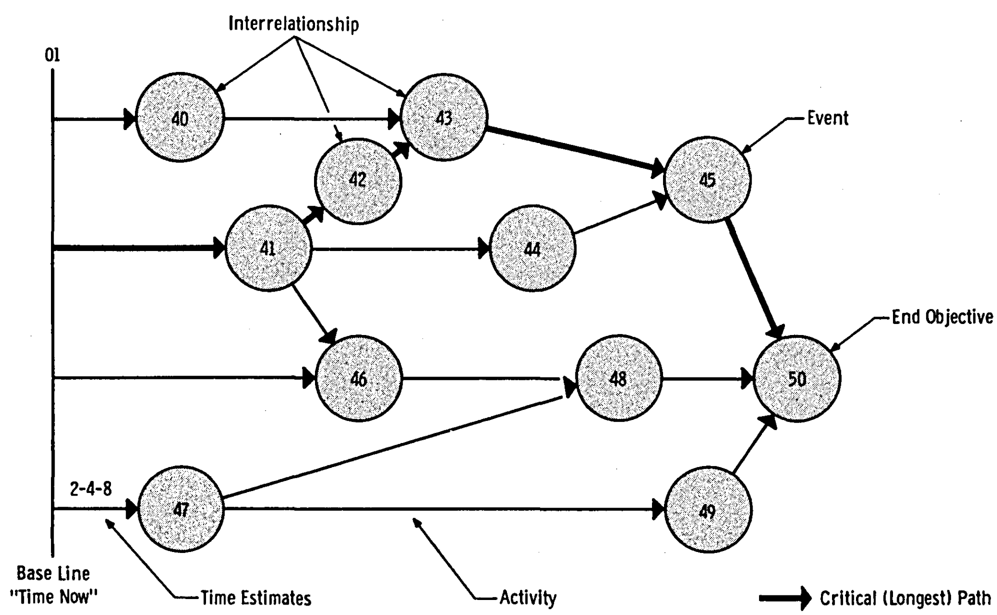
PART ONE: PERT Network Development

The first step in developing a PERT network is the definition of the R&D program objectives. This is accomplished by an analysis of the work statement and by identification of the functional areas or departmental responsibilities. The technical specifications for the program should be clarified by detailed statements that describe the components, assemblies and subassemblies which go into the final product.

Evolving from this step will be a work plan which broadly expresses the technical approach and a time scale for the entire program. The time scale shows when large work groups are involved and the extent to which the activity of one depends upon another.

This preliminary planning is done whether PERT is being used in the development of a technical proposal or for internal laboratory management.

At this point the activity network is developed. It is a pictorial representation of the events which take place in the program. Events are depicted by blocks or circles and are connected by arrows which represent the activities necessary to achieve the events. The network is the basic planning tool of the PERT technique; it consists of those milestone check points that must be accomplished under the currently approved plan and, therefore, is the actual procedure to be followed. Although not specified, the resources needed for the project are implied.



PERT Network of Events and Activities

If a schedule is utilized in planning the network, PERT becomes a reporting tool. If, however, planning is done without the constraint of a schedule, PERT becomes both a scheduling and reporting technique. This results in a more realistic picture of the program.

The network should be developed at the level where the work is to be performed. In developing the detailed network, the team approach has been found to be an effective method. A typical team would consist of from two to twelve people, depending on the size of the product being planned. It could include representatives from Engineering, Quality Control, Manufacturing, Reliability, Purchasing, and specialists from other functional areas. This kind of integration leads to an exchange of ideas that is conducive to good planning. In addition, experience has shown that much of the time required to develop the program plan or network will be used to make decisions which must be communicated to all team members involved. The team evolves the best plan for completing a project. The leader of the team must be thoroughly familiar with the planning requirements established.

Each area responsible for specific activities and events should establish its own individual network, which will be a further detailed breakdown of events on the master network.

Experience also indicates that it is advisable to start with an end objective and then work backward in developing a network. When using a contractual schedule, the completion date tends to fall on the contractual date. The base line for the network is a starting point or the first event in the network. It must be dated, and it must be numbered.

Initial networks are prepared by team members. The events are joined together by activity lines to indicate the chronological order of event accomplishment. Activity lines indicate the interrelationship and sequence of events as well as work effort required between events. The initial networks are then integrated into an overall detailed network. During this operation any discrepancies between output of one group and input of another will be apparent. The completed plan is analyzed by the team for reasonableness.

Three time estimates are required from the technically responsible persons and are assigned to each activity line. These indicate the time required for completing the activity between events. The network is recorded and then may be processed either manually or by a computer depending upon complexity. Necessary replanning is done by the team if the critical path (that is, the longest path in the network) indicates that too much time is required in attaining the end objective. This replanning may involve a change in the length of time allowed to accomplish an event or a change in the objective, and may be based, for example, on changes in technical requirements and resources allocation.

The completed PERT network depicts:

1. The plan to be used and the time required to complete the project.

2. A critical path (the longest path in the network) and other paths that may limit the end objective or completion date.

Event Definition

The term "event" describes the milestone of the PERT network. A further definition of an event is "the start or completion of a task; not the actual performance of that task." Events, therefore, do not require time or resources. They are points in time which may reflect only that a decision has to be made. Since events are points in time when something is to be accomplished, their definition permits a logical analysis of their sequential relationships.

Network events should be sequenced according to technical requirements. The network's degree of detail should be sufficient to permit a timely appraisal of progress at the required levels of reporting. For best results more attention should be given to detailing areas in which the probability of program delay is greatest.

In detailing a network, the exact interrelationship of events that lead to completion must be known. The maximum time between events should be related to the degree of control desired.

After events have been properly selected, their descriptions are placed on a flow chart as shown in the illustration on the following page. In selecting titles for events on the network, the following examples can be used:

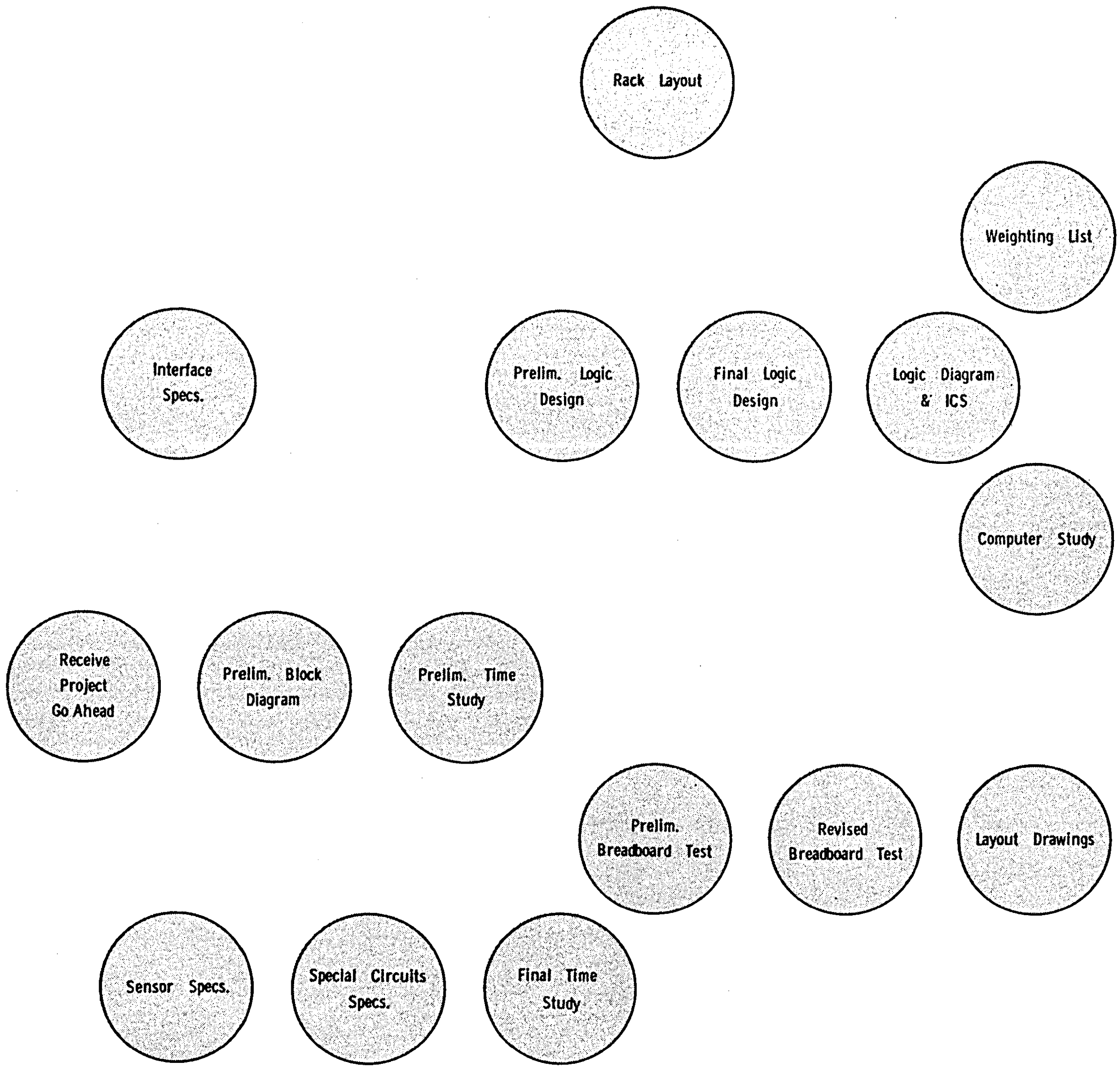
- Key Action Words: Start, complete, ship, receive,

- Description of Action Words: Design, release, place, purchase order, fabricate, assemble, test, decide, approve, check-out.

- Locations: Subcontractors, associate contractors, divisions and locations.

- Identification of Items: Hardware items, drawings, reports, specifications, purchase orders, documents, training aids.

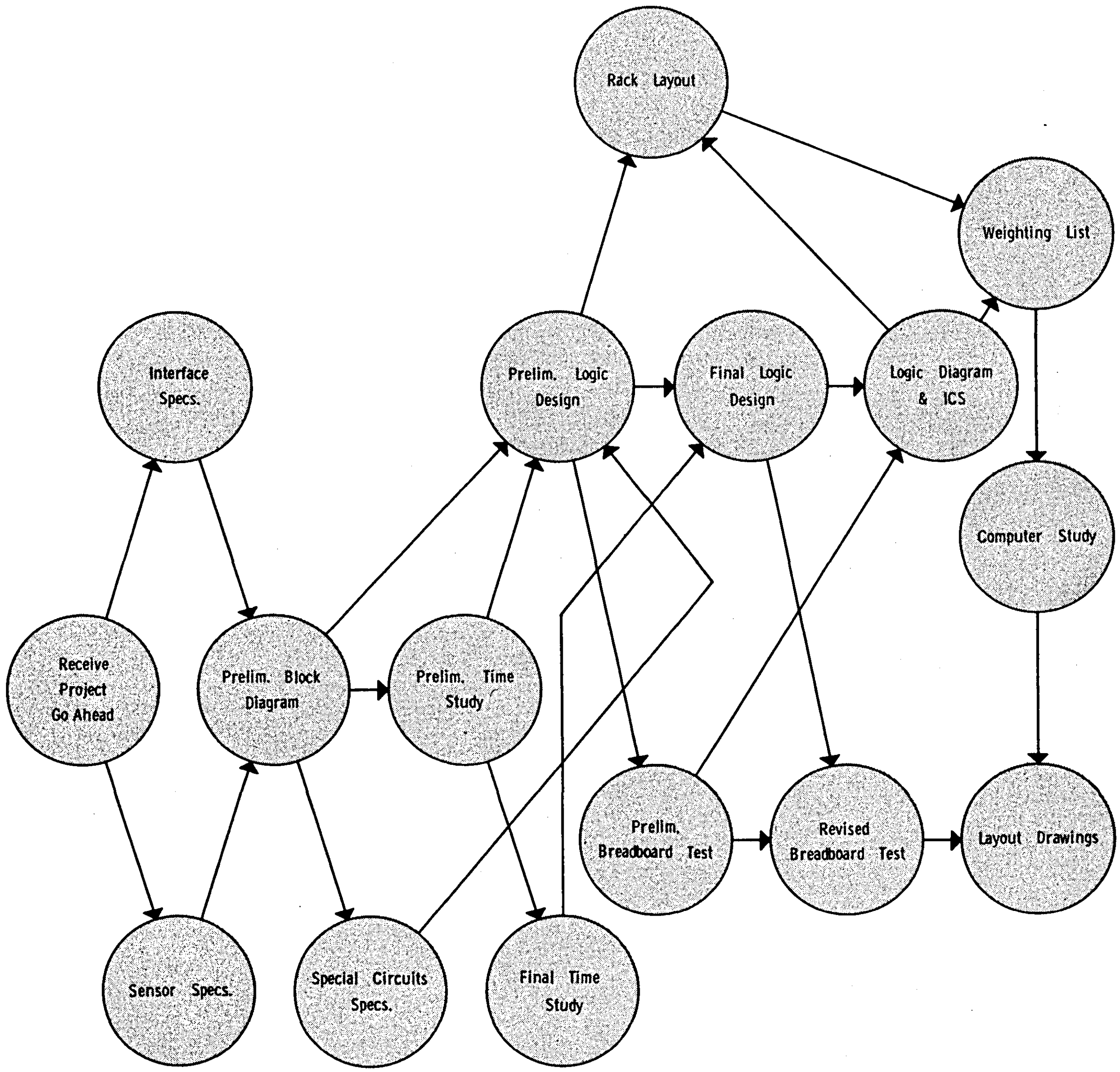
Events are sometimes placed within horizontal bands across the network to show departmental or functional areas. They may also be placed in logical sequence from left to right. A horizontal time scale should not be placed on the chart because the estimator may use it in determining the scheduled elapsed time between events. At this point, the chart is a series of circles or squares containing written descriptions. After all events have been placed on the chart, identification numbers are then assigned to each.



Event Layout

Activities

An activity links two successive events in a PERT network and is represented by an arrow. It cannot be started until the first event has been accomplished. The second event connected by this activity cannot be considered accomplished until all activities leading to the event are accomplished.

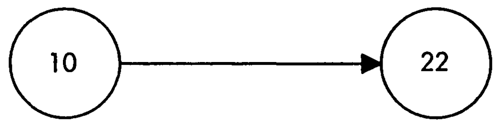


Activity Layout (Connecting Events)

Each activity is defined by its initiating and terminating events. Activities require manpower, material, facilities, space, or other resources. Activities, therefore, represent work; events represent specific accomplishments that are the result of work. Some users are activity-oriented in their planning and control operations while others are event-oriented.

Activities must be precisely defined by qualified personnel since they will subsequently be used as a basis for estimating performance time. Activities may include production, fabrication, testing, development, analysis, administration, research, and decision making. More specific examples of activities are (1) the fabrication which takes place between the delivery of a missile motor chamber and the availability of the specific test vehicle for firing, and (2) the test program which takes place between the first development test and the qualifications of the motor firing. This last activity can be broken down into subactivities which would most likely include development analysis, administration, and decision making.

Connections or arrows are drawn between events according to present plans and expectations.



For example: "According to present plans, events numbered 10 and 22 are related; number 22 cannot be accomplished unless event number 10 is accomplished. The activity between them cannot begin until event 10 is accomplished. Furthermore, it will be impossible to reach that point in progress represented by 22 until the activity between them is completed. More specifically: An inspection of an assembly cannot be started until it is assembled, the assembly cannot begin until the components are fabricated, etc."

Time Estimates

The same person who determines the activity between events should also provide those estimates of elapsed time for performance of the activity. He should have full knowledge of a "fixed resources mix" available to him and should be capable of performing the activity. The three time estimates which are then to be submitted are:

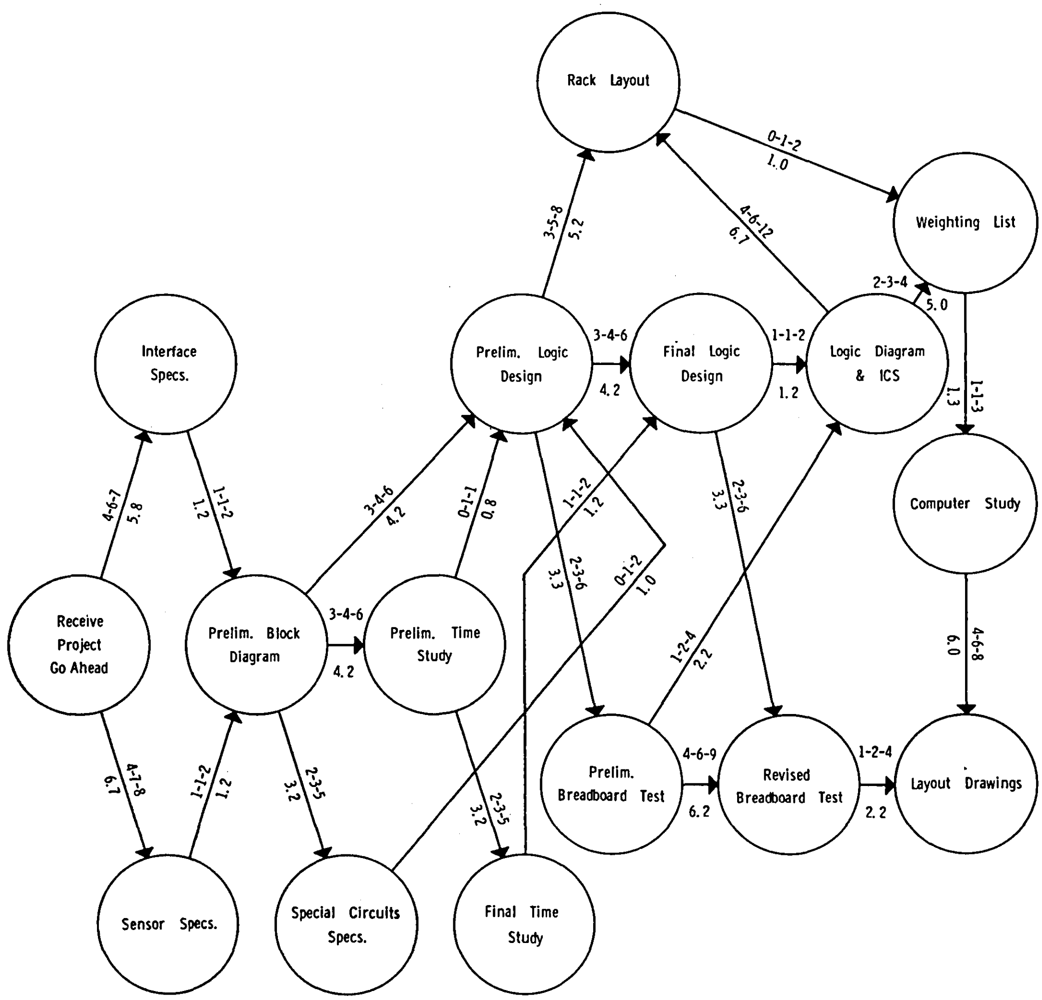
Optimistic Time — the shortest possible time in which the activity can be accomplished.

Most Likely Time — the time estimate which would be made if only one were requested. It is also the time that would occur most often if the activity were repeated under exactly the same conditions many times; or it is the one that would be given most often if many qualified people were asked.

Pessimistic Time — the longest time that the activity would take. It should be a time that is exceeded no more than once in a hundred occasions.

The three time estimates should be based on elapsed calendar time (days, weeks or months including holidays) rather than upon work days, which may or may not represent a standard 40-hour week. Once submitted, time estimates should not change unless there is a corresponding change in work content, rate of application of resources, or more knowledge which would improve estimates. These include revision of plans, introduction of new resources, change in personnel, technical difficulties or breakthroughs, and authorization of overtime.

Time estimates should be entered on the flow charts along the arrows to which they apply.



Completed PERT Network

In estimating the time for each activity, the estimator should record who or what skill is required for the task, quantity needed, facilities and material required and the point at which they are required. It should be assumed that normally available or expected resources will remain constant throughout the program. The importance of a realistic estimate cannot be overemphasized.

The existence of a calendar schedule date makes it difficult to obtain realistic estimates. This restriction inhibits clear thinking about the many factors which must be drawn together in forming estimates. Its effect may be reduced by:

1. Obtaining estimates for activities in various parts of the network in a random fashion.

2. Breaking down a larger network into subnetworks and obtaining estimates for the smaller sections separately.

3. Publicizing scheduled or contractual dates subsequent to the collection of estimates.

PART TWO: PERT Network Computation

A typical PERT computer program performs the following functions:

- Lists the sequence of events.
- Determines the characteristics of the probability curve for performance times of activities.
- Synthesizes network and time data to indicate the relation to program deadlines.
- Indicates critical areas of slack.
- Compares current forecasts against scheduled completion dates and computes the probability of meeting the scheduled dates.
- Provides at any time, for top management, a summary of progress and the outlook for future progress.
- Rapidly computes the effects of alternate courses of action.

In order to feed the network data into the computer, it is first transcribed on an input data sheet and then punched into cards. Control totals should be provided. All entries on the input data sheet are to be checked against the completed network for:

- Accuracy of transcribed time estimates.
- Accuracy of event numbering.
- Accurate transcription of all activities from the network.
- Compliance with recording specifications.

Sample Input Data Sheet

PREPARED BY____________________

PERT

INPUT DATA SHEET

DATE ________ PAGE ________ OF ________

PUNCH HEADER IF APPEARS ☐

HEADER CARD

1–6	7	8–49	50–53	54–59	60–65	66–70	71–73	74–80
0 0 0 0 0 0		NETWORK TITLE	FILE ID	RUN DATE	NETWORK START DATE		CASE	

Column 7: =1 WRITE NEW FILE, =2 MODIFY, =3 RERUN, =4 USE CARDS ONLY

ANALYST MOD CODE-
A = ADD, C = CHANGE, D = DELETE

COMP CODE (REFERS TO SCHEDULED OR COMPLETION) -
BLANK = SCHEDULED, 1 = COMPLETED, 3 = END EVENT

KEYPUNCH
PUNCH AS WRITTEN

MUST BE FILLED IN FOR PROPER PROCESSING

66–68	69–70	71–73	74–80
0 0 0	0 0	CASE	

1 CARD

SEQ NO (2–6)	PRECEDING EVENT (7–18)	19	SUCCEEDING EVENT (20–31)	32	ELAPSED TIME (WEEKS) OPTIMISTIC (33–36)	E (37)	MOST LIKELY (38–41)	E (42)	PESSIMISTIC (43–46)	E (47)	SCHEDULED OR COMPLETED (DATE) MO / DAY / YR (48–53)	55–64	65
0 4						3		3		3			
0 3						3		3		3			
1 2						3		3		3			
1 6						3		3		3			
2 0						3		3		3			
2 4						3		3		3			
2 3						3		3		3			
3 2						3		3		3			
3 6						3		3		3			
4 0						3		3		3			
4 4						3		3		3			
4 3						3		3		3			
5 2						3		3		3			
5 6						3		3		3			
6 0						3		3		3			
6 4						3		3		3			
6 3						3		3		3			
7 2						3		3		3			
7 6						3		3		3			
8 0						3		3		3			
8 4						3		3		3			
8 3						3		3		3			
9 2						3		3		3			
9 6						3		3		3			

2 CARD

ACTIVITY TITLE (7–42)	DEPARTMENT CODE: DEPT (43–48)	GRP (49–65)

KEYPUNCH
1 STANDARD 2 DUPLICATE COL 2-6 OF 1 CARD IN COL 2-6 OF 2 CARD

Printed output which can be obtained as a result of computer runs includes activity listings by: (1) successor and predecessor event numbers, (2) paths of criticality (in this report activities are grouped by equal slack) and (3) schedule or latest allowable date for activity completion.

Examples of these three are illustrated below. Other analyses which can be made as a result of processing this data are activities ranked by the number of paths in which they occur and activities ranked by probability of delay.

PERT SYSTEM PAGE 1

RUN 1 ENDING EVENT DATE 07-18-6
BY SUCCESSOR EVENT NUMBER AND PREDECESSOR EVENT NUMBER.
CHART 4A REVERSE ROTAR ASSY

EVENT		NOMENCLATURE	DEP.	DATE		DATE	PROB	SLACK	EXP.	EXP.
PREDECESSOR	SUCCESSOR			EXPECTED	ALLOWED	SCHD/ACT.			TIME	VAR.
0000-001	0000-101	0 START PERT NET			12-26-6	A01-01-6		- .9	+	
0000-901	0000-102	107 00072 PRELIMINARY DIAG	004	08-26-6	11-22-6			+12.6	+ 5.6	.6
0000-102	0000-103	7 00072 PRELIM TIME STUDY APPRVL	004	09-08-6	12-06-6			+12.6	+ 7.5	.7
0000-901	0000-103	107 00072 PRELIM TIME STUDY APPRVL	008	08-26-6	12-06-6			+14.5	+ 5.6	.6
0010-103	0000-103	7 00072 PRELIM TIME STUDY APPRVL	004	08-28-6	12-06-6			+14.3	+ 5.8	.5
0000-103	0000-104	0 00072 SPEC CIRCUITS DESIGN	008	09-24-6	12-21-6			+12.6	+ 9.7	.9
0000-303	0000-104	0 00072 SPEC CIRCUITS DESIGN	008	08-19-6	12-21-6			+17.7	+ 4.6	.6
0000-901	0000-104	100 00072 SPEC CIRCUITS DESIGN	008	08-26-6	12-21-6			+16.7	+ 5.6	.6

PERT SYSTEM PAGE 1

RUN 1 ENDING EVENT DATE 07-18-6
BY PATHS OF CRITICALITY
CHART 4A REVERSE ROTAR ASSY

EVENT		NOMENCLATURE	DEP.	DATE		DATE	PROB	SLACK	EXP.	EXP.
PREDECESSOR	SUCCESSOR			EXPECTED	ALLOWED	SCHD/ACT.			TIME	VAR.
0001-122	0001-421	109 00070 DEL ROTOR TO TEST	744	04-23-6	04-27-6	03-31-6	.07	+ .6	+39.8	4.9
0001-122	0010-106	0 00070 SCHED DEL	104	04-23-6	04-27-6	04-27-6	.61	+ .6	+39.8	4.9
0010-106	0010-109	00072 PROJECT END		04-23-6	04-27-6	04-27-6	.61	+ .6	+39.8	4.9
0001-301	0000-603	100 00002 TIME STUDY FOR TS	622		02-20-6	A02-13-6		+ 1.0	+	
0000-603	0000-604	100 00002 TS LAYOUT	622		02-20-6	A02-13-6		+ 1.0	+	
0001-901	0000-604	100 00002 TS LAYOUT	622		02-20-6	A02-13-6		+ 1.0	+	

PERT SYSTEM PAGE 1

RUN 1 ENDING EVENT DATE 07-18-6
BY DEPARTMENT, SCHEDULE OR LATEST ALLOWABLE DATE, AND SUCCESSOR EVENT
CHART 4A REVERSE ROTOR ASSY

EVENT		NOMENCLATURE	DEP.	DATE		DATE	PROB	SLACK	EXP.	EXP.
PREDECESSOR	SUCCESSOR			EXPECTED	ALLOWED	SCHD/ACT.			TIME	VAR.
0000-804	0000-705	5 0002 STUDY MADE	425		03-26-6	A02-13-6		+ 5.8	+	
0001-901	0001-902	100 0002 LOGIC DESIGN	425		02-07-6	A02-13-6		- .9	+	
0001-902	0001-903	175 0002 APPROVE DESIGN	425		03-26-6	A02-13-6		+ 5.8	+	
0000-804	0001-405	250 0002 BLACKBOARD TEST	425		04-21-6	A04-14-6		+ 1.0	+	
0001-404	0001-405	250 0002 BLACKBOARD TEST	425		04-21-6	A04-14-6		+ 1.0	+	
0001-904	0001-905	35 0002 DESIGN INCORPORATED	425		07-02-6	A05-22-6		+ 5.8	+	
0002-502	0002-505	0 0002 LAYOUT CHECKED	425		05-21-6	A05-27-6		- .9	+	

Slack

One of the most important attributes of PERT is its use in telling management whether a program is on, ahead of, or behind schedule. This is possible through the determination of slack, which treats the effects of those activities contingent upon any given activity.

Slack is the difference between the expected time (T_E) and the latest allowable completion time (T_L) for each event. It is the amount of time an event can be delayed without affecting the schedule, and it indicates to the manager those areas where manpower and/or funds can be shifted to a more critical area of the network if necessary.

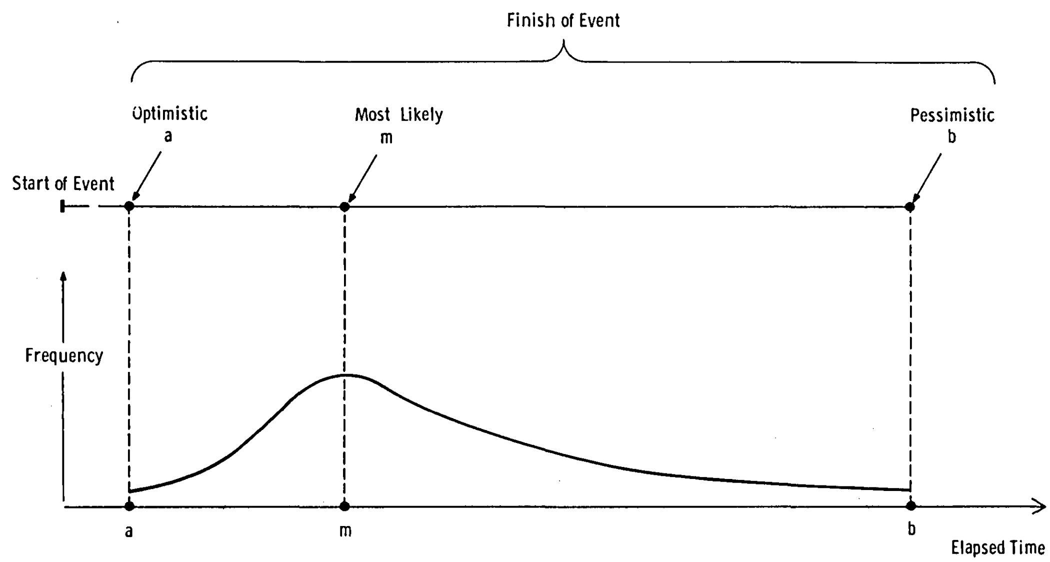
There are three basic steps prior to computing slack:

1. Compute mean time and variance for each activity.
2. Establish calculated expected time (T_E).
3. Establish calculated latest allowable time (T_L).

Establish Mean Time, t_e, and Variance, σt_e^2

The mean and variance of an activity performance time are derived from the three estimates of elapsed time. A set of assumptions makes possible a translation of these estimates into characteristics of a probability distribution of activity performance times. By obtaining three estimates, rather than only one estimate, the time to accomplish a task is expressed in terms of likelihood rather than positive assurance. Likelihood, in turn, can be expressed in terms of statistical probability and distribution curves.

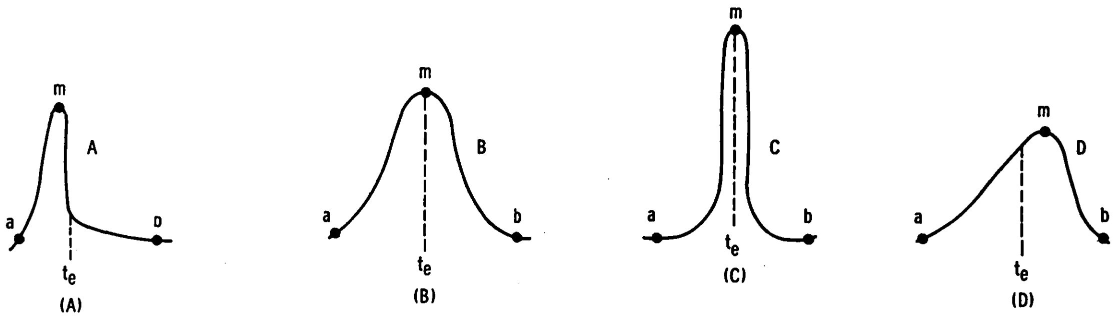
Three time estimates for an activity are depicted along the top line of the illustration below. An activity is started, and its completion is estimated at some future point in time. Thus the points "a," "m," and "b" correspond respectively to the optimistic, most likely, and pessimistic estimates.



Theoretical Frequency Distribution of Performance Times

In the lower portion of the illustration is the curve representing the frequency of occurrence of various times which it is assumed would occur if the activity were to be performed a large number of times. This curve is assumed to have only one peak, the most likely time for completion. Thus the point "m" is representative of the most probable time. Similarly, there is relatively little chance that either the optimistic or pessimistic estimates will be realized. Hence, small probabilities (about one in a hundred) are associated with the points "a" and "b." No assumption is made about the position of the point "m" relative to "a" and "b." It may take any position between the two extremes, depending entirely on the estimator's judgment.

A variety of forms which this distribution can take are illustrated on the following page.



Problem: Given three estimates of elapsed time, find t_e, the expected value (mean) and $\sigma_{t_e}^2$, (variance) of distribution when distribution form varies as shown above

$$\left.\begin{array}{l} a \text{ - optimistic estimate of interval} \\ m \text{ - most likely time of interval} \\ b \text{ - pessimistic estimate of interval} \end{array}\right\} \text{Obtained for Each Interval}$$

Solution: An estimating equation was developed which gives estimate of mean and variance for range of distributions to be encountered

Mean: $$t_e = \frac{a + 4m + b}{6}$$

Variance: $$\sigma_{t_e}^2 = \left(\frac{b - a}{6}\right)^2$$

Determining Mean and Variance of Performance Time Distributions

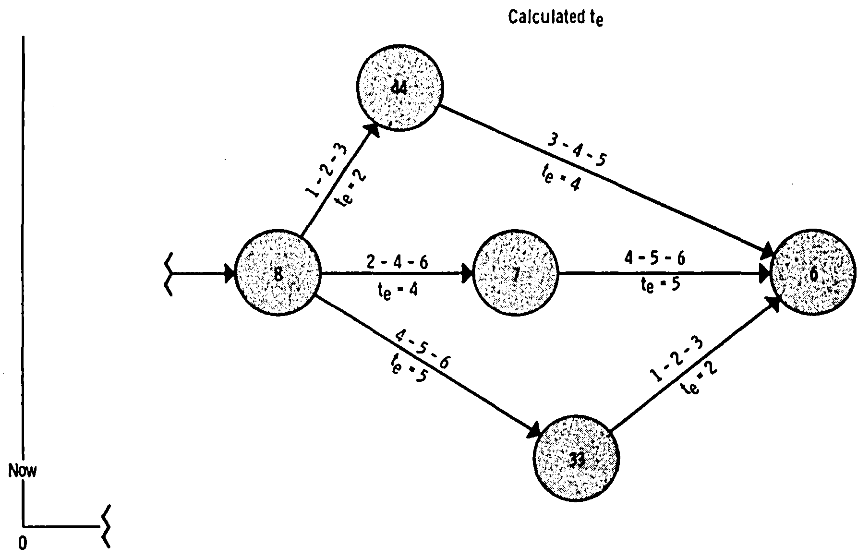
In order to make statistical inferences about the times at which future events will be accomplished, it is necessary to typify intervals between adjacent events with their expected values and variances. The expected value is a statistical term that corresponds to "average" or "mean" in common parlance. The variance is a term that is descriptive of the uncertainty associated with the process. If the variance is large (e.g., when optimistic and pessimistic estimates are far apart), there is great uncertainty as to the time at which the activity will be completed. If the variance is small, the uncertainty is relatively small.

Mathematical investigation of the various distribution types as above, yields the estimating equations at the bottom of the illustration. The application of these formulas to each interval between events will yield the values of "mean time" and "variance."

In the following network, event 8 is the predecessor and event 44 the successor of the activity connecting the two. The activity connecting events 8 and 44 must be completed before event 44 is considered complete, and before the activity connecting events 44 and 6 can start.

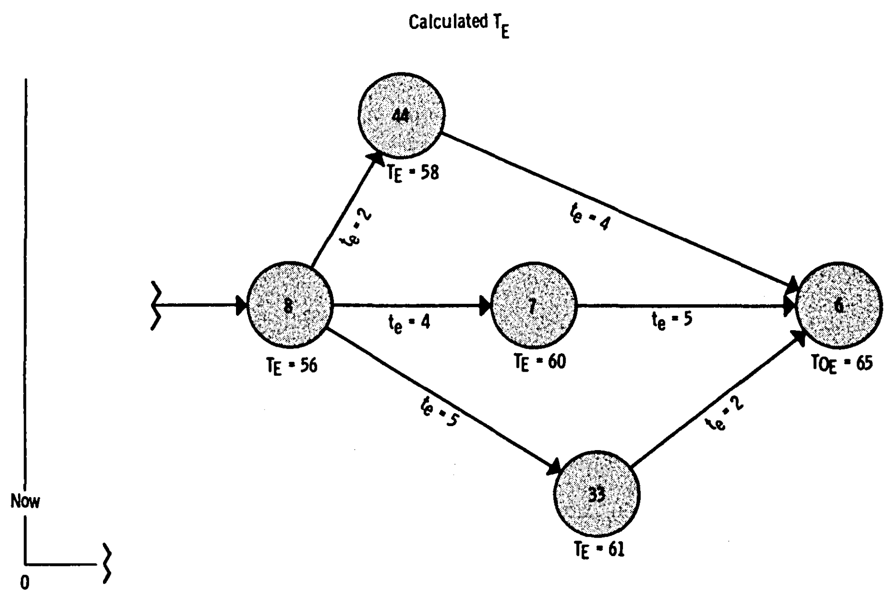
The three time estimates — optimistic, most likely and pessimistic — associated with each activity are shown above the activity arrow. Those estimates correspond to a, m and b respectively in the previous discussion.

From the three elapsed time estimates, mean time, t_e, and its associated variance, σt_e^2 are computed for each activity.



Establish Calculated Expected Time (T_E)

Next the expected time or T_E is calculated for each event in the network. T_E represents the expected time of completion of an event measured from the base line and is obtained by accumulating t_e for the activities preceding the event. Sums of activity mean times (t_e) are determined for all possible paths leading to the event. The largest of these sums is established as the T_E for the given event. In the illustration, t_e for the activity between 8 and 44 is two weeks. This is added to T_E (56) for event 8 to obtain T_E (58) for event 44.



The t_e for activity connecting events 8 and 33 is five weeks. This, added to 56 (T_E for event 8), gives 61 (T_E for event 33). The t_e for activity connecting events 8 and 7 is four weeks. This, added to T_E for event 8 gives T_E of 60 weeks at event 7. The t_e for activity connecting events 7 and 6 is five weeks. This, added to T_E for event 7, gives a T_E of 65 weeks at event 6. The completion of event 6, the end objective, will thus be 65 weeks after the project is begun. Since this is the greatest of the three T_E's computed for event 6, it is the value of TO_E (expected time for completion of the objective event).

Establish Calculated Latest Allowable Time (T_L)

The latest allowable time, T_L, derives from the establishment of some future date, such as a desirable program completion date or a contractual obligation. The T_L for an event is located at a point in time such that if succeeding events are completed as anticipated the scheduled program completion date will be precisely met. T_L is both the latest allowable completion date for preceding activities and the latest allowable starting date for the most critical succeeding activity.

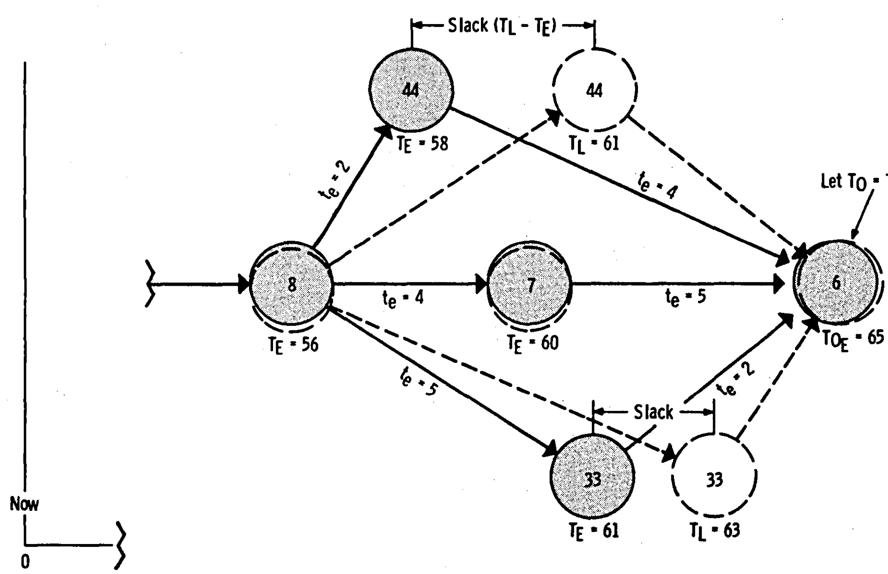
The latest allowable time is calculated conversely to T_E. Activity mean times are cumulatively subtracted from a scheduled date, or TO_E when a date has not been fixed, along the various paths between a given event and the objective event. T_L is selected as the smallest of the possible values thus obtained. For example, set T_L of event 6, the end objective, equal to T_E of event 6 and work backward through the network.

The t_e for activity connecting events 44 and 6 is four weeks. This, subtracted from T_L for event 6, gives a T_L at event 44 equal to 61 weeks.

The t_e for activity connecting events 33 and 6 is two weeks. This, subtracted from T_L for event 6, gives a T_L at event 33 equal to 63 weeks.

The t_e for activity connecting events 7 and 6 is five weeks. This, subtracted from T_L for event 6, gives a T_L at event 7 equal to 60 weeks.

Calculated T_L



The t_e for activity connecting events 8 and 44 is two weeks. This, subtracted from T_L for event 44, gives a T_L at event 8 equal to 59 weeks. Still to be considered is the activity connecting events 8 and 7 and 33. The t_e for this activity (8 and 7) is four weeks; this, subtracted from the T_L for event 7, gives a T_L for event 8 equal to 56 weeks. This, being the smallest of the three T_L's computed for event 8, is selected as the T_L for event 8.

Computing Slack

Having computed both a T_E and a T_L for each of the events, it is possible to compute the slack associated with each event. The formula for slack is $T_L - T_E$.

For events 8, 7 and 6 the value of slack is zero. Events having zero slack are called critical events since any delay in activities between them could cause an increase in the T_E for the end objective, event 6.

Event 44 has three weeks' slack, which indicates that its connecting activities could take up to three weeks longer than estimated, and not delay the completion of event 6.

Event 33 has two weeks' slack.

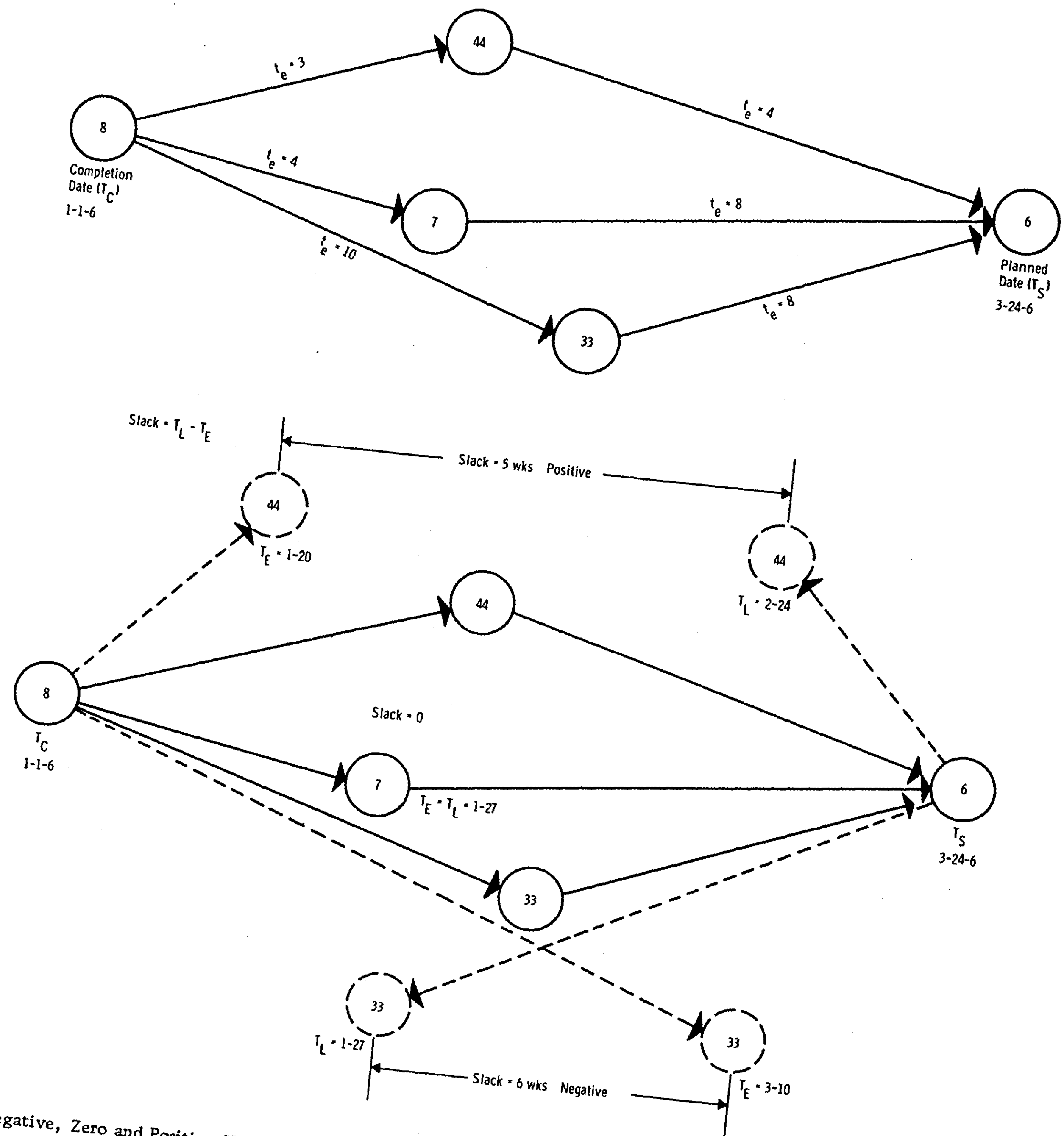
A large amount of positive slack in an event indicates a place where resources might be available for possible trade-offs. A slack value approaching zero indicates that the event in question is likely to be a potential trouble spot. Results indicate that resources may be diverted from activities 8-44 and 8-33 if events 7 and 6 appear to slip.

Critical Path

The determination of slack permits an analysis of the criticalness of the various activities in a program. By grouping activities according to amount of slack, several paths will be defined which vary in their criticalness. It is possible to have two or more critical (and parallel) paths.

Criticalness is measured in times of negative, zero or positive slack. Positive slack indicates an ahead-of-schedule condition, while negative slack indicates a behind-schedule condition, and zero slack indicates an on-schedule condition with a probability of .5. Negative slack occurs when the total activity mean time along a path is greater than the time available to meet program requirements.

One path is often found which is considerably more critical than others and is called the critical path. The critical path of a network is the path between the base line and the end objective that requires the greatest amount of time. It is found by totaling individual mean times (t_e) along every possible path in the network, and selecting the sequence of activities/events requiring the greatest amount of time. When any event on the critical path falls behind its expected date of accomplishment, it can also be expected that the final event will fall behind.

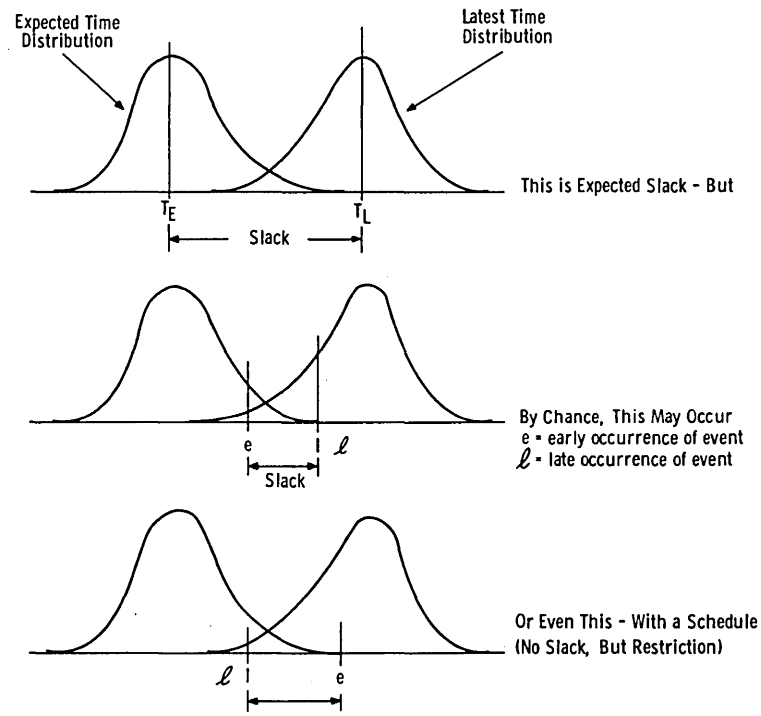


Negative, Zero and Positive Slack

Actual vs Expected Slack

The computed slack time is that which is expected, on the basis of schedules and estimates. Actual or observed slack will generally be different from the expected; its value will depend upon the exigencies of the R&D process and their probabilities. Theoretical analysis allows assumptions to be made about the actual slack that will develop.

In the illustration below, the expected slack is depicted in the top row. However, since change factors are at work, the slack may be smaller than anticipated; this is depicted in the middle row. In the same sense, actual slack may be larger than expected; chance may make the latest allowable time for an event fall earlier than the expected time. Such a situation is depicted in the bottom row.

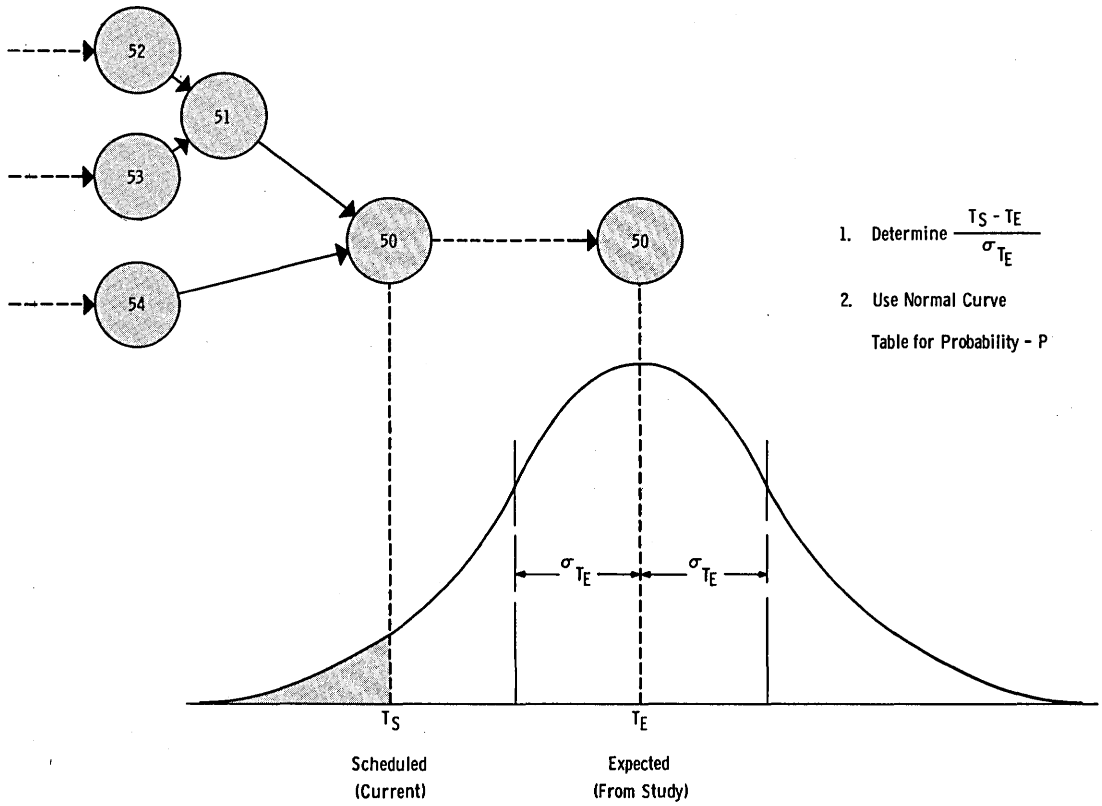


Risk

Often a schedule will be determined for the completion of the objective event and/or other strategic events. The network computation described thus far provides a basis for determining the level of risk involved in meeting such a scheduled date, T_S. PERT can be used to determine the risk involved in completing a given activity, in completing a portion of a program, or in completing an entire program. These calculations permit the adjustment of a schedule so as to arrive at a level of risk acceptable to management.

Risk is defined in terms of probability, which can be computed by the formula shown in the illustration on the following page.

In order for management to use this probability information, late performance must be evaluated, by cost penalties and other kinds of penalties for failure to meet obligations. The probabilities obtained provide only part of the information needed for management decisions.



Estimate of Probability of Meeting Scheduled Date - T_{os}

In the numerator of this formula, the expected time, T_E, is subtracted from the scheduled time, T_S. The quantity in the denominator is determined by summing the variances for the same activities used in arriving at T_E, and taking the square root of this sum. The result of these calculations may be referred to a table such as that found in Part IV to determine the resulting level of probability. An example of these calculations appears in Part IV. The degree of risk, or probability of completing on time, is indicated by the shaded area in the above illustration.

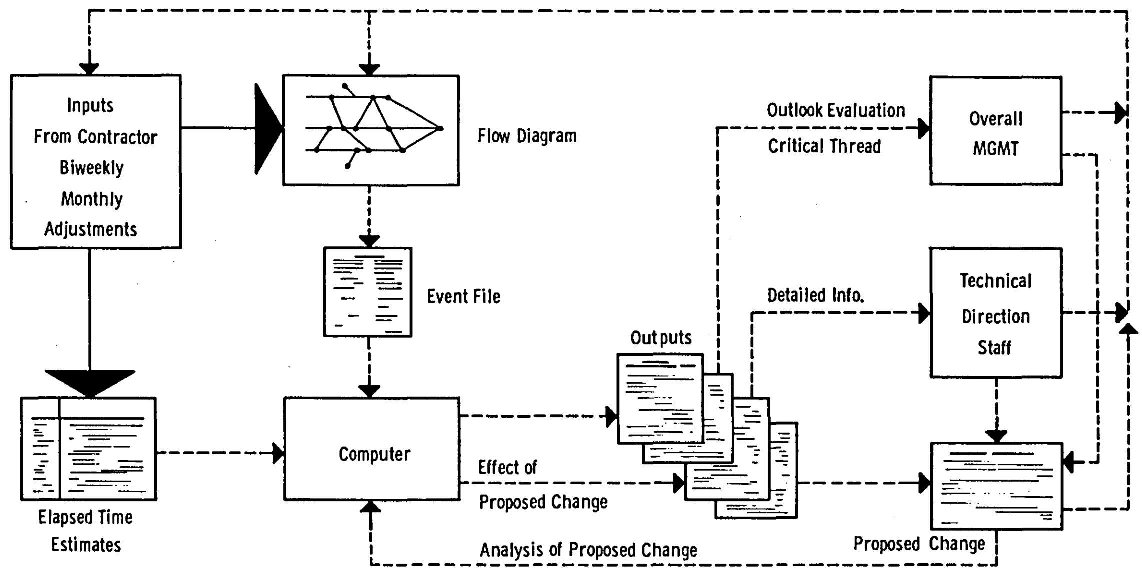
An advantage of PERT is that this same class of calculations of probability can be used for positive management planning to establish dates as well as to adapt to given dates. Once a management policy is established as to what constitutes a desirable level of risk in meeting schedules, a schedule can be developed which is based on that policy and which uses the basic PERT network data. For each activity an appropriate date would be determined having the same given level of risk.

PART THREE: PERT Operation

PERT Cycle

The illustration below depicts the cycle through which PERT supplies a continuous stream of timely program evaluation.

The cycle begins with computer inputs from the R&D group or agency charged with meeting a major end objective. The output reports require analysis. Necessary changes or corrections are submitted for further revision and redistribution.



PERT System in Operation

Periodic requests for additional data will focus attention on relatively critical paths and permit the audit of a few specified events. Re-estimates may be requested for those times that lie along the critical paths. This serves two purposes: (1) emphasis is placed on "tight" areas, and (2) estimates in these areas are marked for critical analysis in an effort to produce more accurate data.

Auditing improves the accuracy of the flow chart because estimates are requested for a small group of selected activities which differ from month to month. All events on the chart will be covered at one time or another.

Analysis

To be effective, the entire project must be incorporated into one network. Otherwise the critical path through connecting events may not be evident.

Analysis is the evaluation of original plans in light of current operating conditions. It will indicate the value of various alternatives and their effect on objectives. To be effective, analysis should start early in a project and continue throughout it.

The effectiveness of an evaluation system is measured by the speed with which out-of-phase situations can be recognized and corrected. To permit this, PERT can be programmed on different computer systems.

Network analysis should be in keeping with an established management policy regarding the level of risk to be assumed in meeting schedule dates. It may occur in the following manner:

1. Analyze each component system within a subsystem independently, and compute all outputs on the basis of the component system only. (For example, analyze ballistic shell, propulsion, etc., each by itself.)

2. Analyze each subsystem, and compute all outputs on the basis of the subsystems only. Analyze to show relationship within each subsystem only.

3. Analyze the complete project, and compute all outputs on the basis of the entire system.

Upon completion of the analysis, top-level management or the technical direction staff may choose to (1) make adjustments and trade-offs in plans, schedules, resources, or performance specifications, or (2) test the effects of different decisions by simulating each on the computer.

Continuous Operation

The dynamics of the R&D development plan require that flow chart information be current. PERT provides for this by requiring in-house managers and subcontractors to submit periodic reports from which changes are posted on a biweekly, monthly and quarterly basis.

At intervals the manager or subcontractor reports on the status of all events that have been completed or scheduled for completion during the past two weeks. In addition he indicates any anticipated changes, problem areas and corrective action to be taken.

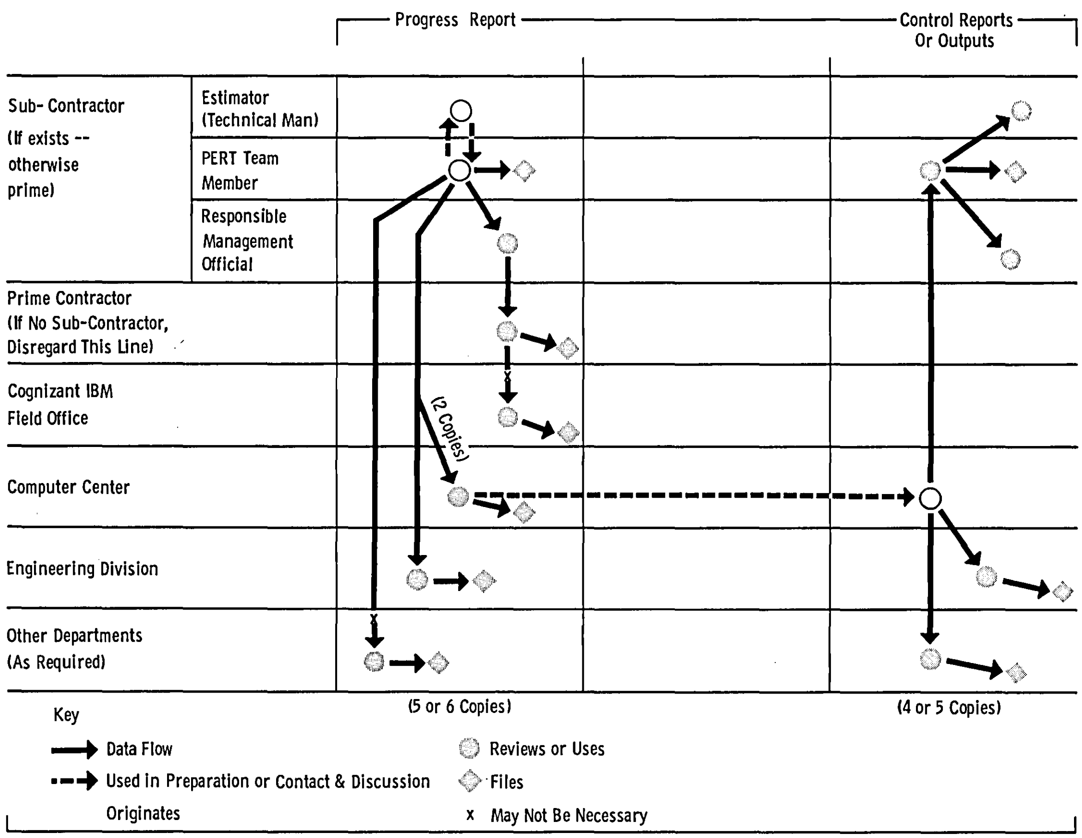
Monthly updating provides the means for examining those activities that are currently most critical. It does not affect the majority of the events in the R&D project. Therefore, it is necessary to evaluate the entire network quarterly.

Essentially the same procedure is followed in revising flow charts quarterly as is used in setting them up originally. A team of representatives critically examines all events and times in the existing flow charts. Revisions and re-estimates are made whenever necessary. All changes are then consolidated according to standard procedures and incorporated in subsequent computer runs.

Results of the quarterly updating provide management with information to re-evaluate the entire development and thus obtain a fresh prognosis.

As PERT is applied to more and more projects, it will be necessary to carefully plan the time at which reports will be submitted. Report cycles should be staggered for the various subsubsystems and components, with the biweekly, monthly and quarterly reports scheduled at different times. Such scheduling of reports should result in a constant workload with little week-to-week variation.

DATA FLOW DIAGRAM - PERT TRANSMITTING BIWEEKLY REPORTS



Data Flow Diagram — PERT Transmitting Biweekly Reports

Revised Plans

After examining initial computer outputs for a particular component or subsystem, management may decide to develop and test hypothetical plans. Those events that are marked as "probably being reached ahead of schedule" indicate possible areas where resource trade-offs might be arranged. Events along the critical (zero slack) path indicate possible areas for reduction in performance requirements or increased resource application.

If the schedule for a major event is jeopardized, it may be possible to replan and thus improve the outlook for meeting the schedule. This replanning could take the form of altering the sequence of events and thus postponing certain activities to a later point in time. For example, it might be possible in certain situations to forego some testing before a flight.

Thus replanning would not force the flight to await all the preliminary tests that were originally planned.

In addition to the changes already mentioned, it is possible to set up the computer so that it will develop a new schedule based on some arbitrary set of criteria.

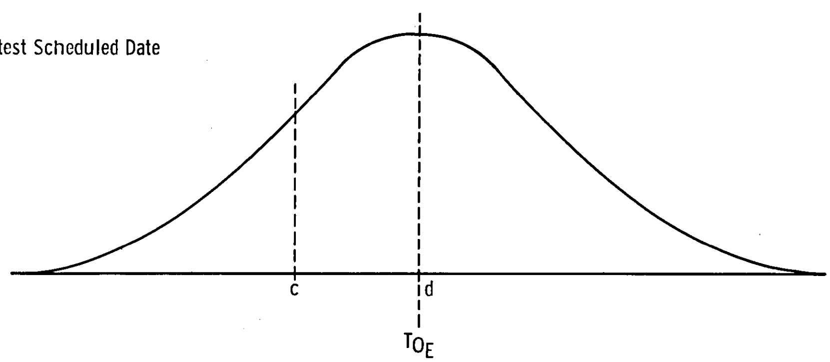
Data on such changes can be read into the computer and the analysis quickly performed.

Rescheduling Method

One reasonable approach to rescheduling is to extend the scheduled date of the objective event. If the scheduled date of the objective event is changed, the latest times for all events must be recomputed.

If the latest time occurs before the expected time, set the scheduled date at the latest time. Otherwise, set the scheduled date for the event at the point which maximizes the probability of its being met within the slack interval. This procedure is illustrated below.

1. Slip Latest Scheduled Date



To a Point Between c & d

At c: Pr Accomplishment = .25 (1 chance in 4)
At d: Pr Accomplishment = .50 (1 chance in 2)

A

2. Recompute All Latest Times
3. For Schedule of Interim Events –

Where $T_L < T_E$ Set $T_S = T_L$

$T_L \geq T_E$ Set T_S to Maximize $Pr\left[e \leq T_S \leq \ell\right]$

After establishing the new schedule, appraise future possibilities by recomputing the probabilities of "no slack" in the system. The use of probability varies among programs. For example, if the probability of no slack is in excess of 0.5, redeployment of resources and/or changes in performance should be considered. If the probability is approximately 0.5, management should closely monitor progress. If the probability of no slack is less than 0.5, progress should receive only routine checking.

By using this method of rescheduling, replanning becomes a continuous part of the PERT program. Therefore, program reduction is accomplished by replanning, elimination of desirable but unnecessary activities, the assumption of technological risk, or the utilization of parallel development.

Scheduling

Although PERT is an effective management control tool, it is also effective as a scheduling tool by using the variations in probability as a basis for determining the expected time.

When a PERT network is developed it must represent the plan for doing the overall task and the schedule must be realistic. Once established, it must be followed by the organization performing the activities.

Intermediate milestone schedules are important since they generate targets or goals to be attained. They are essential for communication between people performing the work, suppliers, program management, top management and the customer.

Establishing and maintaining a good schedule will require the following:

- Constant evaluation of progress.
- Good definition of activities.
- Awareness of the effect created by a delay.
- Events scheduled frequently.
- A feeling on the part of persons who must meet the schedule that they helped to establish it.
- Positive management direction.

PERT will fulfill the first four of the above considerations. The last two factors can be achieved by making the individual or team feel that he has played a part in establishing the goal. The plan must then be accepted by management.

The incorporation of a realistic scheduling system takes the guesswork out of setting schedules and provides many other internal schedules. There should, however, be only one schedule that is generated or checked by PERT.

To be effective, the scheduling technique should be related to an established probability for a desired level of risk. By use of the expected time (T_E), and by knowing the minimum probability that management is willing to accept on schedules for a given project, desired probabilities and the resulting expected times (T_E) for activity scheduling can be determined.

PART FOUR: Sample Network Calculations

The network depicted on page 27 consists of 11 events and 15 activities. To evaluate this network it is necessary to calculate the mean time (t_e) for each work element and its variance from the three time estimates associated with each activity. From this data other computations are easily obtained. Examples of computation for each column are as follows:

- Columns A and B are self-explanatory.
- The values of t_e listed under column C are calculated by the formula

  $$t_e = \frac{a + 4m + b}{6}$$

  where a is the optimistic time, m is the most likely time and b is the pessimistic time. (Example: Calculate t_e for activity times between events 0 and 1.)

  $$t_e = \frac{a + 4m + b}{6} = \frac{1 + 16 + 26}{6} = 7.2$$

- The values of σt_e^2 listed under column D are calculated by the formula

  $$\sigma t_e^2 = \left[\frac{(b - a)}{6}\right]^2$$

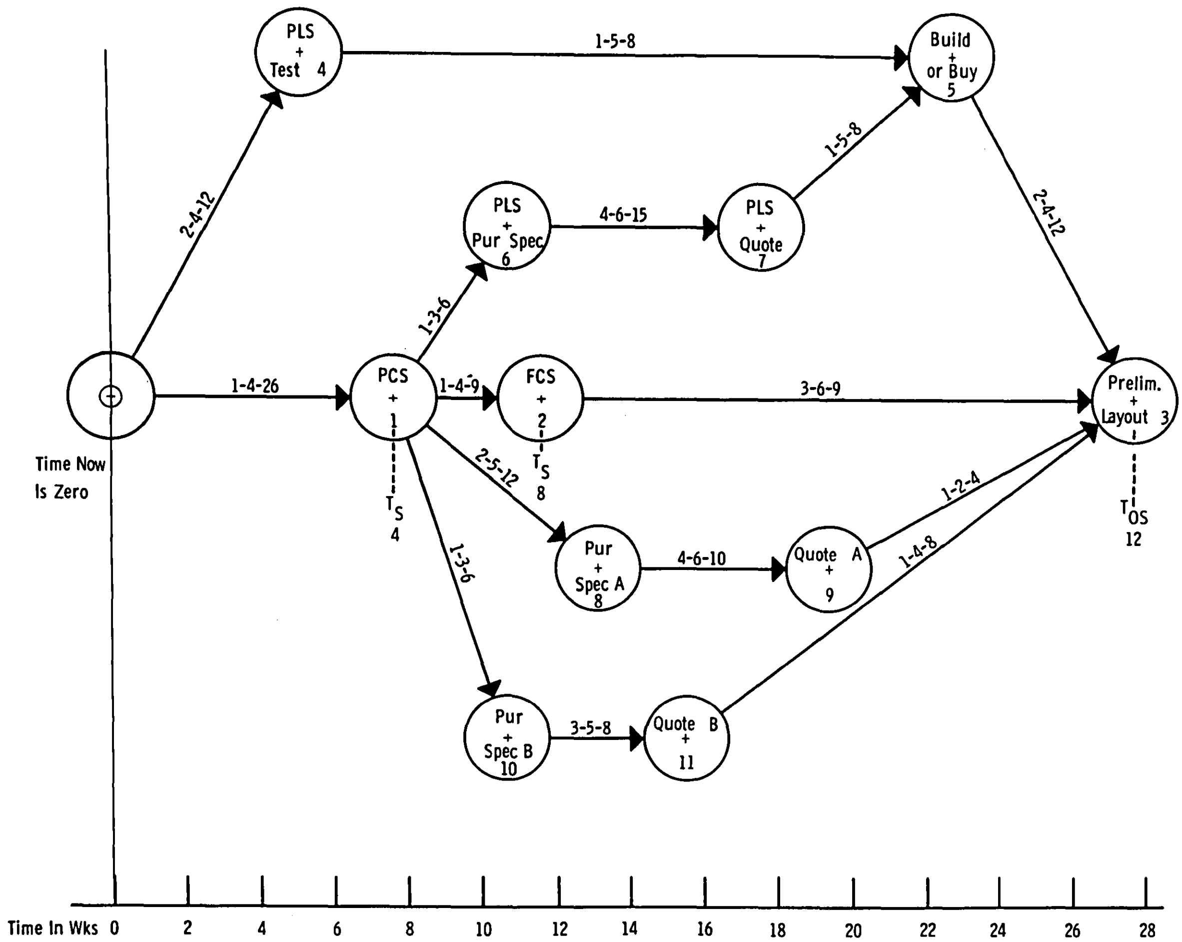
  where b is the pessimistic time and a is the optimistic time. (Example: Calculate σt_e^2 for activity times between events 0 and 1.)

  $$\sigma t_e^2 = \left[\frac{(26 - 1)}{6}\right]^2 = 17.35$$

- The values of T_E listed under column E are calculated in accordance with the procedure on determination of slack.
- The values of T_L listed under column F are calculated in accordance with the procedure on determination of slack.
- The values of $T_L - T_E$ (slack) listed under column G are calculated in accordance with procedure on determination of slack.
- The values of P_R listed under column J are the probabilities of meeting the manager's specified scheduled date for the applicable event. (Example: Calculate P_R for event 2.)

  $$\text{Determine } \frac{T_S - T_E}{\sigma T_E} = \frac{8-(7.2 + 4.3)}{\sqrt{17.35 + 1.77}} = -0.8$$

  In the above formula the expression T_E is further defined by the expression of 17.35 + 1.77, 17.35 being the value σt_e^2 (column D) for the activity between 0 and 1 while 1.77 is the value of σt_e^2 (column D) for the activity between 1 and 2. By use of the normal probability tables on page 30, the P_R value for a Z value of -0.8 is 0.2119.
- Slack time for each event is listed in sequence from low to high under columns M and N. Column M lists the amount of slack for associated event listed under column N.



A	B	C	D	E	F	G	H	J	M	N
Event	Preceding Event	t_e	σt_e^2	T_E	T_L	$T_L - T_E$	T_S	P_R	$T_L - T_E$	Event
3	5	5.0	2.78	27.4	27.4	0.0	12.0	.00	0.0	1
3	2	6.0	1.00						0.0	6
3	9	2.2	.25						0.0	7
3	11	4.2	1.36						0.0	5
5	7	4.8	1.36	22.4	22.4	0.0			0.0	3
5	4	4.8	1.36						6.0	8
7	6	7.2	3.35	17.6	17.6	0.0			6.0	9
6	1	3.2	.69	10.4	10.4	0.0			7.6	10
2	1	4.3	1.77	11.5	21.4	9.9	·8.0	.21	7.6	11
9	8	6.3	1.00	19.2	25.2	6.0			9.9	2
8	1	5.7	2.78	12.9	18.9	6.0			12.6	4
11	10	5.2	.69	15.6	23.2	7.6				
10	1	3.2	.69	10.4	18.0	7.6				
4	0	5.0	2.78	5.0	17.6	12.6				
1	0	7.2	17.35	7.2	7.2	0.0	4.0	.22		

TABLE OF VALUES OF THE STANDARD NORMAL DISTRIBUTION FUNCTION

z	0	1	2	3	4	5	6	7	8	9
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9430	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9648	.9556	.9664	.9671	.9678	.9686	.9693	.9700	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9762	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9874	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000

(Continued)

z	0	1	2	3	4	5	6	7	8	9
-3	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000
-2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0126	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0238	.0233
-1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0300	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0570	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
- .9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
- .8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
- .7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
- .6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
- .5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
- .4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
- .3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
- .2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
- .1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
- .0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

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