

FAR FIELD ANTENNA PATTERN CALCULATIONS BY MEANS OF A GENERAL PURPOSE ANALOG COMPUTER

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Abstract.—The complexity of antenna pattern calculations has, in the past, necessitated the use of the highest speed and largest capacity digital computers, such as the IBM 704.

The core of the problem is the evaluation of a surface integral of a complex, vector integrand of the form

$$\mathbf{E}(\theta, \phi) = \iint \mathbf{f}(z, y, x) \exp \{ -jg(z, y, \theta, \phi) \} dS$$

where \mathbf{E} is a complex vector, \mathbf{f} is an ordinary vector dependent on the horn pattern and the geometry of the antenna surface, and g is a scalar measure of the path length of a ray travelling from the horn, to the reflector and thence to the aperture plane through the focus. The particular "far field point" is defined by coordinates θ, ϕ . dS is the differential of the antenna surface.

To determine the feasibility of carrying out these calculations on an analog computer, the particular case of a circular paraboloid antenna surface was chosen. Both "on axis" and "off axis" horn locations were evaluated for far field points in the principal θ, ϕ planes.

The analog computer programs shown in detail and means of improving its efficiency are discussed.

It is concluded that the analog computer can successfully compete with the IBM 704 computer in this general problem area, with a possibility of a cost reduction factor of from three to ten.

I. INTRODUCTION

The general mathematical model for calculating far field patterns from near field measurements on antennas is well known and has been described.¹ The design engineer has the choice of using either the field distribution over an aperture, or the current distribution over the surface of a reflector. The latter gives a more complete answer. These calculations have in the past largely been performed on digital computers. For example, Ref. 2 describes the calculation of far field radiation for a cheese aerial from phase and amplitude measurements across the aperture of the aerial, by digital computer techniques. Reference 3 is a more general reference of digital computer techniques for antenna pattern calculations.

It is not generally known that analog computing techniques are applicable to this problem, and to illustrate these techniques we will consider the far field radiation pattern of a circular paraboloid reflector, using the current distribution as the input for the problem. To make the calculation more complex, the contour of the reflector edge projected on a plane perpendicular to the axis of the reflector is elliptical or, stated in another way, reflector edge does not lie in a plane. A three dimensionalized view of the antenna is shown in Fig. 1. In Fig. 2 the quantities of interest and their relation to the antenna surface are shown. Figure 3 defines the spherical coordinates r, θ, ϕ of the point P at which we shall evaluate the "far field."

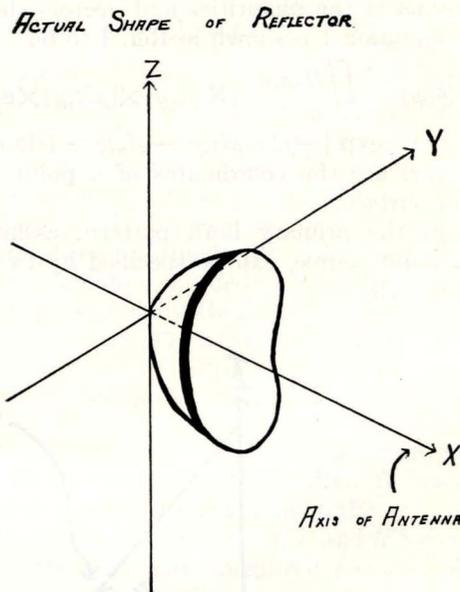


Fig. 1—Three dimensional view of antenna.

II. MATHEMATICAL FORMULATION

The problem, stated mathematically, is the evaluation of the surface integral of a complex vector of the form

$$(1) \quad \mathbf{E}(\theta, \phi) = \iint \mathbf{f}(z, y, x) \exp \{ -jg(z, y, x, \theta, \phi) \} dS$$

where $\mathbf{E}(\theta, \phi)$ is the field at the far point having

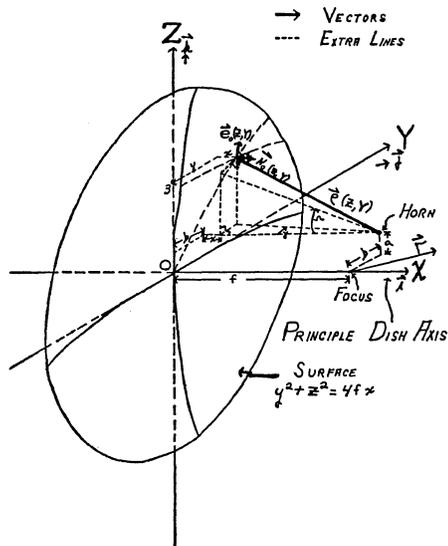


Fig. 2—Quantities at dish.

coordinates (θ, ϕ) , \mathbf{f} is a vector dependent on the horn pattern (current distribution) and the antenna surface and g is the phase function, which is a scalar measure of the path length of a ray travelling from the horn, to a point on the reflector and from the reflector parallel to the direction of r , to the aperture plane. The aperture plane is perpendicular to r and passes through the focus.

In terms of the quantities and vectors shown in Fig. 2, equation 1 is shown in Ref. 1 to be

$$(2) \quad \mathbf{E}(\theta, \phi) = \iint \frac{H(z, y)}{\rho^2(z, y)} \{ \mathbf{N}_0(z, y) \times [\rho(z, y) \times \mathbf{e}_0(z, y)] \} \exp \{ -j2\pi[\rho(z, y) - \rho(z, y) \cdot \mathbf{i}r(\theta, \phi)] \} dS$$

where y, z , are the coordinates of a point on the reflector surface.

$H(z, y)$ the primary horn pattern, assumed to have a point source, can be specified by two angle functions, viz.

$$(3) \quad H(z, y) = f(\Omega)g(\gamma)$$

where

$$(4) \quad \tan \gamma = \frac{y - b}{-(x - f)}$$

$$(5) \quad \tan \Omega = \frac{z - a}{-(x - f)}$$

The functions $f(\Omega)$ and $g(\gamma)$ are the experimentally measured patterns which are the basic inputs for the problem. Typical functions are shown in Fig. 4.

The constants a and b are the offset distances of the horn from the focus, as shown in Fig. 2. The constant f is the distance from the origin to the focus.

The equation for the circular paraboloid antenna surface is

$$(6) \quad z^2 + y^2 = 4fx$$

Since the edge of the reflector does not lie in a plane, it is necessary to specify the projection of the edge on the zy plane. For the particular antenna, the projection chosen is an ellipse whose equation is given by

$$(7) \quad z/(z_{\max})^2 + (y/y_{\max})^2 = 1$$

where z_{\max} and y_{\max} are the maximum values of the coordinates z and y on the reflector surface.

The unit normal \mathbf{N}_0 to the reflector surface can be obtained from standard calculus texts, such as Ref. 4, and is given by

$$(8) \quad \mathbf{N}_0 = \frac{2f\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{[(2f)^2 + y^2 + z^2]^{\frac{1}{2}}} \equiv \frac{2f\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{N}$$

The vector ρ from the horn to a point on the surface is given by

$$(9) \quad \rho = (x - f)\mathbf{i} + (y - b)\mathbf{j} + (z - a)\mathbf{k}$$

where the magnitude of ρ is

$$(10) \quad \rho = [(x - f)^2 + (y - b)^2 + (z - a)^2]^{\frac{1}{2}}$$

Further, \mathbf{e}_0 is a unit vector perpendicular to ρ and

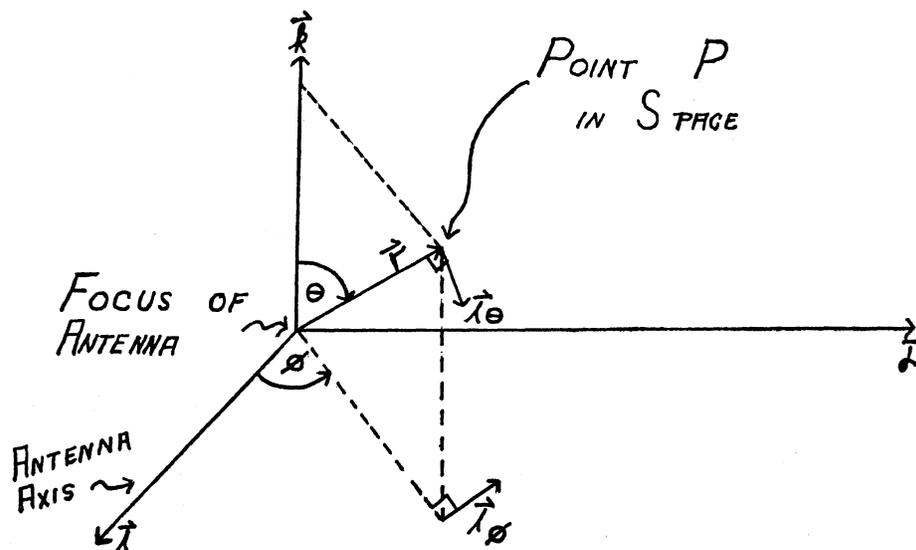


Fig. 3—Spherical coordinates of field point, P , with respect to focus of antenna.

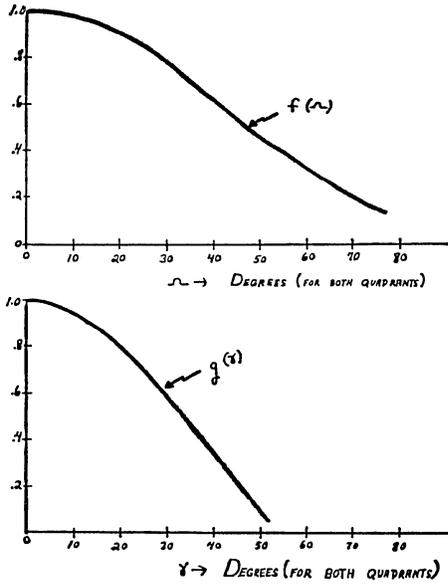


Fig. 4—Typical input functions of f and g .

lies in the vertical plane. It follows from the above definitions and the angles shown in Fig. 2 that

$$(11) \quad \rho \times \mathbf{e}_0 = \rho(\sin \gamma \mathbf{i} + \cos \gamma \mathbf{j})$$

Substitution of (11) and (8) into the amplitude factor of the integrand of (2), and expanding the vector product yields

$$(12) \quad \frac{H}{\rho^2} [\mathbf{N}_0 \times (\rho \times \mathbf{e}_0)] = \frac{H}{\rho N} [z \cos \gamma \mathbf{i} - z \sin \gamma \mathbf{j} + (2f \cos \gamma + y \sin \gamma) \mathbf{k}]$$

From the differential calculus (see Ref. 5 for example), it can be shown that, for a surface specified by (6) that

$$(13) \quad dS = (N/2f) dy dz$$

The phase angle of the integrand, $2\pi(\rho - \rho \cdot \mathbf{i}_r)$ is g of (1), where \mathbf{i}_r is shown in Fig. 3. The expansion of the dot product is

$$(14) \quad \rho \cdot \mathbf{i}_r = (x - f) \sin \theta \cos \phi + (y - b) \sin \theta \sin \phi + (z - a) \cos \theta$$

The vector \mathbf{E} of (2) is composed of real and imaginary parts which can be written, after substituting (13) and (12) into (2) as

$$(15) \quad \text{Re}(\mathbf{E}) = \iint_S \frac{H}{2\rho f} [z \cos \gamma \mathbf{i} - z \sin \gamma \mathbf{j} + (2f \cos \gamma + y \sin \gamma) \mathbf{k}] (\cos g) dy dz$$

$$(16) \quad \text{Im}(\mathbf{E}) = -\iint_S \frac{H}{2\rho f} [z \cos \gamma \mathbf{i} - z \sin \gamma \mathbf{j} + (2f \cos \gamma + y \sin \gamma) \mathbf{k}] (\sin g) dy dz$$

where the two integrands above differ only in the \cos and \sin functions of g and a minus sign.

Now, on the analog computer we can integrate continuously with respect to one variable. We can choose the variable z to be that independent variable. This is one of the major differences between the analog and digital techniques for this problem.

The integration with respect to the second independent variable y , however, must be performed as a discrete summation on the analog computer, similar to the discrete summations used on the digital computer.

Let the integrand of (15) be \mathbf{F} , so that we can write

$$(17) \quad \text{Re}(\mathbf{E}) = \sum_{i=1}^n \Delta y_i \int_{z_i} \mathbf{F}_i dz$$

By proper choice of logic one integrator can perform the integration and discrete summation, thereby converting an analog integrator into an "integrator accumulator." Of course, a similar expression can be written for (16). Note that since \mathbf{F} is a function of y , it must carry the index i , and that the limits of the continuous integration are also a function of i .

The vector components of (17) must be calculated separately, since the D.C. analog computer is a scalar computer. These are respectively,

$$(17a) \quad \text{Re}(\mathbf{E} \cdot \mathbf{i}) = \sum_{i=1}^n \Delta y_i \int_{z_i} (\mathbf{F}_i \cdot \mathbf{i}) dz$$

$$(17b) \quad \text{Re}(\mathbf{E} \cdot \mathbf{j}) = \sum_{i=1}^n \Delta y_i \int_{z_i} (\mathbf{F}_i \cdot \mathbf{j}) dz$$

$$(17c) \quad \text{Re}(\mathbf{E} \cdot \mathbf{k}) = \sum_{i=1}^n \Delta y_i \int_{z_i} (\mathbf{F}_i \cdot \mathbf{k}) dz$$

There are 3 similar equations for the imaginary part of \mathbf{E} . The dot product indicated in (17a), for example, can be expanded with the aid of (15) to yield

$$(18) \quad \text{Re}(\mathbf{E} \cdot \mathbf{i}) = \sum_{i=1}^n \Delta y_i \int_{z_i} \frac{Hz \cos \gamma (\cos g) dz}{2\rho f}$$

The six quantities that are necessary for computing the far field are $\text{Re}(\mathbf{E} \cdot \mathbf{i})$, $\text{Re}(\mathbf{E} \cdot \mathbf{j})$, $\text{Re}(\mathbf{E} \cdot \mathbf{k})$, $\text{Im}(\mathbf{E} \cdot \mathbf{i})$, $\text{Im}(\mathbf{E} \cdot \mathbf{j})$, and $\text{Im}(\mathbf{E} \cdot \mathbf{k})$. All of these quantities are obtained from 15, 16 and 17 or its equivalent for the imaginary portion.

The size of the increments Δy_i in (17) and (18) have not been specified. In order to reduce the number of points y_i needed to evaluate the discrete summation, a 16 point Gaussian quadrature formula was used, so that (17) becomes

$$(19) \quad \text{Re}(\mathbf{E}) = \sum_{i=1}^{16} W_i \int_{z_i} \mathbf{F}_i dz$$

The weighting factors W_i and the corresponding argument y_i can be found in Ref. 6. A similar expression is used to evaluate $\text{Im}(\mathbf{E})$.

Thus, having obtained the \mathbf{i} , \mathbf{j} , and \mathbf{k} components of both the real and imaginary parts of \mathbf{E} from (19) and a corresponding expression for $\text{Im}(\mathbf{E})$ we can then find the ϕ and θ components of \mathbf{E} , E_ϕ and E_θ respectively. These two components of \mathbf{E} , are of interest to the design engineer. It follows from vector analysis and the definitions of \mathbf{i}_ϕ and \mathbf{i}_θ of Fig. 3 that

$$(20) \quad E_\phi \equiv \mathbf{E} \cdot \mathbf{i}_\phi = \text{Re}(\mathbf{E} \cdot \mathbf{i}) (-\sin \phi) + \text{Re}(\mathbf{E} \cdot \mathbf{j}) \cos(\phi) + \text{Im}(\mathbf{E} \cdot \mathbf{i}) (-\sin \phi) + \text{Im}(\mathbf{E} \cdot \mathbf{j}) (\cos \phi)$$

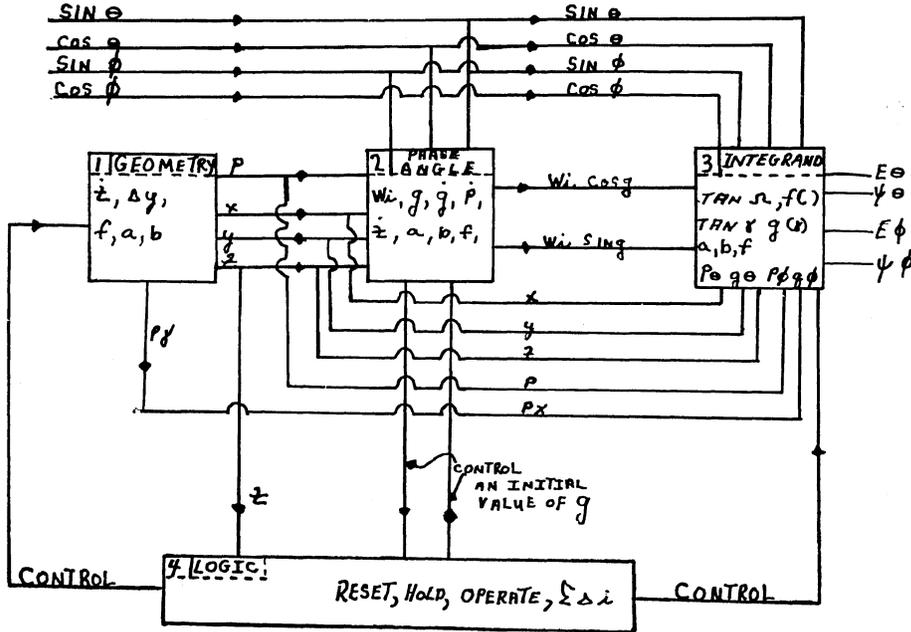


Fig. 5—Far field antenna pattern computation. Block diagram.

$$\begin{aligned}
 (21) \quad E_\theta &\equiv \mathbf{E} \cdot \mathbf{i}_\theta = \text{Re}(\mathbf{E} \cdot \mathbf{i}) (\cos \theta \cos \phi) \\
 &\quad + \text{Re}(\mathbf{E} \cdot \mathbf{j}) (\cos \theta \sin \phi) \\
 &\quad + \text{Re}(\mathbf{E} \cdot \mathbf{k}) (-\sin \theta) \\
 &\quad + j[\text{Im}(\mathbf{E} \cdot \mathbf{i}) (\cos \theta \cos \phi) \\
 &\quad + \text{Im}(\mathbf{E} \cdot \mathbf{j}) (\cos \theta \sin \phi) \\
 &\quad + \text{Im}(\mathbf{E} \cdot \mathbf{k}) (-\sin \theta)]
 \end{aligned}$$

Since E_ϕ and E_θ are both complex quantities, we speak of the magnitude and phase of each of them. The magnitude of E_ϕ is, from the algebra of the complex variable,

$$(22) \quad |E_\phi| = [p_\phi^2 + q_\phi^2]^{\frac{1}{2}}$$

where p_ϕ is the real part of the right hand side of (20) and q_ϕ is the imaginary part of the right hand side of (20). The phase angle of E_ϕ , is ψ_ϕ , where

$$(23) \quad \psi_\phi = \text{arc tan}(q_\phi/p_\phi)$$

Using similar definitions applied to (21) and substituting the subscript θ for ϕ in (22) and (23) we define $|E_\theta|$, p_θ , q_θ and the phase angle ψ_θ .

The above equations and definitions are all that are necessary for the explicit definition of the problem. In addition to these, there are several additional equations which (a) relate the problem independent variable z to the computer independent variable t , (b) relate the time rates of change of several variables and (c) define auxiliary terms. These are as follows

$$(24) \quad z = -c_i + t$$

where c_i is the value of z obtained by the substitution of a particular value y_i of (19) into (7). For any one specific integration with respect to z , such as in (19), we can then write, using (24)

$$(25) \quad \int_{-c_i}^{+c_i} F_i dz = \int_0^{2c_i} F_i dt$$

where the right hand side of (25) is the actual integral instrumented by the analog computer.

The phase function g of (1) and as defined previously is the critical portion of the computation and requires a special formulation for the calculation of the sine and cosine functions of g . This occurs because g can go through as many as six or eight cycles in any one sweep of z . A continuous resolution technique, as described in Ref. 7, was used. This requires the generation of \dot{g} , where the dot refers to differentiation with respect to time on the analog computer. From the definition of g ,

$$(26) \quad \dot{g} = 2\pi [\dot{\rho} - \dot{x} (\sin \theta \cos \phi) - z \cos \theta]$$

This, in turn, requires $\dot{\rho}$, \dot{x} , and \dot{z} . From (10), one can write

$$(27) \quad \dot{\rho} = \frac{(x-f)\dot{x} + (y-b)\dot{y} + (z-a)\dot{z}}{\rho}$$

From (24) it is seen that z is the coefficient of t (unity in this case). Since y remains constant for one sweep in z (or t), $\dot{y} = 0$. Using this latter piece of information, \dot{x} can be found from (6) as

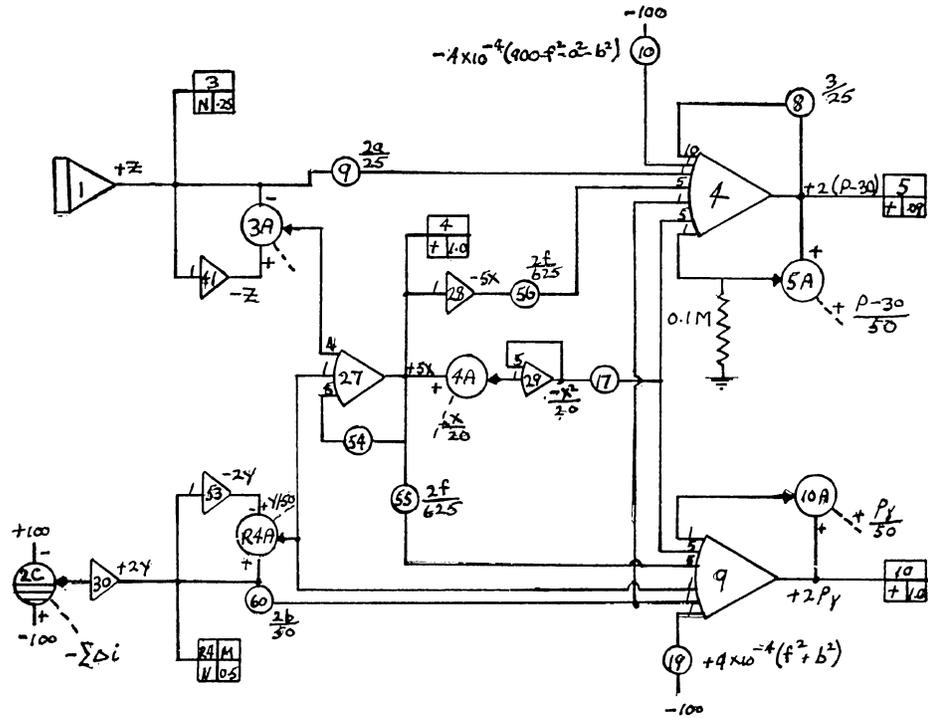
$$(28) \quad \dot{x} = \frac{z\dot{z}}{2f}$$

This completes the mathematical formulation of the problem for the analog computer.

III. ANALOG COMPUTER CIRCUITS

The programming was divided into four basic areas. First, there was the computation of the geometrical relationships associated with the shape of the radar dish. This includes the generation of the input quantities z and y plus the computation of the radius vectors ρ and ϕ .

Fig. 6—Antenna calculation.
Geometry of dish.



Secondly, there was the calculation of the sine and cosine of the phase angle g . The computation of the derivative of the phase angle was also required in this section.

The third basic area of the computation was the generation of the near field patterns and the complete build up of the integrand including the introduction of the terms associated with the angles ϕ and θ to determine the actual field intensity in the θ and ϕ directions.

The logic circuits necessary to control the machine to compute the double integral constituted the fourth block. Included in the block is the computation of the initial value of g at the beginning of each strip going across the dish plus all other switching associated with individual strips.

These four basic units of computation can be seen on the block diagram of Fig. 5.

Assuming that z is available at the output of an integrator and y is available in discrete increments, the first block of computation, namely, the geometry, may be generated. These are (6), (7), (10), and an auxiliary computer equation.

$$(29) \quad \rho r = [(x - f)^2 + (y - b)^2]^{\frac{1}{2}}$$

Equation 7 was used to calculate the initial conditions for the z integrator at each increment of y . Equation 6 was solved for x since the values of z and y are known.

The equations for ρ and ρr were simplified by the use of (6) to eliminate the z^2 and y^2 terms.

The scaled computer circuit to solve these equations is shown in Fig. 6. For details concerning analog computation see Ref. 8. The padded servo

cup 2C generating $(-2y)$ will be described in the fourth block.

The calculation of the phase angle g , the second basic block of computation, is coupled with the logic circuits. A number of quantities, the generation of which will be described in a later paragraph will be assumed available. The equations used to generate the angle g were (26), (27), (28) and the knowledge that $\dot{y} = 0$. Substitution of these into (26) produces the actual computer equation

$$(30) \quad \dot{g} = 2\pi \left[\left(\frac{\dot{z}}{2f} \right) \left(\frac{xz + zf - 2af}{\rho} \right) - \left(\frac{\dot{z}}{2f} \right) (z \sin \theta \cos \phi) - \dot{z} \cos \theta \right]$$

The scaled computer diagram for (30) plus the continuous resolver for the phase angle g (see Ref. 7) is shown in Fig. 7. Included is a diode function generator which supplies the weighting function, W_z . The incremental input to the DFG will be described in the section on logic. The servo cups 7A and 8A were scaled as shown to increase the accuracy. The relay contacts shown for the continuous resolver are required to perform the double integration efficiently. The drive signal for the relays is described with the logic.

The third and largest block of the computation is the generation of the near field pattern functions and buildup of the integrand. The equations required for the generation of the six basic quantities, $\text{Re}(\mathbf{E} \cdot \mathbf{i})$, $\text{Re}(\mathbf{E} \cdot \mathbf{j})$, $\text{Re}(\mathbf{E} \cdot \mathbf{k})$, $\text{Im}(\mathbf{E} \cdot \mathbf{i})$, $\text{Im}(\mathbf{E} \cdot \mathbf{j})$, $\text{Im}(\mathbf{E} \cdot \mathbf{k})$ were the expansion of (19) and its equivalent for the imaginary part, using (3), (4), (5), (8)

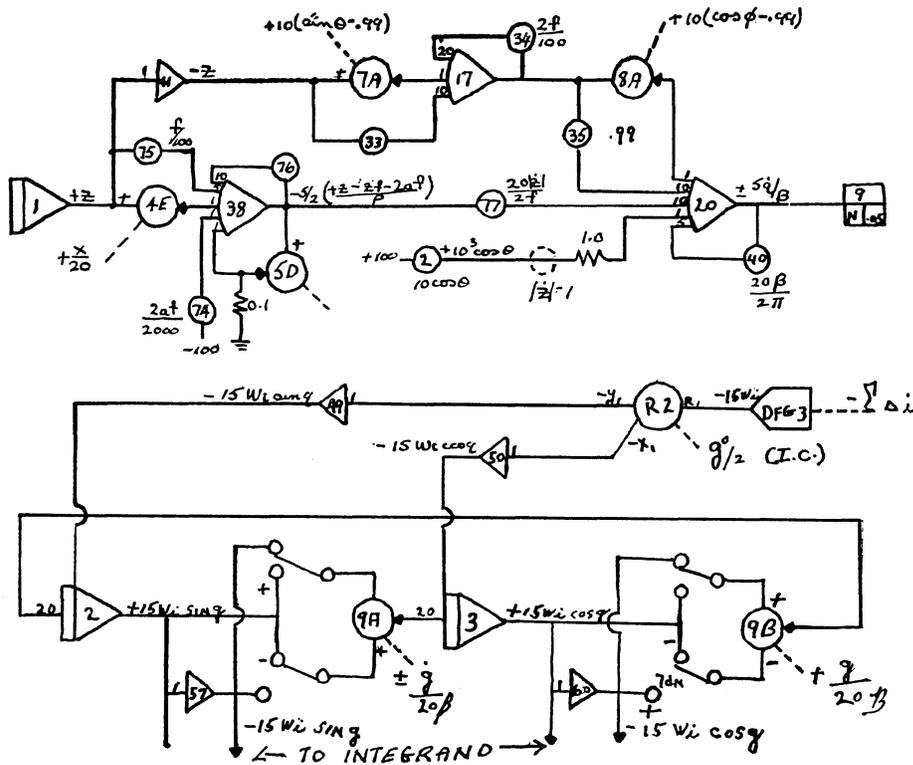


Fig. 7—Antenna calculation.
Phase angle.

and the results of (11) to (18). The near field pattern functions were generated as a function of the tangent of the angles since this was simpler to compute from the available quantities. The six basic quantities are then further resolved by means of (20), (21), (22) and (23) to yield the magnitude and phase of the main and cross polarization components, E_θ and E_ϕ .

Since it was desirable for the first study to limit the equipment used to 60 operational amplifiers, the real parts of the field intensity were computed employing $\cos g$, and then stored, while the same equipment was then used with $\sin g$ to compute the imaginary parts. Due to the lack of equipment available, it was found necessary to compute the resolution of the real and imaginary parts by hand but this was done while computing the next point. The computer diagram for the integrand is shown in the upper portion of Fig. 8.

The fourth block of the computer programming involved the necessary relay amplifiers and circuits to automatically control the computer through the calculation of one point. It was required that the control circuits should consume minimum time, be as foolproof as possible, and not contribute appreciable error to the computation. To minimize the time required for resetting the computer for the next strip, it was decided to start at one edge of the dish, integrate continuously in z to the opposite edge, and then reset to start the next strip at the same edge at which the last strip stopped. This requires establishing an initial condition on the z

integrator which alternates between plus and minus z_{\max} . It also requires employing two different relationships for z with respect to time, rather than the one shown in (24). These are

$$(24a) \quad z = +z_{\max} - t \quad (dz = -dt)$$

$$(24b) \quad z = t - z_{\max} \quad (dz = dt)$$

When these expressions are substituted into the original equations it is necessary to alternately reverse the sign of \dot{z} and the initial condition of the z integrator. The quantity \dot{z} appears explicitly only in the computation of \dot{g} and z . Referring to Figs. 7 and 9, since $|\dot{z}|$ was used wherever \dot{z} appeared explicitly, the sign of the output of amplifier 20, for example, reverses from one strip to another. The relays 5 and 7 are required so that the sense of the continuous resolve will be correct.

To control the computer automatically, it was necessary to employ relay amplifiers to sense when the value of z equaled z_{\max} , throw the computer into hold, and then reset the necessary circuits to compute the next strip. When the computer was prepared to compute the next strip, it was necessary to automatically put the computer into an operating mode and repeat the cycle. Since the phase angle g can go through many cycles from the beginning of one strip to the beginning of the next, it was necessary to remove increments of 360° from the computed value of g before the electromechanical resolver could determine the initial conditions for the continuous resolver circuit. Additional circuits were required to establish new values for z_{\max} , y ,

and W_i as the computation moved from one increment to the next. These circuits are shown in Fig. 9.

This diagram is divided into three main sections *A*, *B*, and *C* sections, where *A* is the computer sensing and control plus the alternate reversal of \dot{z} , *B* is the circuit to change the z_{\max} , y , and W_i conditions for each strip, and *C* is the circuit to compute the initial value of g for each strip.

The best way to describe the operation of these circuits is to follow the actions from the beginning of the first strip to the end of that strip and then to the second strip. The logic will then continue to cycle automatically to the end of the computation for one far field point.

Let us first examine section *C*, since the computer cannot operate until the correct initial value of the phase angle g is computed. The expressions required to compute g are (14) and the definition of " g " immediately preceding (14).

The angle g may be rewritten in degrees as

$$(31) \quad g = 360 [\rho - \sin \theta (x \cos \phi - f \cos \phi + y \sin \phi - b \sin \phi) + z \cos \theta - a \cos \theta]$$

An analysis of this equation will show that the phase angle is always greater than fifty revolutions. Therefore, fifty revolutions may be subtracted as a constant as shown on amplifier 45 in Fig. 9. If the remaining revolutions required to bring the value of g within $\pm 180^\circ$ is called C , it is merely necessary to remove integer values of C until the output of amplifier 11 in Fig. 9 is less than +90 volts. This is accomplished by wiring relay amplifier 37 as a square wave generator which alternately charges capacitor C_1 , and discharges it into amplifier 19 producing step changes of five volts at the output. Amplifier 11 is limited at +100 volts to keep it from overloading until the correct number of revolutions have been removed by amplifier 19. When amplifier 11 becomes less than +90 volts, amplifier 37 stops pulsing, and resolver 2 is positioned correctly so that the initial values of $\sin g$ and $\cos g$ are established on amplifiers 2 and 3 of Fig. 7. The clipping circuit on the input to the resolver keeps it from overloading when amplifier 11 has an output greater than +90 volts.

In the Reset Mode of the computer the following conditions exist:

1. Relays 1 and 3 in minus position
2. Relay 6 in minus position
3. Relays 4, 5 and 7 are in plus or minus position
4. Hufco relay 1001 in normally open position
5. Amplifier 11 conducts through pot 16, diode, and relay 1 up to set relay 2 to + position.
6. All integrators in Reset Mode
 - (a) Amplifier 5 has zero output—condition set for first strip.
 - (b) Amplifier 1 may be either $\pm z_{\max}$ for first strip.

If the control switch is now moved from the Reset

to the Operate position, the Hufco relay will release and relay 6 will throw to the positive position. This will apply a five volt step to the output of amplifier 19 and thereby remove the first revolution from amplifier 11. Approximately one tenth second later relay 6 will return to the minus position and recharge capacitor C_1 . Relay 6 will repeat the cycle until amplifier 11 becomes less than +90 volts at which time relay 6 will be thrown negative and stop pulsing. Amplifier 11 will then position resolver 2 and the correct initial conditions will be setup on the continuous resolver. At the same time the diode on pot 16 will stop conducting and, approximately 1-2 seconds later, relay 2 will throw to the minus position storing the initial conditions. The latching diode from amplifier 23 to 22 will then stop conducting and approximately one tenth second later relays 1 and 3 will throw to the positive position, thereby throwing the remainder of the computer into operate. Since amplifier 22 now has a negative output, relay 6 will be held in the negative position by the pulse latch diode from amplifier 22 to 37.

When the magnitude of the voltage at amplifier 1 is equal to z_{\max} (out of amplifier 25), relays 1 and 3 will again throw negative and place the computer in the "hold" mode. Amplifier 22 then feeds through the .05 capacitor and trips relay 2 to the plus position, latching amplifier 22 and throwing amplifiers 1, 2, and 3 to the reset condition. The outputs of amplifiers 6, 7, and 8, however, are being held during the operation, thereby allowing the computation of the sum of the integrals (19).

The following explains what happened in the logic circuit after relay 2 threw to the plus position: When relay 2 tripped to the plus position, capacitor C_2 was discharged into amplifier 5, producing a negative step of 6.25 volts at the output of 5 which moved servo 2 to supply the input conditions for the second strip. Amplifier 5 also changed the weighting function out of DFG-3. When relay contact "3 up" threw to the negative position, capacitor C_3 overcame the output of pot 43 and latched the flip-flop relays, 4, 5, and 7 into the opposite position, changing the relationship of z to time (i.e. from (24a) to (24b) or vice versa). Since amplifier 22 now has a positive output voltage, the pulse latching diode stops conducting and approximately one half second later the first revolution is again removed from amplifier 11. Amplifier 19 has previously been reset by the relay 1 dn contact while the computer was in operate. The pulse circuit will continue until the new value of g is computed and the cycle repeats itself. When the computer goes into hold at the end of the last strip, the next step change out of amplifier 5 lets the diode out of pot 14 conduct, latching the computer into "hold." The real parts of the integrand may then be stored or read out and the computer manually returned to "Reset," which resets the logic.

Since the problem was confined to a 60 amplifier computer, the computation was somewhat less

TABLE I
TIME ESTIMATE FOR
VARIOUS PORTION OF ANALOG COMPUTER FUNCTIONS FOR THE
ANTENNA PATTERN CALCULATION

<i>Activity</i>	<i>Time</i>	<i>Remarks</i>
Set θ and ϕ pots	1 Min.	—
Operation of logic	1 Min.	Real part
Computing	2 Min.	Real part
Store reals and reset	0.5 Min.	—
Operation of logic	1 Min.	Imaginary part
Computing	2 Min.	Imaginary part
Readout	0.5 Min.	—
Total	8 Min.	—

efficient than is easily obtainable. To compute one point required approximately eight minutes. During this time, the point was manually plotted by hand. Since the sine and cosine of ϕ and θ were entered on pots, these values had to be computed by hand and manually adjusted from one point to the next. An approximate breakdown of the running time per point is shown in Table I.

The efficiency of the analog computer could be improved by a factor of two with the addition of only 3 integrators and rearrangement of the multiplication generating the inputs to amplifiers 6, 7, and 8. The multiplication of the integrand by sine and cosine of g could be done at the inputs to 6, 7, and 8, instead of the input to amplifier 18. Hence, all that would be required to generate all six components of \mathbf{E} would be 3 additional integrators and 1 additional quarter square multiplier chassis. Then, switches would be inserted between the outputs of 6, 7 and 8 and the resolution circuits for p and q , so that either 6, 7 and 8 were connected, yielding, say the p 's or the additional 3 integrators (say 6', 7' and 8') are connected to yield the q 's. This switch would be thrown once, at the end of each point, to yield both real and imaginary components within 30 seconds of each other (allowing readout time), rather than recomputing the imaginary components in a separate computer run. This would reduce total compute time to about 4.5 minutes per point.

Additional improvements could be made in the operation of the logic and in the computing cycle itself, to reduce the overall time by about a factor of 5 or 10. This would reduce the analog computer time per point to something like 30 seconds to one minute.

IV. RESULTS

Since this was a new type problem as far as the analog computer was concerned, it was decided to make a test run using a uniform illumination on the reflector with the primary horn on focus. The results

of this type distribution are known analytically. The results of the calculation by the analog computer are shown in Fig. 10 for the main polarization, E_θ . They agree very well with the magnitude of the analytical curves with some small deviations in the side lobe levels, as far as 30 db down from the peak. However, calculations of the phase indicated that the analog output was correct to only 30 or 25 db below the peak.

Computed results of both the analog and 704 have been plotted and are shown in Figs. 11, 12, and 13. Figures 11 and 12 are the principle plane cuts for the feed on focus and Fig. 13 is the vertical pattern for the off focus case where $a = .234$. Figure 11 is the elevation pattern and shows agreement between analog and digital down to approximately 20 db below the peak. From that point the difference becomes greater as the pattern level de-

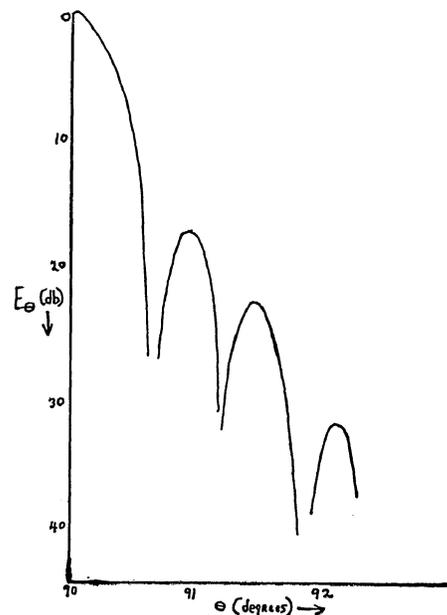


Fig. 10—Test case. Analog results for uniform reflector illumination, primary horn at focus.

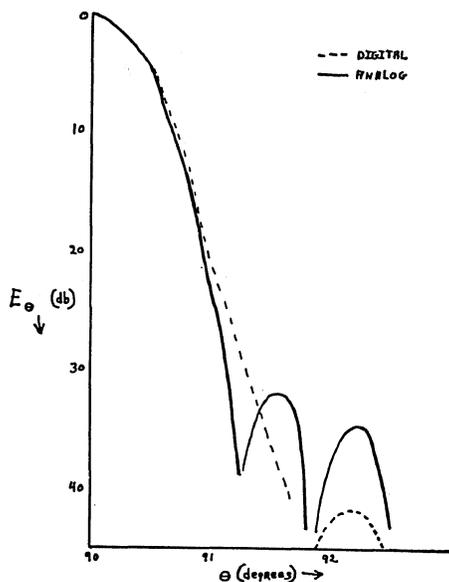


Fig. 11—Elevation pattern for non-uniform illumination and primary horn on focus. (Principal plane.)

creases. This was expected as outlined above. Figure 12 again shows good agreement down to a level of 20 db after which small deviations begin to occur. The fact that only 15 points were used with the analog computer indicates that valid comparisons between the analog and digital results should only be made for ϕ less than 2 degrees.

V. CONCLUSIONS

In comparing the time necessary to calculate each far field point it should be pointed out that neither machine was programmed for optimum time. The ratio of times between machines as originally programmed was about 13 to 1, the analog computer taking longer time. However, it is felt that by optimizing the programs for each machine the time ratio per far field point could be reduced to approximately 5 to 1. To convert these time ratios to cost figures, one should bear in mind that the analog computer that we are speaking of would sell for about \$100,000, and the digital computer that we are comparing it with might sell for \$2,000,000 more. Of course, exact costs vary from installation to installation, but based on the above estimated costs of the two computers, it is reasonable to assume that the analog computer could do this calculation for about 1/3 to 1/2 the digital cost. The analog program would be even cheaper for long, narrow antennas, since the continuous integration is always performed with respect to the longest dimension.

Special studies could be performed on the analog computer quite readily. One such study could be the perturbation of the antenna surface calculation with a random noise generator, in an amount equal to the known manufacturing tolerances, and obtain,

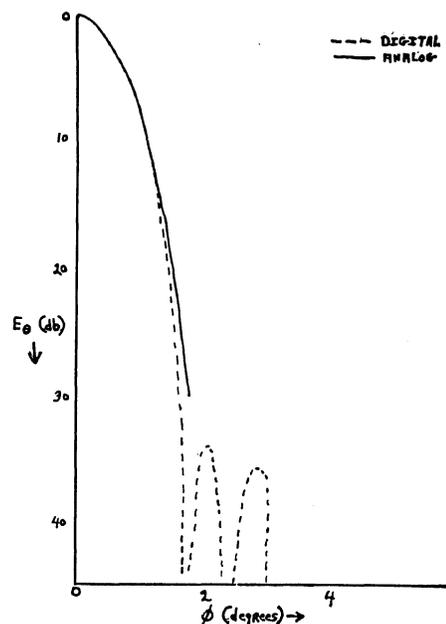


Fig. 1—Main polarization pattern for non-uniform illumination and primary horn on focus. (Principal plane.)

thereby, a quantitative measure of the differences in the field pattern between a "real" antenna and an "ideal" antenna.

The primary purpose in calculating antenna patterns is to arrive at a reflector design which, when fed by the proper feed horn, will form the specified beam in the far field. With computers available to ease the numerical calculations, many more preliminary designs may be investigated before the final design is set, resulting in the optimum final design. With this in mind the analog computer could be used to make the preliminary investiga-

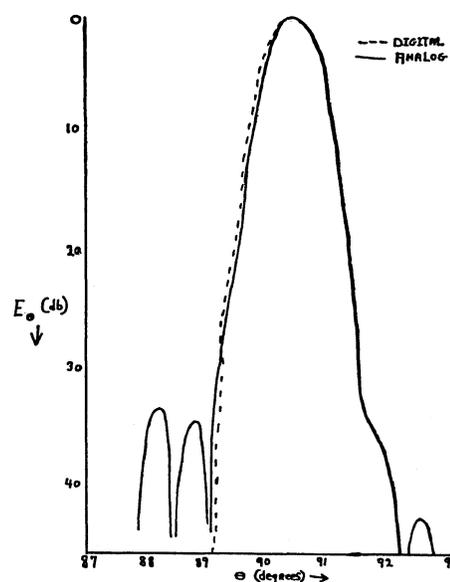


Fig. 13—Vertical pattern for "off focus" case.

tions with the final design made by a digital type machine, thus reducing the cost of the overall calculations. By reprogramming the analog computer for optimum speed and accuracy it might be possible to use the analog for complete design.

ACKNOWLEDGMENT

The work described herein was sponsored by the General Electric Company, Syracuse, New York.

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Reprinted from Volume XV
PROCEEDINGS OF THE NATIONAL ELECTRONICS CONFERENCE
Hotel Sherman, Chicago, Illinois, October 12, 13, 14, 1959.