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The logo for International Telephone and Telegraph (ITT), consisting of the letters 'ITT' in a bold, stylized, black font.

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This issue in brief

Pentaconta A1 Switching System

Making effective use of the well-established Pentaconta switching equipment and techniques, the A1 system has been developed to fit in the public telephone network of North America.

The vertical selector of the standard Pentaconta crossbar switch when used as a line unit is divided into 3 parts, the top is for trunks, the middle section for lines, and the lower part is a junctor section. Connections between some lines and trunks can be established over only a single vertical bar and for a wider range of lines and trunks through the operation of 2 vertical bars over the junctor sections of two switches.

The link switches for connecting junctors to lines are divided into 2 vertical sections (lines) and into 4 horizontal sections (junctors). Each switch can handle 192 junctors from 6 line units.

After a small trial installation to prove the system, a computer program of exact simulation based on the complete system logic was run to determine traffic capacity. Printouts were provided on the number of calls completed, types of connections made, average use of elements, and much similar data. Computer calculations were based on generated traffic that was collectively and individually random in time of origination and exponential in holding time about a mean of 120 seconds. The early part of each computer run was discarded as a transient preceding the steady-state conditions.

A second large installation in Las Vegas, Nevada, was based on this extensive analysis and is performing satisfactorily in all respects.

Herkomat Electronic-Control Private Automatic Branch Exchange

A telephone switching system that employs electronic components and reed switches to the exclusion of electromechanical devices can be described as quasi-electronic. The practicability and advantages of such a system, which uses reed relays both as crosspoint elements and as switches at suitable electronic control points, has been demonstrated by the trial switching systems supplied to the German Post Office. Using hermetically sealed reed contacts bearing the trademark "Herkon", these HE-60 L systems have shown an improved reliability factor compared with crossbar exchanges whilst occupying less space.

The quasi-electronic technique has now been adopted for the Herkomat private automatic branch exchange to bring all these advantages to the PABX system.

A 24-channel Pulse-Code-Modulation Junction Carrier System

Presently the British Post Office orders and installs 24-circuit pulse-code-modulation systems in large quantities for the junction network. The traffic carried is between a local exchange and a group switching center, local and tandem, and between local exchanges; often these requirements are combined in one traffic route.

The basic principles of pulse-code modulation are now well known, and papers have appeared describing the system technique. Mostly they have dealt with contemplated experimental development or field-trial systems, the major exception being a series of articles on the Bell Telephone System T1 carrier that has been in regular commercial use for several years. This paper describes a new pulse-code-modulation system now in high volume production, which was the first of its kind to be installed on a large scale for regular use in the United Kingdom telecommunications network.

Mastergroup and Supermastergroup Carrier Multiplex Equipment

Telephone carrier systems of 2700-channel capacity are presently being used in increasing quantities on main communication arteries linking important traffic centers.

The mastergroup and supermastergroup equipment is in conformity with International Standard Equipment Practice (ISEP), and designed to accord with the CCITT-recommended frequency-translating scheme 1. That is, higher-order modulation stages of the terminal multiplex system may comprise 300-channel mastergroups and 900-channel supermastergroups.

The equipment completes, with the existing ISEP versions of channeling, group, and supergroup equipments, a whole range of transistorized equipments designed to cover a variety of carrier multiplex systems up to 2700 channels.

It takes into account as many as possible of the requirements of the various telephone administration and operating companies.

Doppler VOR Ground Equipment

VOR stands for "VHF Omnicrange" a term used to describe a system of navigation using very-high-frequency omnidirectional radio beacons for determining the course of an aircraft. This system provides good bearing information over flat terrain but if the installation is sited, because of airway requirements, near reflecting objects, or the receiving aircraft flies over mountainous territory, errors occur due to propagation disturbances.

These disturbances can be effectively reduced by increasing the antenna base line but this leads to bearing ambiguity.

Doppler VOR enables the antenna base line to be extended whilst retaining unambiguous bearings. Trials using this technique show, in comparison with conventional VOR, a remarkable improvement in course information even in difficult siting terrain or where the receiving aircraft is flying over a mountainous route.

Air-Navigation Horizontal Situation Display

The increasing complexity and speed of modern commercial and military aircraft require assistance to the pilot in informing him of rapidly changing conditions. An integrated avionics system does this by combining the outputs of various independent sensors into a coordinated report that can be understood quickly by the pilot. These reports are presented as visual displays that may cover specific flight modes such as take-off, cruising, landing, and attack.

The field of view around an aircraft makes up the horizontal situation display. This display equipment is under the control of a navigation computer. It uses a transparent drum around which photographic microcharts are fastened. A folded optical system projects part of a chart through the axial center of the drum onto the viewing screen. The chart is moved along its axis and is rotated in either direction by digital longitude and latitude inputs to present the aircraft's present position and surrounding terrain. A ground-track reticle with an image of the aircraft, and a compass reticle, are similarly controlled by independent computer inputs. A Dove prism is rotated to change the orientation of the chart in the forward or north direction.

A cathode-ray direct-view storage tube projects radar, television, infrared, locally developed symbols, and other data on the viewing screen through an optical system that combines or alternates this information with the chart image.

The chart and cathode-ray images are projected from the rear onto a screen having low reflectance. The addition of a plastic Fresnel lens concentrates the light towards the pilot.

Computation of Urban Trunking Networks with Alternate Routing by Computer

A computer program has been produced for the computation of urban trunking networks with alternate routing. This program is designed to be a tool for network planners which will provide a result that represents an economic optimum.

The program computes:

- the number of trunks between all exchanges and tandem exchanges based on a traffic matrix,
- a list of costs (for cables, tandems, etc.),
- the overall grade of service.

The program uses an iterative process and employs a new method of computing Erlang's loss formula by continued fractions. This technique has been successfully applied to a series of applications including one where the problem of an imaginary town of 250 000 subscribers needing an increase of 150 000 subscribers occupied only one-and-a-half hours of computer time.

Numerical Evaluation of the Erlang Function Through a Continued-Fraction Algorithm

There exist in the literature various approaches for the evaluation of the Erlang function for both integer and non-integer values of the parameter. These approaches have permitted the computation of tables. However, different methods may be more appropriate to the treatment of special problems, for example, the optimization of telephone networks.

Previously known methods make no use of a continuous algorithm. This latter method would make it possible to reach the required accuracy by selecting the appropriate number of terms from a convergent series, a continued fraction, or even an asymptotic expansion. The continuous algorithm also makes treatment of the problem easier by a computer.

European Color Television Standards — The SECAM System

The creation of European color television standards from the American NTSC system was outlined in an article published in the preceding issue and brief descriptions of the NTSC and PAL systems were given. Both these systems use subcarrier quadrature-amplitude modulation. The present article is concerned with the SECAM system, which utilizes frequency modulation. A number of improvements have been incorporated in the Oceanic-Radio color receiver including a new technique for combining the red and blue chrominance signals to produce green that prevents variations in supply voltage from affecting the ratios of the chrominance signals during the process. Another

improved technique uses a coincidence gate to isolate the identification signals and so prevent spurious signals from affecting the operation of the channel changeover circuits.

According to the article's general conclusions, the multiplicity of international standards is detrimental from an economic standpoint, although transcoding from one standard to another now raises no major technical problems.

Traffic Imbalances in Small Groups of Subscribers

The problem treated is the statistical determination of the deviation in traffic in a small group of sources from an assumed value. A mathematical method is presented with practical rules for solving some specific cases encountered in traffic engineering.

Delay Problems Relating to an Invariable Number of Connections per Hour, Having Invariable Holding Time, and Carried by a Single Outlet

This article follows a previous one published in Volume 41/3, 1966 of this magazine. It related to other similar studies, notably those by Fry and Erlang, which assume a constant average number of connections per hour and an invariable holding time. Here it is further assumed that the number of connections is exactly the same for every hour. The mathematical treatment leads to various useful expressions among which are:

- the delay distribution function,
- the average total delay per hour, and
- the average number of delayed calls per hour.

Awards, Honors

A **Queen's Award for Industry** for 1968 has been received by STC for achievements in the export of submarine cable systems.

In the last ten years STC have received orders for submarine cable systems to a value of about £100 million. Three divisions of the company are concerned with this business — Submarine Systems Marketing, Submerged Repeaters, and Submarine Cable and they share the Award.

Queen's Awards were instituted by Royal Warrant in 1966 as a way of recognising outstanding achievement in industry, either by increasing exports or technological innovation. Recommendations are made to the Queen by the Prime Minister.

The Lord Rutherford award (Joint IERE/IEE), was presented to **C. P. Sandbank, R. Harcourt and J. Froom** on December 5, 1967 for their paper entitled "Acoustic Amplification in Semi-Conductors" published in *The Radio and Electronic Engineer*, Vol. 31, No. 3, March 1966.

A conference award was presented to **C. P. Sandbank** on February 14, 1968, on behalf of the 1968 International Solid-State Circuits Conference, University of Pennsylvania, in recognition of his special contribution to ISSCC 67.

Casabona and Spagnoletti Honored

The Institute of Electrical and Electronics Engineers in its annual advancement of outstanding members to its Fellow grade has honored the following two engineers of the International Telephone and Telegraph System.

Anthony M. Casabona of ITT Federal Laboratories "for his pioneering technical and managerial contributions to navigation and instrument landing systems".

Philip H. Spagnoletti of Standard Telephones and Cables "for contributions to the design of short-wave broadcasting equipment and single-sideband telecommunications equipment".

They thus take place among the 2 percent of the 160 000 members of the Institute who comprise this greatly respected grade of membership.

Book Review

Cutting the Cost of Quality

Philip B. Crosby, Quality Director for the International Telephone and Telegraph Corporation, is the author of this book, which is divided into the following 10 chapters.

- *P* is for Profit
- The Attitude of Defect Prevention
- The Cost of Quality
- The Quality Department — How Good Is It?
- The Quality Department — What Does It Do?
- Superior Quality — What Is It?
- Defect Prevention — The Nuts and Bolts
- Defect Prevention vs "Compensation"
- Goal Setting
- Corporate Quality Control.

The book is 8.5 by 11 inches (22 by 28 centimeters) and contains 242 pages. It is published by Industrial Education Institute, 221 Columbus Avenue, Boston, Massachusetts 02210. The price is \$ 10.00 per copy.

Pentaconta A1 Switching System

R. Y. SIMS

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1. Introduction

The Pentaconta* A1 switching system was designed principally for North America and provides the features, sophistication, and growth flexibility so necessary in this rapidly changing market. Overall cost is low due to the use of efficient Pentaconta relays and multiswitches as well as an advanced form of matrix. Interconnecting facilities approach the theoretical maximum with full availability to trunks and registers regardless of group size. Maintenance is minimal due to a centralized self-checking marker that monitors the establishment of each call.

The Pentaconta multiswitch matrix provides for through signaling and talking paths as well as connections to registers and senders. Code translation, alternate routing, as well as route advance are marker controlled. Failure of a marker or common circuit to advance a call will result in a punched card at the trouble recorder.

Multigroup Centrex working and Tel Touch (push button) pulsing from subscribers sets are standard. One of the more important features of the system is the use of the most-direct-path selection technique. This allows the selection of the most physically direct path through the multiswitches, resulting in a saving of speech matrix equipment.

2. Background

A small trial office was installed in the village of Rio Grande in Puerto Rico in 1965. This office proved the feasibility of the most-direct-path selection technique. It was then decided to proceed with designs for a full range of offices. Due to the matrix complexity and inability to get consistent traffic capacity results by conventional mathematical methods, a full-traffic simulation was instituted by computer. This exact-replica simulation was conducted in Paramus, New Jersey, at the Data Information Systems Division in 1965. The results that established the system capacity are part of the discussion presented here.

A second installation, the North 3 office of the Central Telephone Company in Las Vegas, Nevada, was then produced and installed and is performing satisfactorily in all respects. A typical rack suite is shown in Figure 1.

3. General System Organization

A block diagram of the system is shown in Figure 2. The basic building block of the system is the line unit shown in Figure 3. A line unit is made up of 8 frames mounting Pentaconta multiswitches of 16 vertical selectors each. When the office size exceeds the capacity of 7 line units, junctor link units are added. Each junctor link unit is made up of 3 frames, which also mount Pentaconta multiswitches of 16 vertical selectors. Regardless of office size all trunks always appear on the line units. Any marker can control the complete path regardless of



Figure 1 - Typical rack suite at Las Vegas central office.

office size. In very large offices the common marker may be split into two parts: one controls establishment of connections of lines and trunks to registers and the other controls the establishment of line-to-trunk, trunk-to-line, or line-to-line connections. When split in this manner the markers are called dial-tone markers and completing markers.

Ringin arrangements are flexible and are controlled by the marker. The actual application of the ringin current at the trunks may be done by 6-, 8-, or 10-wire 7-select-bar multiswitches with 10 to 20 vertical selectors, or by relay selector units when the ringin requirements are simple.

Number translator groups are so arranged that any number may be assigned to any line location in the office. Each number group is capable of handling 1000 directory numbers. Each register is capable of recognizing 4 local-office codes, allowing the theoretical group size to be 40 000 numbers.

The registers and senders are conventional but may be arranged to allow overlapping of the digits received with the digits transmitted. Where multifrequency pulsing is used, or where it is not desirable to overlap reception and transmission, the complete digits may be passed from the register to the marker and thence to the sender. When registers are arranged for Tel Touch operation,

* Trademark of International Telephone and Telegraph Corporation and its associate companies.

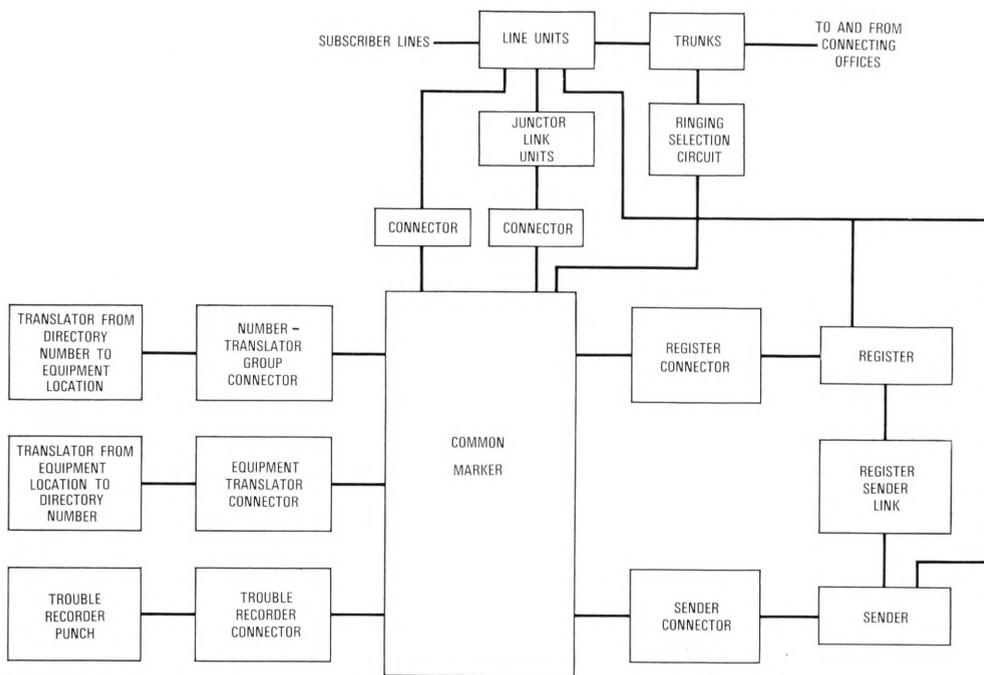


Figure 2 - Block diagram of A1 system.

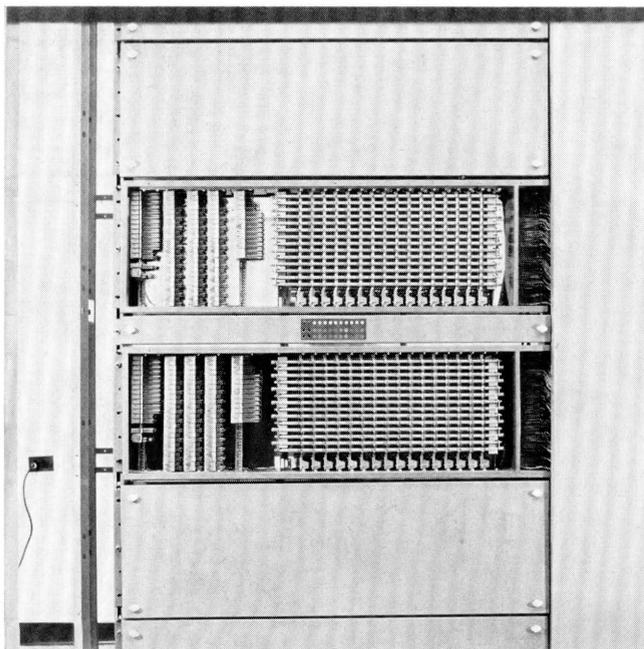


Figure 3 - Line unit.

they also accept either dial or Tel Touch pulsing from the same subscriber line.

4. Description of Components

The basis of the design is standard Pentaconta apparatus but minor changes and additions have been made in almost all apparatus except in the relay itself. Some typical components are shown in Figure 4.

A new formed-type frame mounts the vertical selectors, which are always 16 in number. The selectors are standard Pentaconta types except that the vertical multiples of the line and link units are broken, the line unit into 3 parts and the link unit into 2 parts.

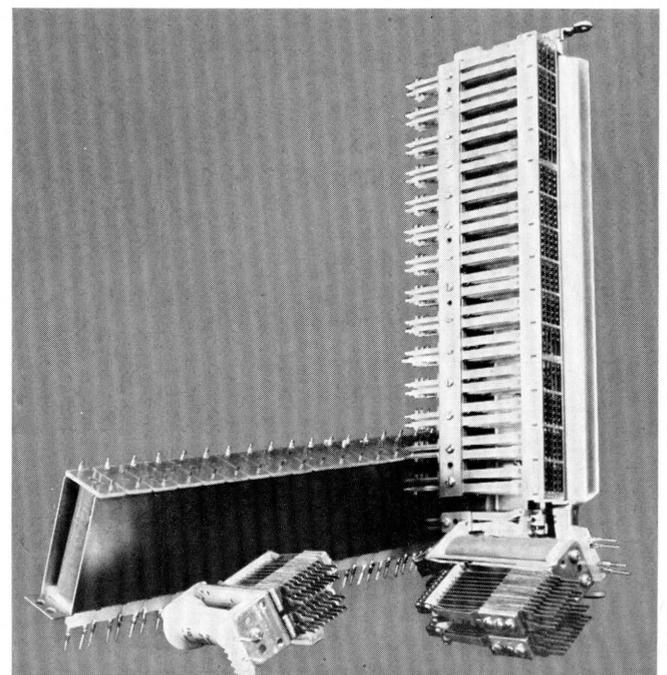


Figure 4 - Some typical components.

distributed under the trademark of Penta Reed relay. Each coil is a standard Pentaconta oval and may contain either 1 or 2 dry reed contacts. All spark suppressors are of the resistor-capacitor type, usually mounted on the rear.

5. Switching Matrix

Each line unit provides terminations for 288 lines, 8 registers, 8 senders, and 80 trunks and is capable of carrying 36 erlangs of traffic. When traffic per line permits, extension units may be provided by using 8 identical additional line-unit switch frames to build out to 576 lines or with 3 additional line units (24 additional line-unit switch frames) to build out to 1152 lines. When 1152 lines are used, line capacity will be 0.03 erlang of originating, plus terminating, traffic. Since the basic line unit may be coupled with 6 others before a link stage is required, the line capacity will vary without line frames from 2016 to 8064 at full fill and from 0.12 to 0.03 erlang per line originating plus terminating, respectively.

In larger offices or those where traffic is heavier, the line units are combined into groups of 6 each and called "frame groups". The frame group then becomes the basic building block. These groups are interconnected by means of junctor link units up to a maximum of 15 frame groups requiring 120 junctor link units.

6. Assignments on the Line-Unit Switches

The line-unit switch frame is made up of a multiswitch that always uses 16 vertical selectors, 32 or 36 line and cutoff relays, and miscellaneous control and connection relays. A line unit can accommodate from 256 to 288 lines, in steps of 4, depending on the proportion of the line-unit switch frames that are equipped with 32 or 36 line and cutoff relays.

Figure 5 shows a typical vertical selector. The upper section is the trunk section, the middle is the line section, and the lower is the junctor section. In each section one bar is required for "doubling". In any section the operation of a bar, other than the doubling bar, connects 6 wires to the vertical multiple. The doubling bar selects either the left or right group of 3 wires by operating either up or down for left or right. Since the function of the doubling bars is required for access to any of the switch sections, its operation is assumed in this discussion of assignments.

Tables 1 and 2 show the assignment relationship and the accessibility of various elements. A shorthand picture of the multiswitch is used that is understood to mean that any square on any switch carrying the same number is connected by a wired horizontal multiple. This can be illustrated by considering the possibilities of a connection between a particular line in group 210/215 and a particular trunk in group 300/311. It will be noted that two *single vertical* connections are possible on switch 0 using verticals 12 and 13.

Connections requiring *two verticals* are possible using verticals 10, 11, 14, 15, 16, and 17 of switch 0 for access to junctors and then verticals 06 and 07 on switch 0 or verticals 00, 01, 14, and 15 on switch 2.

Further consideration will reveal that the same access situation exists regardless of the line or trunk chosen. Every line cannot reach every trunk on a line unit by a single vertical, but every line can reach every trunk if junctors are used and in a similar way every trunk can reach every line. This is true regardless of the size of the trunk group even up to 7200 trunks. The simulation showed that with 75 percent local traffic, 47 percent of the connections were made on a *single vertical*. This was with

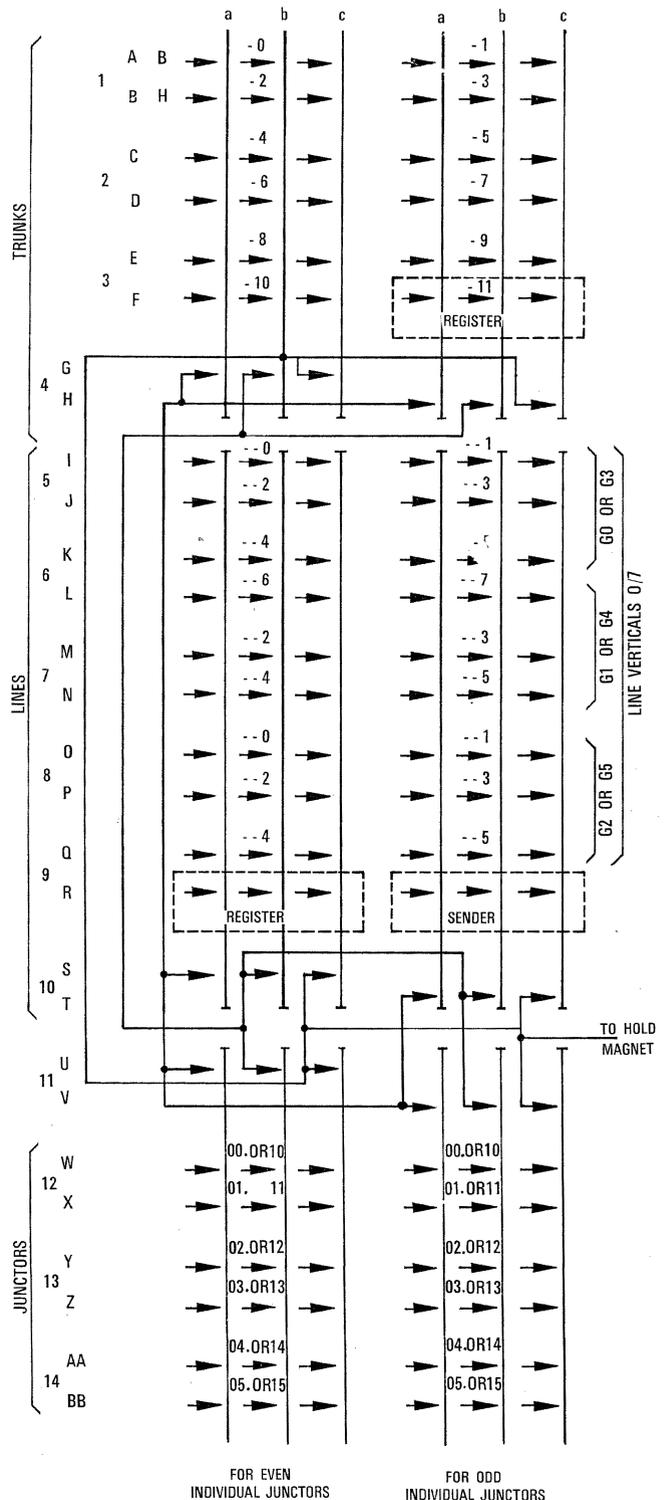


Figure 5 - Typical vertical selector of a line unit.

Table 1 - Switch assignments

		Switch 0								Switch 2																								
		00	01	02	03	04	05	06	07	10	11	12	13	14	15	16	17	00	01	02	03	04	05	06	07	10	11	12	13	14	15	16	17	
Trunks or Registers		000/011	100/111	200/211	300/311	200/211	300/311	000/011	100/111	300/311	200/211	100/111	000/011	100/111	000/011	300/311	200/211	4 Bars																
Lines		000/005				010/015				320/325				330/335				6 Bars																
		100/105				110/115				420/425				430/435																				
		200/205				210/215				520/525				530/535																				
Register		0				1				2				3																				
Sender		1				2				3				0																				
Junctors in junctor groups 0/5		0-1	2-3	4-5	6-7	0-7	2-5	4-3	6-1	0-3	2-1	4-7	6-5	0-5	2-7	4-1	6-3	4 Bars																
		Switch 4								Switch 6																								
		00	01	02	03	04	05	06	07	10	11	12	13	14	15	16	17	00	01	02	03	04	05	06	07	10	11	12	13	14	15	16	17	
Trunks or Registers		400/411	500/511	600/611	700/711	600/611	700/711	400/411	500/511	700/711	600/611	500/511	400/411	500/511	400/411	700/711	600/611	4 Bars																
Lines		040/045				060/065				360/365				370/375				6 Bars																
		140/145				160/165				460/465				470/475																				
		240/245				260/265				560/565				570/575																				
Register		4				5				6				7																				
Sender		5				6				7				4																				
Junctors in junctor groups 0/5		0-1	2-3	4-5	6-7	0-7	2-5	4-3	6-1	0-3	2-1	4-7	6-5	0-5	2-7	4-1	6-3	4 Bars																

Table 2 - Switch assignments

		Switch 1								Switch 3																								
		00	01	02	03	04	05	06	07	10	11	12	13	14	15	16	17	00	01	02	03	04	05	06	07	10	11	12	13	14	15	16	17	
Trunks or Registers		000/011	100/111	200/211	300/311	200/211	300/311	000/011	100/111	300/311	200/211	100/111	000/011	100/111	000/011	300/311	200/211															4 Bars		
Lines		340/345				350/355				060/065				070/075																				
		440/445				450/455				160/165				170/175																				
		540/545				550/555				260/265				270/275				6 Bars																
Register		4				5				6				7																				
Sender		5				6				7				4																				
Junctors in junctor groups 0/5		0-1	2-3	4-5	6-7	0-7	2-5	4-3	6-1	0-3	2-1	4-7	6-5	0-5	2-7	4-1	6-3														4 Bars			
		Switch 5								Switch 7																								
		00	01	02	03	04	05	06	07	10	11	12	13	14	15	16	17	00	01	02	03	04	05	06	07	10	11	12	13	14	15	16	17	
Trunks or Registers		400/411	500/511	600/611	700/711	600/611	700/711	400/411	500/511	700/711	600/611	500/511	400/411	500/511	400/411	700/711	600/611														4 Bars			
Lines		300/305				310/315				020/025				030/035																				
		400/405				410/415				120/125				130/135																				
		500/505				510/515				220/225				230/235				6 Bars																
Register		0				1				2				3																				
Sender		1				2				3				0																				
Junctors in junctor groups 0/5		0-1	2-3	4-5	6-7	0-7	2-5	4-3	6-1	0-3	2-1	4-7	6-5	0-5	2-7	4-1	6-3														4 Bars			

1792 lines each originating and terminating a total of 0.14 erlang of traffic.

Further reference to Tables 1 and 2 will reveal that a multiple situation as shown in Figure 6 exists. Switches 0, 2, 4, 6 have access to junctor group 0/5 while switches 1, 3, 5, 7 have access to junctor group 10/15. Switches 0, 2, 1, 3 have access to trunk groups 000 through 311, while switches 4, 6, 5, 7 have access to trunk groups 400 through 711.

From this, the conclusion may be drawn that a given set of switches, say 0, 2, 1, 3, may have access to 40 trunks directly and to 40 trunks by junctors. This situation holds throughout.

In trunk assignment it is desirable to spread all trunks evenly over the line switch appearances. This increases the probability of most-direct-path connections and distributes the traffic as well. Where trunk groups are small, even distribution may not be possible but neither is it important. In growth, each line unit brings with it additional trunk terminations; where necessary the existing plug-in trunks may be redistributed.

At this point some reflections on most-economical-path selection are in order. First choice is *single vertical*, then use of verticals in the same line unit by junctor, and finally by junctor and junctor link. In each case physical nearness is the key because nearness and minimum equipment usage are synonymous.

7. Junctor Link Units

Link switching in A1 terminology means junctor switching and as such serves to connect the junctors (a 3-wire metallic tip, ring, and sleeve) of the same or differ-

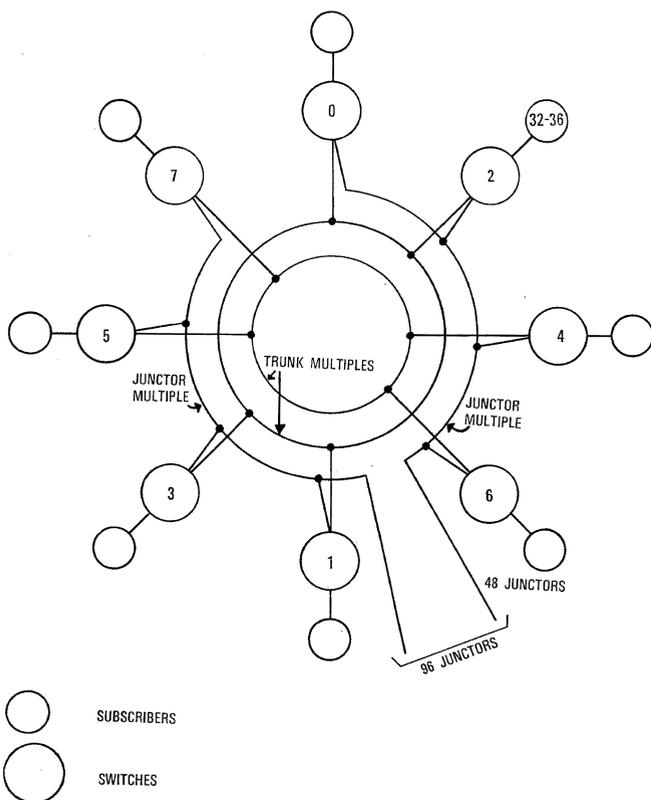


Figure 6 - Line unit multiples.

ent frame groups using a single vertical. The link unit is always made up of 3 link-switch frames. Each frame contains a 14-bar 16-vertical 6-wire switch. The switch is split into 2 parts vertically and 4 parts horizontally as shown in Figures 7 and 8. Since each line unit has 96 junctors emanating from it and since there are 6 line units to a frame group, all junctors of 1 frame group can appear on 1 link unit. In practice, 8 link units are required for 2 frame groups to maintain proper occupancy levels. For 7 frame groups 38 link units are required and for 15 frame groups 120 link units are required.

Most-direct-path selection confines the link stage traffic to incoming calls from other offices, to the terminating side of local calls, and to that part of all types of calls that overflows. In the line stage only single vertical connections can be made and they connect junctor path to junctor path.

Where an office is initially equipped with junctor link stages no line junctor distributing frame is required. Instead each junctor link bay is delivered equipped with distributing fields.

Expansion from nonjunctor link size to junctor link size requires that the initial frame groups, but not the

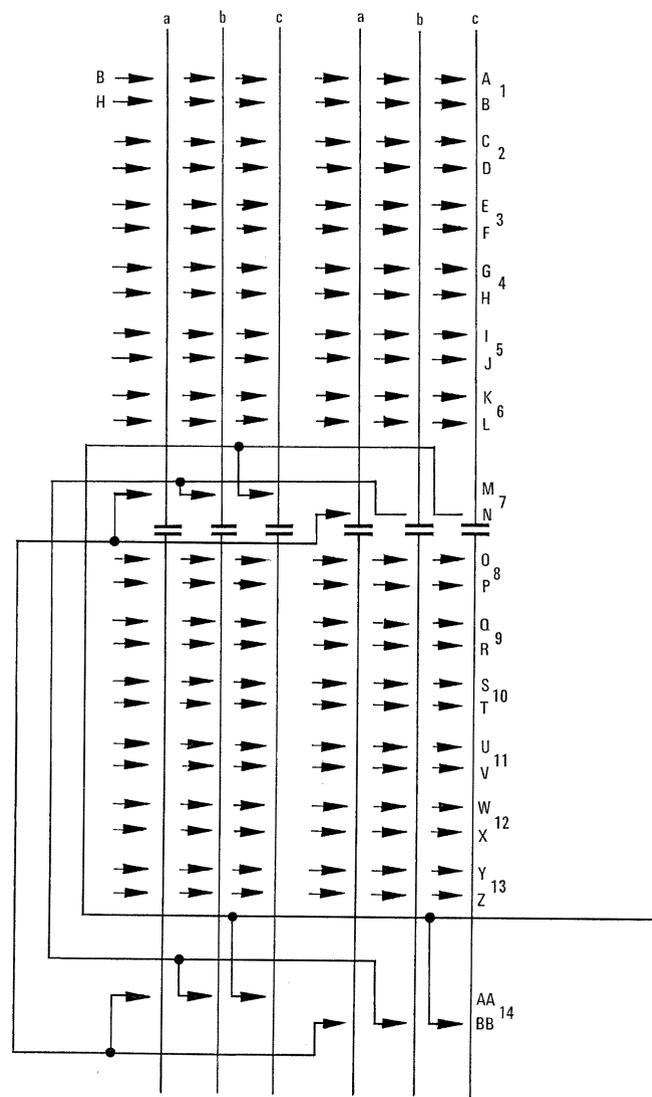


Figure 7 - Typical junctor link vertical.

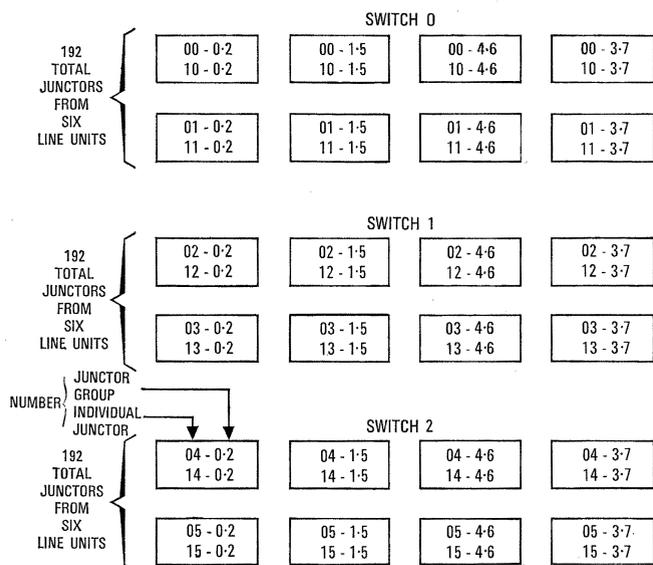


Figure 8 - Typical link unit composed of 3 switches.

subsequent additions, be equipped with line junctor distributing frames.

8. Expansion

In offices not equipped with junctor link stages, the line junctor distributing frame has 6 jacks each having 48 pins. Later, as the office grows, movement of these jacks automatically rearranges the junctors. Growth may be accommodated in this manner until the maximum non-link size of 7 line units is reached. These 7 line units may represent 2016 to 8064 lines "depending on the traffic" as explained in Section 5.

9. Traffic Capacity

The capacity of the system depends on the degree of saturation that can be achieved in the utilization of the connecting paths between the inlets and outlets of the speech matrix and on the delays that may be accepted in the common equipment. The common equipment is that which provides for connection, registration, logic, and storage of the controlling data.

It was assumed and later verified that calculations of delay would not materially affect system configuration. For this reason first investigations were centered about inlet and outlet matrix configurations.

The verification of matrix capacity had two parts: the simulation of the nonlink system by exact replica and the calculation of the link system by Kittredge-Molina techniques based on junctor occupancies derived from the simulation.

10. Simulation

The complete system logic was set up in a computer program with printouts so arranged as to provide counts of calls completed, the type of connection made, the average usage of elements, and much other data required in establishing the system capacity as well as the reliability of each run.

Since the loading of the system had to come from sources that as nearly as possible simulated real life, considerable effort was spent in deriving a truly lifelike source tape.

The case for 7 line units assumed 256 customers per unit. This required origination from 1792 subscriber lines. The characteristic of the traffic generated was that it be collectively and individually random in time of origination and exponential in length of holding time about a mean average of 120 seconds. Using random numbers and exponential derivations for the program, output tapes were printed and plotted. Figures 9 and 10 show unsatisfactory and satisfactory distributions of generated traffic. Further, it was decided to provide an unbalanced load as might occur from improperly distributed private-branch-exchange lines. The traffic generator was thus unbalanced so that the 112 groups of 16 lines each instead of generating 0.89 percent of the total would be distributed as shown in Table 3 and Figure 11.

Four source programs were evolved corresponding to 0.11, 0.125, 0.14, and 0.15 erlang and each tape was verified and corrected to guarantee a proper distribution.

Using the generator tapes for source and analyzing the carried load, it was determined that 12 minutes were required for stabilization. This meant that in a run of 2 hours and 12 minutes (simulated real time) the first 12 minutes had to be discarded.

Following the establishment of proper source generators, office parameters and calls logic were built into the system. The office parameters were from Table 4 while calls logic was derived from flow diagrams.

- Printouts from the machine served three purposes.
- Generated checks that could be used to eliminate programming errors.
- Provided enough detail to show the inner workings of the system; that is, the percentage of calls completing on a single vertical or the number of calls completing on recycle.
- Established the quality grade of the system.

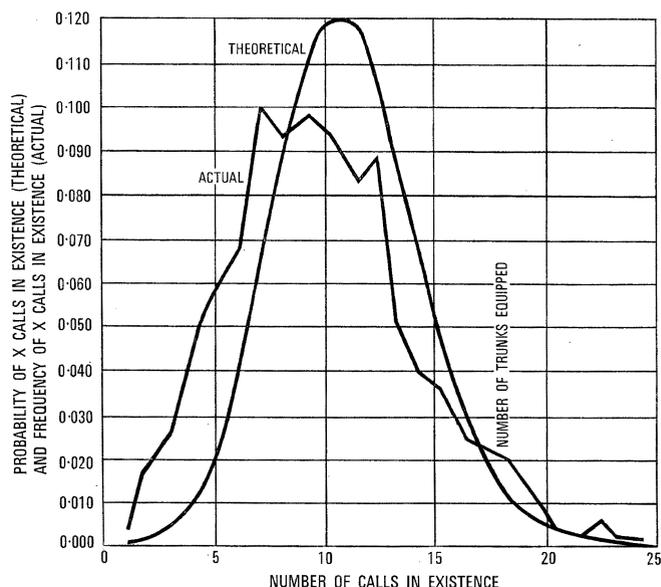


Figure 9 - Unsatisfactory traffic simulation for 7 line units generating 0.125 erlang per line. The 610 outgoing calls on route A produce 10.9 erlangs.

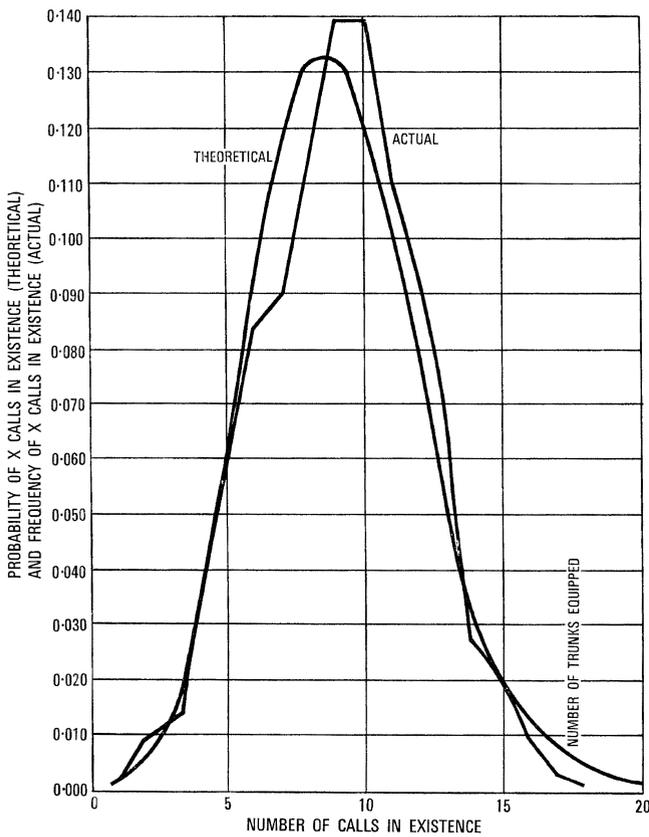


Figure 10 - Satisfactory traffic simulation for 7 line units generating 0.125 erlang per line. The 557 outgoing calls on route A produce 9.00 erlangs.

Table 3 - Unbalanced groups

Number of Groups of 16 Lines	Percent of Total Per Group	Percent Cumulative
23	0.6	13.8
16	0.7	11.2
14	0.8	11.2
10	0.9	9.0
12	1.0	12.0
16	1.1	17.6
21	1.2	25.2
112		100.0

A coded list of 58 items was devised as the required output data; a portion of one typical list is given in Table 5.

Tables 6 and 7 show a typical printout. Numerous runs of this type were made with the more-significant results shown in Figures 12 and 13.

11. Junctor Linkages

The analysis of the link switching technique was made using the Kittredge-Molina general form

$$p = [1 - (1-a)(1-b)(1-c)]^N$$

where *p* represents the probability of blocking and *a*, *b*, and *c* are the occupancy of the various elements.

Reference to Figure 14 will reveal in linear graph form the general line-to-trunk situation as it exists in the A1 system. The simulation discussed in Section 10 was concerned with the traffic flow in A and B of Figure 14.

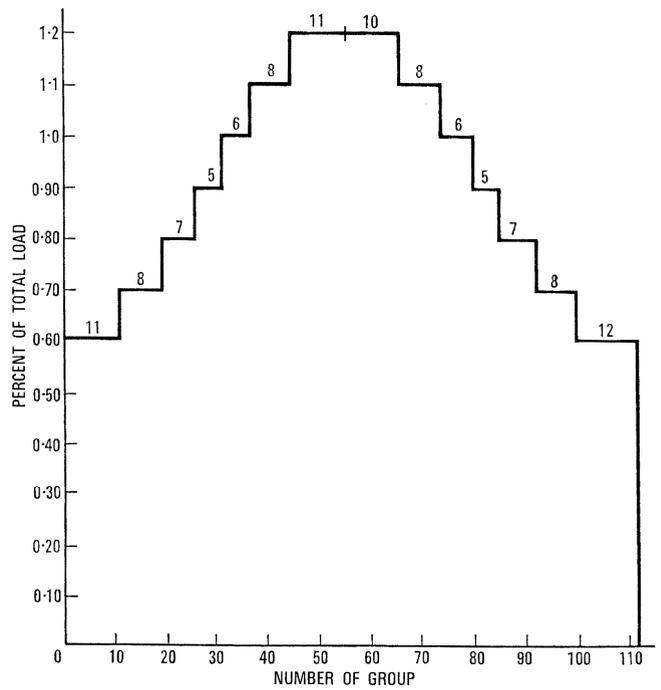


Figure 11 - Unbalanced groups of lines.

Table 4 - Office Parameters

Line Units Per Office	Traffic Per Line		Local Trunks	Trunks Per Route			Registers
	Hundred Call Sec-onds	Erlangs		Outgoing	Incoming	2-Way	
1	4.0	0.116	20			8	7
1	4.5	0.125	22			8	7
1	5.0	0.139	23			9	7
1	5.5	0.153	25			9	8
1	6.0	0.167	25			9	8
7	4.0	0.116	96	17	17		26
7	4.5	0.125	108	18	18		28
7	5.0	0.139	120	19	19		30
7	5.5	0.153	132	21	21		32
7	6.0	0.167	132	21	21		32

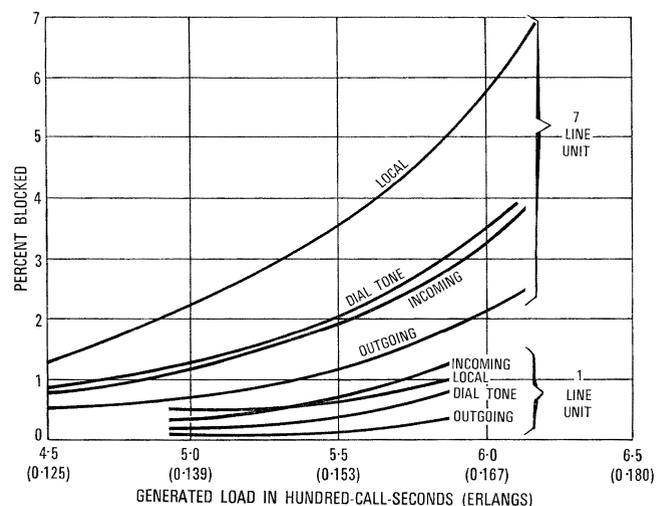


Figure 12 - Generated load versus percent blocking.

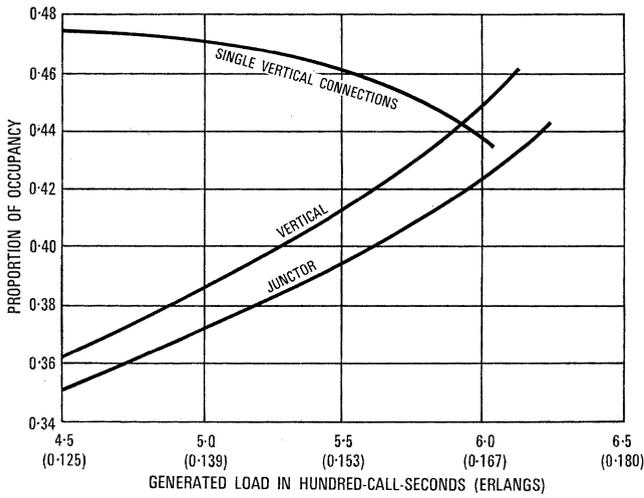


Figure 13 - Effect of generated load per line.

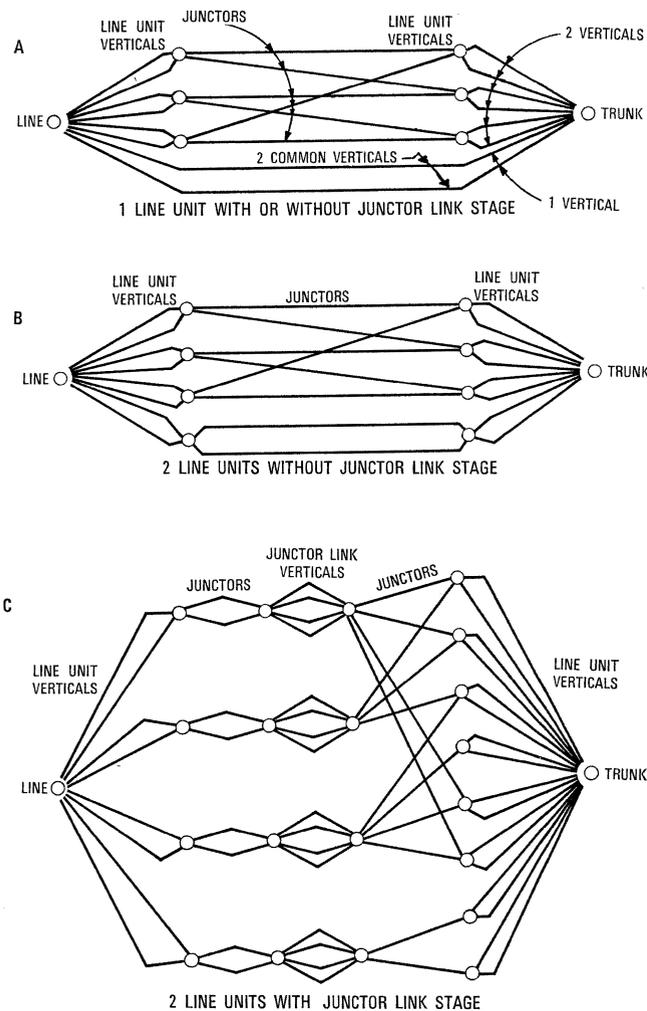


Figure 14 - General line-to-trunk arrangements.

Figure 14 C illustrates the junctor link switching problem in one of its simplest forms. In this discussion, we ignore recycle, retest, and other access, such as line to register.

The specific form for calculation of the diagram in Figure 14 C is

$$p = \{1 - (1 - c^4) \{1 - [1 - (1 - a^2)(1 - b^2)]^2\}^2\}^4.$$

Table 5 - Output data items

<i>Number of Calls Offered</i>	
(1) Line to line	5 087
(2) Line to trunk	1 652
(3) Trunk to line	1 752
<i>Number of Connections Placed</i>	
(4) On 1 vertical	9 873
(5) On 2 verticals and a junctor	11 030
(44) Line-to-line calls with holding time less than register time	508
(45) Line-to-trunk calls with holding time less than register time	87
(46) Line-to-trunk calls with holding time less than register time	91
(47) Line-to-trunk 1st selection calls with holding time less than sender time	102
(48) Line-to-trunk 2nd selection calls with holding time less than sender time	1
(50) Calls originated from busy lines	11
<i>Occupancies</i>	
Average number of lines busy	192.19
Average number of registers busy	13.56
Average number of local trunks busy	66.51
Average number of route A trunks busy, outgoing 8.45; incoming 8.23	
Average number of route B trunks busy, outgoing 9.02; incoming 8.40	
Average number of route C trunks busy, outgoing 7.41; incoming 8.65	
Average number of verticals busy	321.55
Average number of junctors busy	118.56

Application of this form requires extrapolation of the junctor occupancy as found in the simulation.

The simulation showed that in a 7-unit office, 45 percent of the originating and 2 percent of the terminating connections were made on a single vertical. Since those calls not made via a single vertical must be carried by junctors, it is possible to establish the originating and terminating junctor load and with this sum, plus an allowance for unbalance, the total junctor load. Using this technique the load may be established for the junctors in any size of office.

Since all the line-junctor load between line units must flow through the junctor linkages, proper element occupancy may thus be calculated.

As a final step, a generalized small computer program (with an algorithm for the probability calculation) was made. This allowed quick resolution of various configurations once occupancy was established.

Study of the final design indicated that the loss at rated line unit levels (36 erlangs) would not exceed 0.02 on a local call in any size up to 15 frame groups.

Table 6 – Typical tape printout of busy conditions

Time Frame	Lines	Registers	Local	Route A		Route B		Route C		Vertical	Junctors	Links
				Out	In	Out	In	Out	In			
15 720	187	16	128	7	4	9	9	10	10	309	198	0
15 780	207	13	148	5	7	11	9	9	10	348	232	0
15 840	191	11	134	3	5	11	11	9	12	324	216	0
Avg.*	192.43	13.55	133.59	8.40	8.16	8.92	8.32	7.50	8.61	326.01	237.55	0

* Sum of above items per number of time frames (after 12 minutes).

Table 7 – Tape printout of blockages by block numbers (N)

Blockages (N Block)			
(1)	5087	(18)	0
(2)	1652	(19)	0
(3)	1754	(20)	0
(4)	8677	(21)	0
(5)	12190	(22)	0
(6)	0	(23)	0
(7)	0	(24)	0
(8)	22	(25)	0
(9)	0	(26)	204
(10)	0	(27)	8
(11)	0	(28)	13
(12)	3	(29)	63
(13)	0	(30)	37
(14)	0	(31)	188
(15)	0	(32)	37
(16)	497	(33)	0
(17)	17	(34)	0
(35)	68	(36)	201
(37)	0	(38)	40
(39)	0	(40)	0
(41)	0	(42)	0
(43)	41	(44)	502
(45)	87	(46)	91
(47)	102	(48)	1
(49)	0	(50)	10

Sum of holding time (busy calling lines) = 2338.

12. System Delays

In a system of this type there are two kinds of delays, those set by the design and those controlled by the quantities or equipment (markers and registers). Calculations have been made for both, but the latter take the form of tables for the use of the customers. In design, a typical case is the access to a marker by a line unit. The Cromelin constant delay formulas [1] reveal that this delay will average between 40 to 50 milliseconds depending on the number of frame groups. Furthermore, this calculation shows that no queuing of consequence occurs because the line unit is limited to generating 36 erlangs. In a similar way, calculations show that a marker accessing a line unit can expect a delay of 60 milliseconds. Neither was there significant delay found in the accessing by any of the common circuits or the junctor link. Again using Cromelin constant delay, studies of the marker indicated that close to the theoretical maximum efficiency was reached when the quantity numbered 4.

The marker capacity was calculated to be as follows:

Number of Markers	Occupancy in Erlangs
1	0.4
2	0.6
3	0.7
4	0.75

Since marker quantities are holding-time based and since most connector quantities are directly affected by marker quantities, high speed in marker operation even on line-to-register or trunk-to-register connections is most essential.

Registers may be provided on either a loss basis or a delay basis depending on choice of the customer. Referring to Figure 15, it can be seen that for 300 hundred-call-seconds (8.33 erlangs) offered load, either 14 or 19 registers would be required depending on the basis used. Traffic administration based on dial-tone speed-

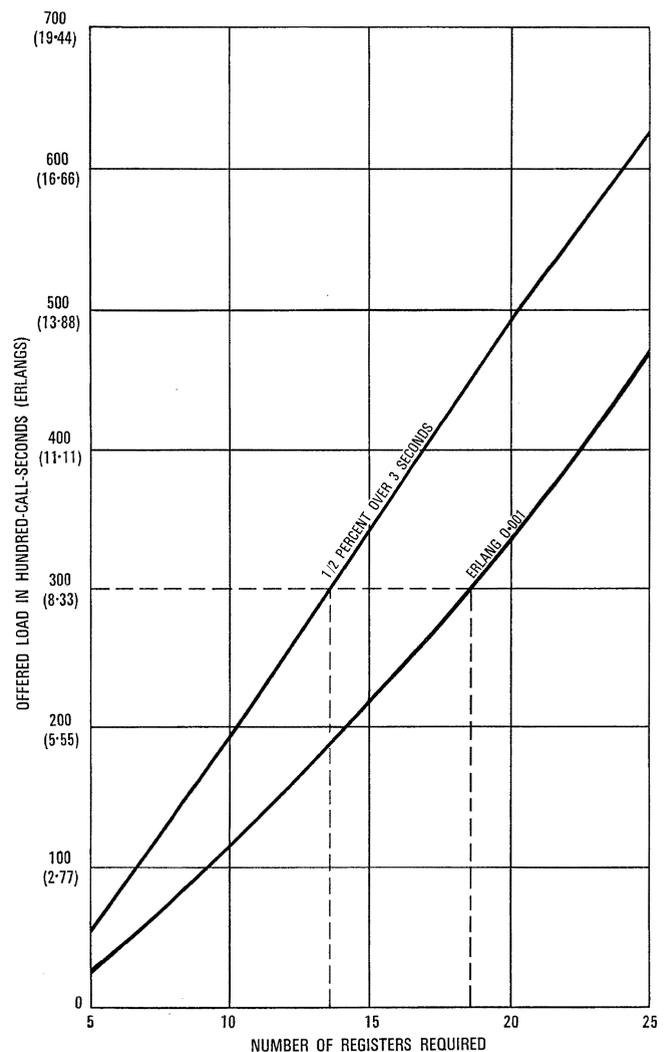


Figure 15 - Registers required on either a delay or lost-call basis.

measuring devices must be used if delay-basis engineering is to be acceptable.

13. Maintenance

The techniques of maintenance are conventional and are based on the principle that neither traffic overloads nor customer or operator irregularities shall produce trouble records. After establishment each connection is thoroughly monitored and tested before release by the marker. Marker access to each transmission path for testing is by use of a relay in each trunk or feeding bridge. Since only one marker is allowed to control one line unit, it is possible to multiple these paths and connect them to the marker in an economical way. Having provided this relay arbitrarily designated as *F*, it then becomes convenient to use it for many control purposes.

The markers used in this system make several attempts to serve a call before accepting the fact that the call cannot be saved. If a call is not completed because of traffic congestion then recycle or retest or both are used to shift the junctor group or the trunk group to a free combination. This action has the effect of providing additional traffic capacity without additional multiswitch verticals. If the failure is due to an equipment problem such as a broken wire or a dirty contact, a second trial is instituted using different common equipments. Failure of all the attempts results in setting the call to an overflow tone but only after a record of the trouble is printed on a punched standard card.

Registers are so arranged that they may be caused to remain out of service when a timeout occurs. Since it is conceivable that all registers might be affected by the same problem, only a small portion are allowed to assume this condition while all others are automatically released.

A marker which is unable to complete its functions will timeout very rapidly but will always restore itself and leave a record on the punched card and give an alarm.

Senders encountering unserviceable trunks render a record of this condition via the register and marker and the trouble-recording robot. The punched card identifies the trunk by number and the sender's point of failure in progression. This type of trouble will often be a fault in a distant office.

Subscriber line faults such as permanent signals may be set either to line lockout or to a programmed series of tones and howlers automatically applied in a timed sequence. If the automatic system does not clear the line in a predetermined time (usually 10 minutes), a card is punched identifying the line in trouble and later a connection may be established to a test desk.

Audible and visual alarms indicate all fuse operation and all control equipment timeout. Counting meters are provided for both the traffic and plant departments. In general the traffic meters show flow and overflow in the various routes while the plant meters show the number of seizures of the common circuits and the number of timeouts and second trials.

Figure 16 shows the marker test panel. Using this panel any condition recorded as a fault may be exactly reproduced using the identical speech matrix elements indicated in the record. This means that the path in trouble through to the trunk or feeding bridge is again seized just as it was on the call that encountered trouble.

14. Traffic Administration

Almost all meter registrations originate with the common marker. The principal index of matrix performance is failure to match, but counts of test, retest, cycle, and re-

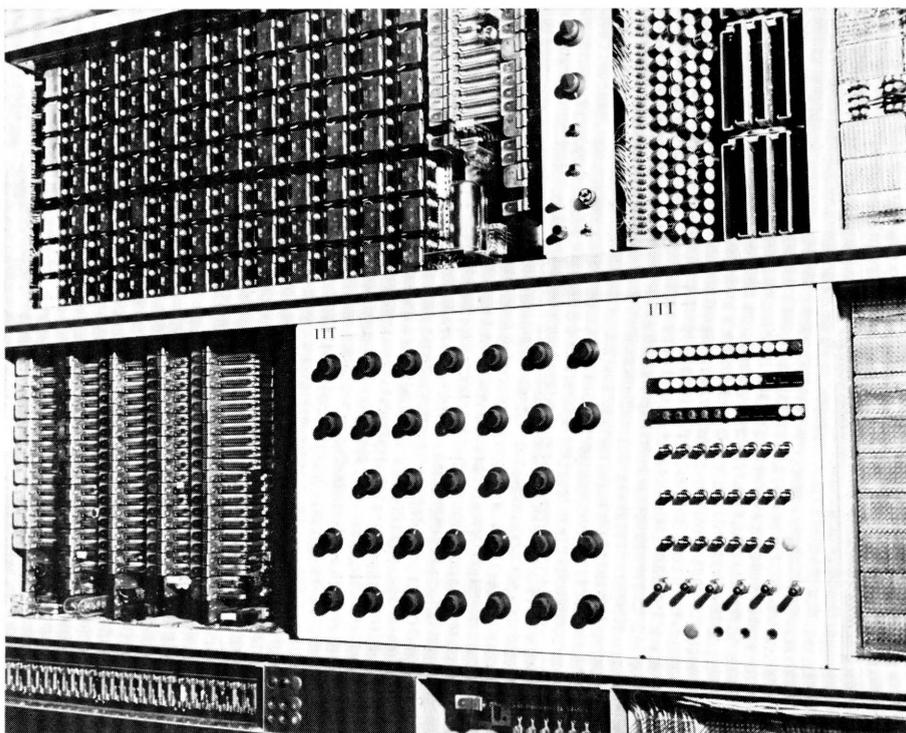


Figure 16 - Marker test panel.

cycle give indications of the level of busy elements encountered. For load balance, the marker may be set to score when any particular group of 8 line-unit verticals has more than 5 busy indications.

When it is desirable to engineer registers on the basis of the 10 highest traffic days (see Figure 15), a circuit must be provided that will measure and record the speed with which registers are able to furnish dial tone.

15. Message Charging

Standard traffic meter operation is by means of sleeve pulses. Toll calls make use of a translation from equipment location to directory number. These translators are of a conventional diode type. During light load, the marker interrogates the number translators as well as the equipment-location translator and requires that they match.

16. Centrex Features

All of the features of standard private automatic branch exchanges may be made available using this equipment in its Centrex configuration. If a large building has sufficient extensions to require a complete system, as many as 100 customer groups could be served each with its own attendant. When the system is located in such a building and gives this type of service, the service is characterized as *CU* or customer's Centrex service.

Where a large private automatic branch exchange is near a central office, it is reasonable to bring all of the lines into the central office. This form of service is referred to as *CO* or central office Centrex service.

Centrex service provides all of the normal features of private automatic branch exchanges some of the more important of which follow.

- Identified outward dialing.
- Direct inward dialing.
- Call transfer.
- Intra private-branch-exchange dialing.
- Conference circuits.
- Tie-line calls.

The first system installed provided all of these features for United States Nellis Air Force Base. The local city directory provides 7-digit numbers to reach the operator or to reach certain individuals on the base directly. For calls among stations on the base, 4 digits are used. Stations on the base reach the operator by dialing 0. Outside calls from the base use the digit 9 then on receiving a second dial tone, either 7 or 10 additional digits.

Most of the traffic reaching the attendant is incoming calls for a listed number. To release the operator from unnecessary supervision, once incoming calls are served, the calls are caused to disappear from the board and are returned only if a request for transfer is initiated by the called extension.

The attendant position (Figure 17) is accessed by attendant trunks and an additional linkage. These trunks act as parking or waiting locations and remain connected as long as the call is in progress. The attendant may set up a call and then release it or hold the call in her position if it requires supervision. Each attendant trunk is equipped with a relay that recognizes a request for trans-



Figure 17 - Attendant position.

fer. Local and incoming trunks are also equipped for transfer.

To transfer a call, either a transfer button on the subscriber set is pushed or its switch-hook is depressed momentarily. The associated trunk circuit will request a transfer circuit, which will be provided through the linkage in the same way as an ordinary subscriber is provided a register except that the marker recognizes that a special 6-wire link is required.

The transfer circuit has 2 line appearances which can demand registers and connect alternatively to the new line dialed. Transfer from line to line can continue indefinitely.

When more than one Centrex group is involved it is necessary to store the identity of the group in the trunk. This action is accomplished via the *F* relay described in Section 13. When the marker operates this relay in the course of setting up a call the identity of the group being served is stored in a 2-out-of-5 code in the trunk. Two digit stores are required when more than 10 groups are served.

The appeal of Centrex service is largely dependent on local tariffs, conditions of ownership, and physical characteristics of the area being served.

17. Conclusion

Future development of the *A1* system continues with the use of miniature crossbar multiswitches and electronic control circuits. It is also expected that the use of miniature relays and repeat coils will allow miniaturization of trunks.

All of these advances are expected to increase the value of the most-direct-path selection technique.

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He served in the United States Navy as a submarine officer working in conjunction with Bell Telephone Laboratories on the early development of guided torpedoes. On release from naval service he joined Bell Telephone Laboratories, where he was concerned with the development of direct distance dialing, centralized automatic message accounting, automatic number identification, and automation for person-to-person calls and various switching projects.

On joining International Telephone and Telegraph System in 1963, he founded its Caribbean Research and Development Laboratory, which was moved to Chicago in 1967. He is now an executive of the Puerto Rico Telephone Company.

Herkomat Electronic-Control Private Automatic Branch Exchange

A. HEZEL

Standard Elektrik Lorenz AG, Stuttgart, Germany

1. Introduction

The use of electronic control of reed relays for telephone switching has been called quasi-electronic switching [1, 2]. In these systems, the hermetically sealed contacts [3] bearing the trademark "Herkon" are not restricted to the crosspoint matrix but are used as switches in the control system as well. The effectiveness of these Herkon contacts has been established by trial installations of the HE-60 L quasi-electronic switching system in the public networks of Stuttgart [4] and Vienna [5] and in their use in several hundred translators in the long-distance switching network of Germany [6]. A logical extension of such switching systems has now been made to private automatic branch exchanges [7]. A prototype was displayed at the International Traffic Exhibition in Munich in 1965 and a factory-produced system under the registered trademark of "Herkomat" was shown at the International Industrial Fair in Hanover in 1967.

2. Systems Structure

The Herkomat* II PABX is available in various sizes ranging from 3-30-4 to 10-100-12 (the figures indicating the numbers of trunks, extensions, and internal junctors). The subsets can be equipped with the conventional dial or with a set of push buttons for push-button calling.

The Herkomat system has the following characteristics which relate to the use of the quasi-electronic techniques.

2.1 Speech-Path Switching in Space-Division Multiplex

To ensure the optimum transmission characteristics, the switching network was equipped with reed crosspoints [9]. Each crosspoint in the switching grid consists of one coil and four reed contacts type H 50 [15]. Five such crosspoint elements are then mechanically combined into a crosspoint strip [10].

2.1.1 Grouping

The grouping of a PABX with 50 extensions using dial switches is shown in Figure 1. Up to size 8-50-6, the subscriber switching grid has a single stage with full availability, so that the 50 extensions have direct access to all internal junctors, to the auxiliary junctor, and to all trunk junctors. In the larger systems up to 10-100-12, the subscriber switching grid is a three-stage link arrangement. The extensions are connected to the first stage and all junctors to the third stage.

2.1.2 Hunting in the Subscriber Switching Grid

In systems with single-stage crosspoint arrangements, the crosspoint elements are operated by direct marking in a conventional way. In three-stage arrangements, a guide-wire method employing an offering and a seizing potential is used [11]. Figure 2 shows the principle of interconnecting the guide wires in the three-stage link arrangement.

A calling extension causes a z contact to apply the offering potential to all free guide wires of all the switching stages in that crosspoint matrix of the A stage to which this extension is connected. The marker selects a free junctor connected to that C stage reached by the

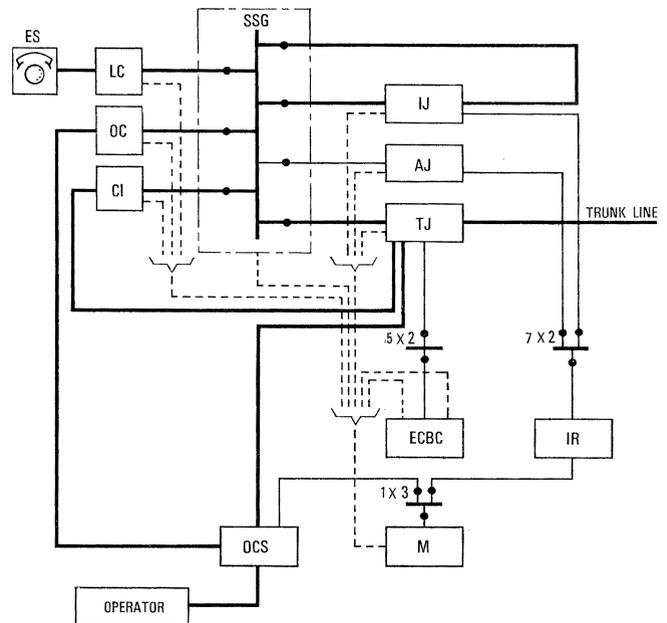


Figure 1 - Block diagram of a Herkomat PABX 5-50-6 with conventional dial selection.

The following abbreviations are used in the text and drawings:

ES - Extension station	TJ - Trunk junctor (external)
LC - Subscriber circuit	AJ - Auxiliary junctor
OC - Operator circuit	IJ - Internal junctor
CI - Inquiry line circuits	SSG - Subscriber switching grid
OCS - Operator's connection set	RE - Restriction of extension
ECBC - External-call barring circuit	CR - Code receiver
IR - Internal register	ER - External register
M - Marker	

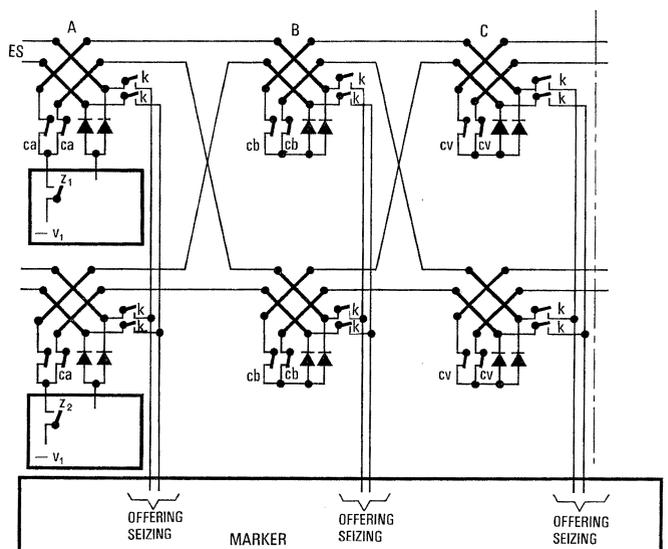


Figure 2 - Guide-wire arrangement (schematic).

* Trademark of International Telephone and Telegraph Corporation and associated companies.

offering potential. The closing of the *k* contacts in the corresponding crosspoint matrix of the C stage switches all guide wires connecting this crosspoint matrix to the B stage, through to the marker. The marker then selects a free guide wire and applies an offering potential to it. This causes the closing of the *k* contacts in the corresponding crosspoint matrix of the B stage.

In the B stage, the same procedure is repeated. In the A stage, the marker marks the calling extension.

Subsequent to the selecting and seizing processes, the crosspoint elements *KE* are operated section by section (Figure 3). Thus the seizing relay of the junctor closes with its contact *cv*, which is the holding-current circuit for the crosspoint element *KE* of the C stage. After the marker has operated the crosspoint element, the corresponding link relay *CB* in the B stage operates and prepares the holding-current circuit in the B stage. In the course of the further program steps, the relay *CA* in the A stage and finally the cut-off relay *T* in the extension circuit are energized. The operated relay contacts *ca*, *cb* and *cv* indicate the busy state of the links and junctors.

2.2 Central Logic and Program Control

In addition to operating the crosspoint elements in establishing the connection, the marker also centrally controls complex functions. For instance, the marker tests a desired connection to ascertain whether it is free or busy; it also checks the effectiveness of the first ringing pulse. Normally, since various combinations of process steps are necessary for the various functions, several programs have been prepared so that the marker may use the optimum program for each function.

All the functions and operations are controlled by

clock pulses on which all the programs are based (for instance, *originating internal traffic*, *terminating internal traffic*) whenever the marker is started. The clock pulse is derived from a 425-hertz tone generator with a stabilized frequency output. If a program should be disturbed, the signal from the calling extension cannot be erased; this is how the marker recognizes a fault before going on to investigate alternative connecting paths. In this way, all the functions pertaining to a program are checked for completeness.

All the functional units that can activate the marker are periodically scanned at a frequency of 10 kilohertz. When the marker has been activated, it receives a code for the program to be used.

Clock-pulse-controlled timing circuits determine the occupancy time of the code receiver and the protective times in the internal and external registers. In push-button selection systems, even the emission of dial pulses to the public network is controlled by this clock system.

2.3 Systems for Dial Signaling or Push-Button Selection

The Herkomat subsets can be equipped either with the conventional dial or with push buttons for push-button selection. Apart from this, the two versions offer no discernible differences.

For push-button selection, the so-called diode-grounding method is employed. It is based on the standard code specified by the German PTT for PABX. A central code receiver translates the push-button selection signals [12] and supplies the result for further processing to the internal register. Since the public network employs a dialing system, external registers are employed for push-button selection. The external register is available to the trunk junctor only while the connection is being established

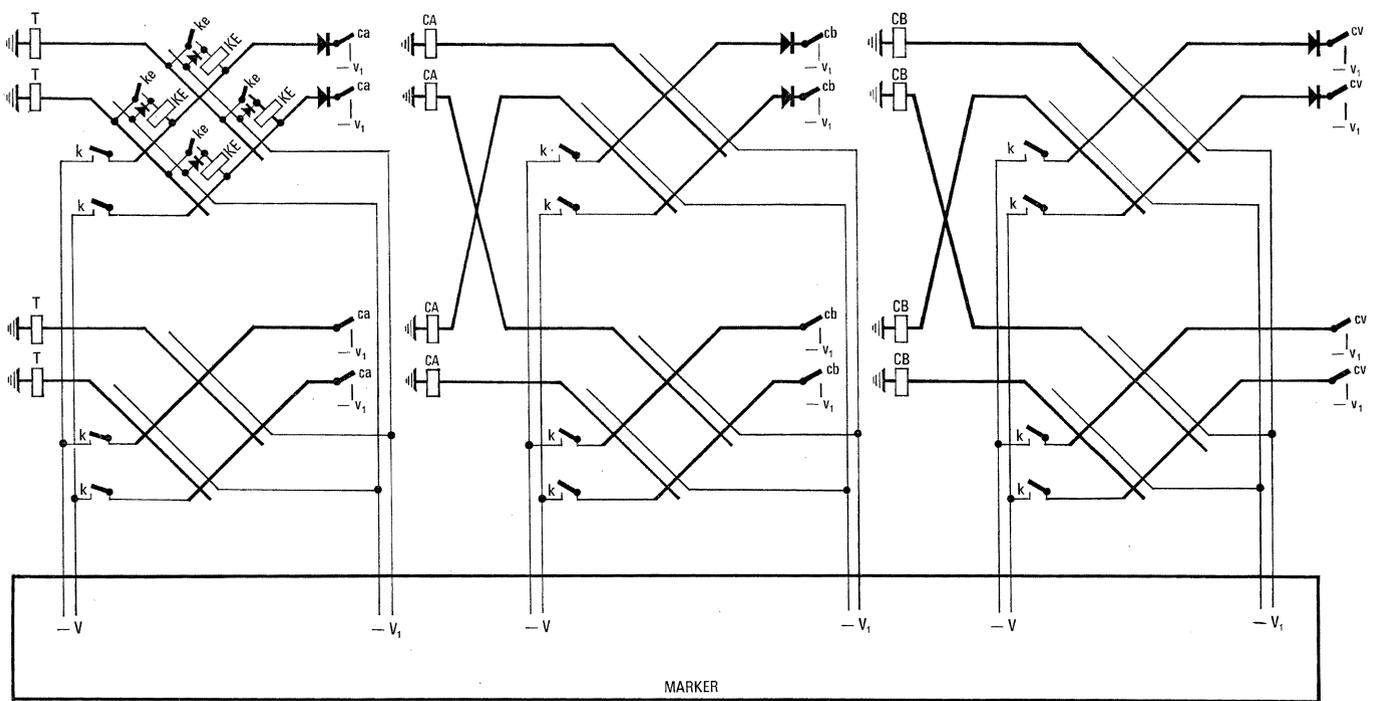


Figure 3 - Holding wire arrangement (schematic).

and translates the push-button selection signals into the dial pulses required for the public telephone network. The selected number of the called subscriber is stored in an intermediate buffer in the external register.

2.4 Electronic Ringing-Current and Signal Generator

The ringing current generator is designed for 50 hertz. The tone generator (425 hertz) also supplies the time base for clock control of the system; its frequency is therefore carefully stabilized. Counter networks divide the frequency and thus generate all the clock pulses required for the system, including those for the tone signals in accordance with the International Telegraph and Telephone Consultative Committee (CCITT) recommendations.

3. Traffic-Handling Procedure

The sequence of events in a system is largely the same for dial and push-button selection. Therefore, a push-button selection system will be explained first, and the differences between it and a dial-switch system will be discussed subsequently.

3.1 Internal Traffic

When the handset of an extension is lifted, the marker *M* (Figure 4) is activated by the subscriber circuit *LC*. A continuously running scanner in the marker identifies this starting signal and comes to a stop. The program *outgoing internal traffic* that is controlling the cycle of events proceeds and an internal junctor *IJ* and an internal register *IR* are connected to the calling extension through the subscriber switching grid and the internal-register switching grid. The calling subscriber now receives the proceed-to-dial tone from the internal register.

As soon as the internal register has recognized the operation of a push button, it connects to the central code receiver *CR*. This code receiver translates within 30 milliseconds the code combination associated with the operated push button and supplies the information to the internal register, where it is stored. Now the connection to the code receiver is immediately interrupted so that the code receiver is free for other calls. The internal register monitors the push buttons for possible additional operations. If a new push button is operated, the above procedure is repeated and additional storage cells in the internal register accept and store the push-button selection signals.

When the number of the called subscriber is completely stored in the internal register, this register starts the marker. The starting signal is identified by a scanner that causes the internal register to be connected to the marker and the program *incoming internal traffic* to proceed. The marker causes the called extension to be connected through the subscriber switching grid to the second output of the internal junctor. It tests the busy state of the desired connection via the operated switching grid. If the called extension is free, the marker causes the ringing generator to be connected and controls the duration of the first ringing pulse.

3.2 Outgoing Trunk Traffic

When the handset of the calling extension is lifted, a free internal register is reached. When the decoding of the first push-button signal indicates a trunk call, the internal register starts the marker and provides it with the corresponding information. The marker identifies the calling extension and checks whether or not it is unrestricted for external calls. When a trunk junctor *TJ* (Figure 4) and an

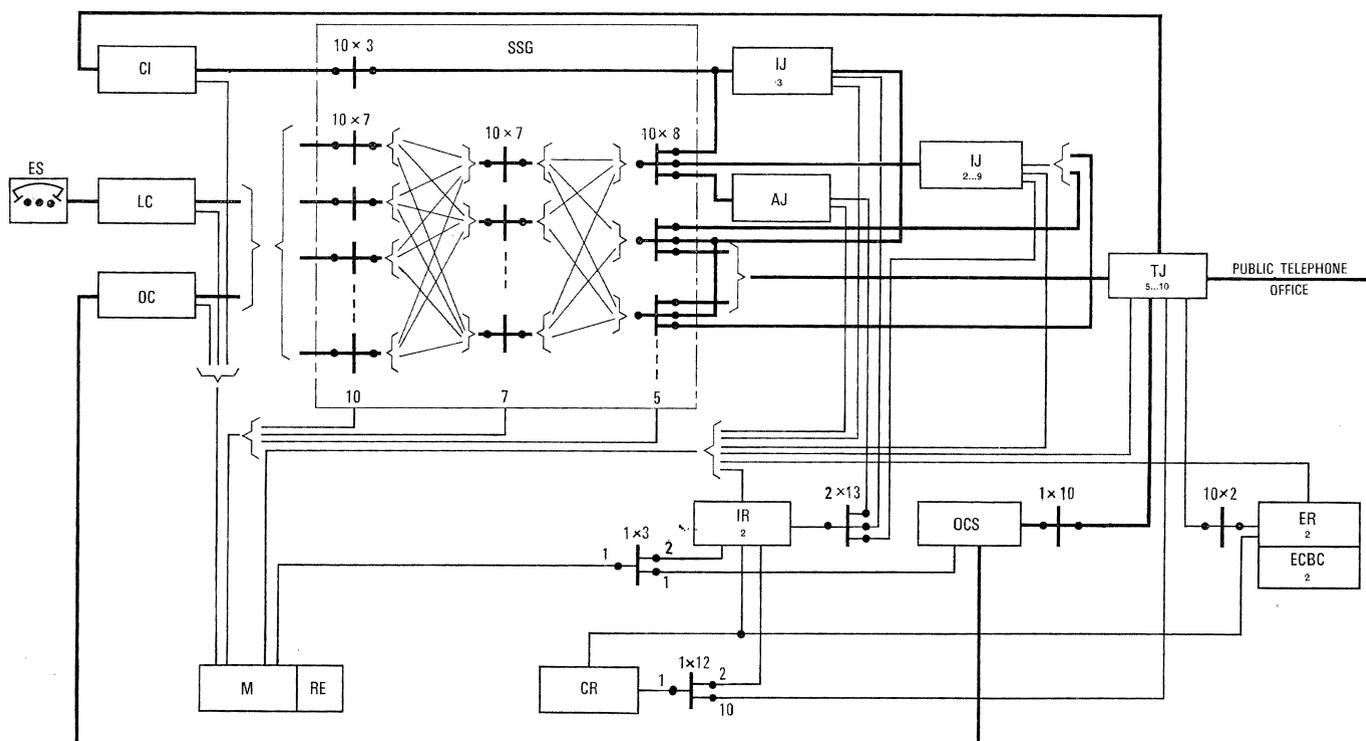


Figure 4 - Block diagram of Herkomat PABX 10-100-12 with push-button selection.

external register *ER* are available, the number of the calling extension is then stored in accordance with the program *outgoing external traffic*; the existing connection to the internal register is released and a new connection is made to the trunk junctor. The trunk junctor connects to the external register and causes the proceed-to-dial tone to be transmitted to the calling extension.

When the calling subscriber now begins the push-button selection, the operation of a push button is identified in the trunk junctor. This connects to the central code receiver, which processes the information and passes it directly to the external register. Here the individual digits are stored and coded into series of dialing pulses with the aid of a pulse sender. The pulses are transmitted through the trunk junctor to the public exchange office. The memory of the external register can store ten digits and operates in a cyclic procedure. After the stored dialing signals have been transmitted, the external register is disconnected when at least 8 seconds have elapsed since the last operation of a push button.

If the marker finds no free trunk junctor, or if the calling extension is barred for trunk calls, the internal register is immediately released and the calling subscriber receives the busy tone from the internal junctor. If no external register is free, the marker is disconnected. The calling subscriber may wait for an external register to become available, but only for the 10 seconds required for disconnection of the internal register.

3.3 Incoming Trunk Traffic

An incoming trunk call is identified by both optical and acoustical signals; the operator depresses a key and is thus connected to the trunk line. The incoming trunk call is routed to the called extension with the aid of a set of push buttons. Operating a push button causes the trunk junctor to connect to the code receiver, which routes the decoded digit information to the connection set *OCS*. As soon as the called number is complete, the connection set starts the marker which directs the subscriber switching grid to the desired extension in accordance with the program *incoming trunk traffic*. Checking of the called extension, to ascertain whether the latter is barred for terminating trunk calls, is performed via the subscriber switching grid; if the called extension is barred, the operator is informed by a corresponding signal.

If the called extension is busy, the operator may connect himself to the existing connection. As long as this state lasts, a warning tone indicates it to both subscribers. The calling external subscriber may wait for the called subscriber to become free, in which case the system automatically establishes the connection.

3.4 Internal Inquiries and Call Transfers

While a trunk call is going on, an extension subscriber may operate the office key in the usual way to make an inquiry at another extension. As a result of the key depression, the trunk junctor connects to the code receiver which identifies the code combination and transmits corresponding information to the trunk junctor. Then the subscriber is connected to an inquiry line circuit. As

described in Section 3.1, an internal junctor and an internal register are occupied. The trunk junctor switches through and the extension is fed from the internal register until the internal inquiry connection to the other extension has been established. In all other respects, the establishing of the connection is accomplished as in ordinary internal traffic. By another depression of the office key, the subscriber may terminate the internal inquiry and re-establish the connection to the waiting external subscriber.

If the external call is to be transferred to another extension, the subscriber simply dials the new extension number and replaces the handset. The marker started by the trunk junctor then initiates the program *transfer of a city call*. When the check for non-restriction has been successful, the second extension number is stored while the connection to the first extension is released. Then the new connection is switched through.

In the single-stage subscriber grid (Figure 1), the inquiry line circuits *CI* are connected as ordinary extensions. In the three-stage subscriber switching grid (Figure 4), the inquiry line circuits reach the internal junctors, preferably available for the inquiry traffic, through a concentrator. These internal junctors have access to all crosspoint matrices to which trunk junctors are connected. When an internal inquiry occurs, these internal junctors in principle always use that crosspoint matrix to which the trunk junctor belonging to the inquiry call is connected. In the case of a call transfer, at least the links used for the inquiry are available even during a busy hour; the call can thus be transferred at any time.

3.5 Other Facilities

Of the wide range of service features, some of the more interesting are listed below. Certain of them belong to the standard equipment, while others are available at extra cost.

— *Classes of restrictions for extensions.*

There are eight different classes; one extension may belong to several classes.

— *Cutting-in for extensions.*

This feature is fixed in the system only by class association independently of the subscriber station. The subscriber in question causes the cutting-in by dialing a certain single-digit number.

— *Numbering of extensions.*

Apart from the usual two-digit numbers, one- and three-digit numbers are available.

— *Tie-line traffic.*

The systems can be equipped with the well-known devices for tie-line traffic. In push-button selection, the coded selection information has to be translated into dial pulses just as in outgoing external traffic. This is accomplished by the external register. If the system of the called subscriber also has push-button selection no such translation is necessary.

3.6 System Differences with Dial Switches

Essentially the internal traffic of a system with dial switches is similar to that of one with push-button selection. Since the dial information consists of a series of

pulses, no code receiver is necessary. The internal register accepts the dial pulses directly into its memory.

In the outgoing external traffic, the dial pulses are directly routed to the public office so that no external register is needed.

4. Design Details

The use of electronic components and reed relays considerably affects the design of this PABX [13] (Figure 5). Special features include simple design, rack wiring arranged on the front side, and the construction and arrangement of the distribution board, which also serves as a jumpering board. Since PABX's with up to 100 extensions are often installed in office rooms, their cabinet shape and color have been matched to this environment.

The Herkomat exchange with up to 50 extensions is accommodated in a cabinet with the following dimensions:

Width	820 millimeters (32 inches)
Depth	486 millimeters (19 inches)
Height	1887 millimeters (75 inches)

Systems for up to 100 extensions require two cabinets with the dimensions:

Width	820 millimeters (32 inches)
Depth	486 millimeters (19 inches)
Height	1482 millimeters (58 inches)

The cabinet is a welded structure of sheet steel. Below the detachable cover, the extension and trunk-lines frame can be rotated for convenient access. Here also

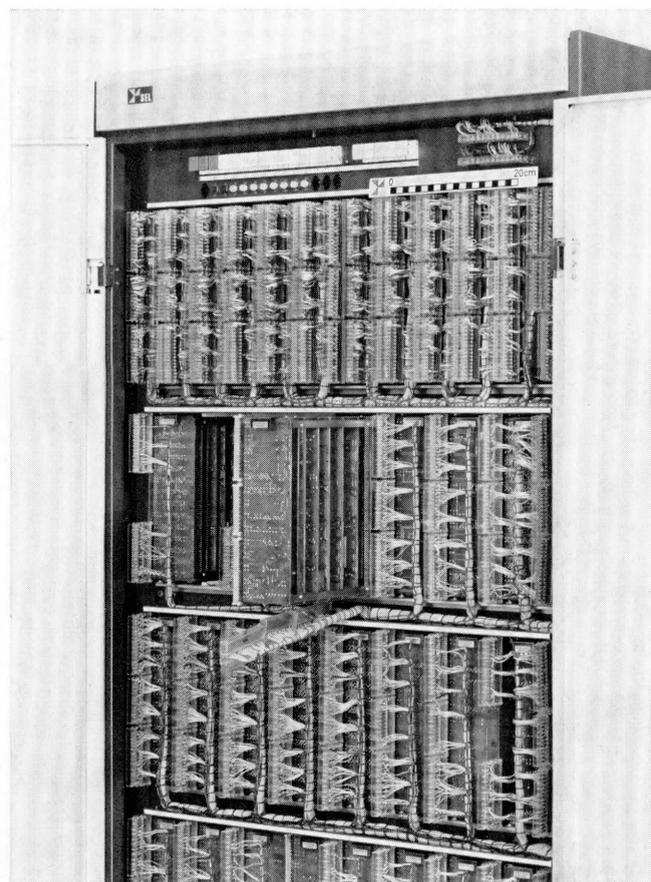


Figure 5 - Herkomat PABX 5-50-6, one control unit is partly pulled out.

are provided the terminals for the operator's desk and for cabinet interconnection for systems with two cabinets. The side walls protrude to the back so that they protect connecting cables when the cabinet is installed close to the wall of an office room.

A cabinet has four or five compartments formed by aluminium rails and guide rails of fiber-glass reinforced plastic. Two guide rails fix the plug-in units.

The same design principle is used for the switching network units and the control units. These units have terminals on their front panels, a base area of 262 millimeters (10 inches) \times 394 millimeters (15 inches), and require a division of 61 millimeters (2.5 inches) or 86 millimeters (3 inches) in the compartments.

As an example, Figure 6 shows a control unit. Sub-unit boards are vertically mounted on the unit board. Both boards are made of phenolic paper and have uniform rows of holes and printed wiring soldered by the flowsoldering method. All the electrical outputs of the sub-units terminate on connectors that connect to the wiring of the unit board. A metal enclosure fixes the two free corners of the sub-unit board. This design principle results in a stable construction resistant to mechanical distortion.

Switching grid units are of a similar design. In addition to the unit board, they have a second board which forms an additional connector plane. In place of the sub-units, these units are equipped with crosspoint strips, each combining five crosspoint elements into a mechanical unit.

The subscriber switching grid is so subdivided among the grid units that, in a single-stage crosspoint arrangement, two of these units contain that part of a grid required for ten extensions, that is, a switching-grid section up to size 10×21 in the 5-50-6 system. This arrangement was so made that one of the units consists of a 10×8 matrix together with ten subscriber circuits while the other unit consists of a multiswitch up to size 10×13 . Both units are wired to form the above described grid

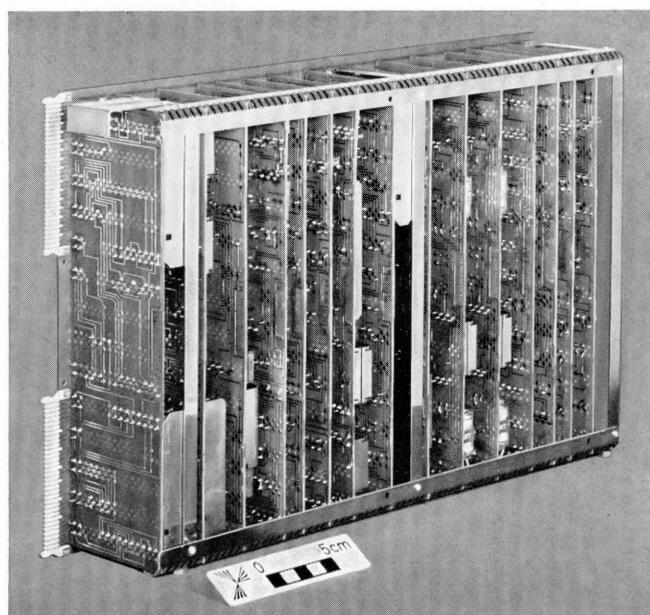


Figure 6 - Control unit of the Herkomat PABX.

section. In systems with three-stage crosspoint arrangements, each unit has a crosspoint matrix of the stages A, B or C in the size 10×7 or 10×8 where the A-stage units again consist of ten subscriber circuits.

Depending upon the size of the PABX the power supply units will have different output powers. However, the housing is of the same size. These power units are mounted on the base of the cabinet and are plug-in types.

The cabinet cable has solderless wrapped connections and is dimensioned according to requirements. It includes the whole internal wiring, feeders from the power supply to the units and all connections to the distribution board. All function outputs terminate on 33-point or 25-point female connectors to which the units can be plugged while the subscriber outputs end on two-point male connectors which are plugged into the female connectors of the distributor frame on site. In a case of subsequent jumpering, the subscriber outputs involved need only to be re-plugged on the jumpering frame.

5. Operator's Desk

The operator's desk (Figure 7) has a plug-in type connecting cable. It can be connected to the cabinet either



Figure 7 - Operator's desk of a Herkomat 5-50-6 exchange with push-button selection comprising meters and luminous-number extension identification.

directly or through a junction box with extension cable. The set is relatively small, simple to operate, and uniform for all extension stages. The handset is also plugged-in. The desk can be equipped with luminous numbers for subscriber identification and with a meter for each trunk line.

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A 24-channel Pulse-Code-Modulation Junction Carrier System

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1. Introduction

Although the transmission of analog functions such as telephone speech by purely digital signals was first suggested many years ago it is only in recent times, with the advent of semiconductor devices and particularly integrated circuits, that the technique has become economically viable for civic use. Presently the British Post Office is ordering and installing 24-circuit pulse-code-modulation systems in large quantities, mostly for the junction network. The network traffic is between a local exchange and group switching center, local and tandem, and between local exchanges but combinations may be found in one traffic route.

Pulse-code-modulation techniques [1] are well known, and many papers have appeared describing system techniques. Mostly these papers have discussed systems contemplated, under experimental development, or on field trial. The major exception was a series of articles on the Bell Telephone System T1 carrier that has been in commercial use for several years. This present paper describes a new pulse-code-modulation system now in high-volume production that is the first to be installed for large-scale regular use in the United Kingdom telecommunications network.

2. Basic Parameters

The basic system facilities are similar to their experimental predecessors [2]. There are necessary differences in detail because of British Post Office specification requirements and also radical implementation differences due to the rapid evolution of technique and component technology.

Basically the system provides 24 speech channels each with associated signaling facilities. Each speech waveform is sampled at 8 kilohertz and each sample amplitude is quantised into one of 128 levels. Since this number is inadequate for high-quality transmission of speech signals the effective volume range is extended by amplitude companding using a nonlinear codec*. The extension in volume range now provided is 24 decibels.

To generate 128-code characters, 7 binary digits are used without restriction on combinations. The code is a symmetrical binary giving an improved signal-to-noise ratio over the most important segment of volume range reducing noise generated by random digital errors. A symmetrical binary code uses the most significant digit in the code character to indicate if the sample is positive or negative. The remaining digits indicate amplitude without regard to polarity.

For the transmission of signaling information an eighth digit is included in digit-position-one of the channel time-slot. This digit is used in the time-division-mode to provide for up to three signaling (or data) channels per

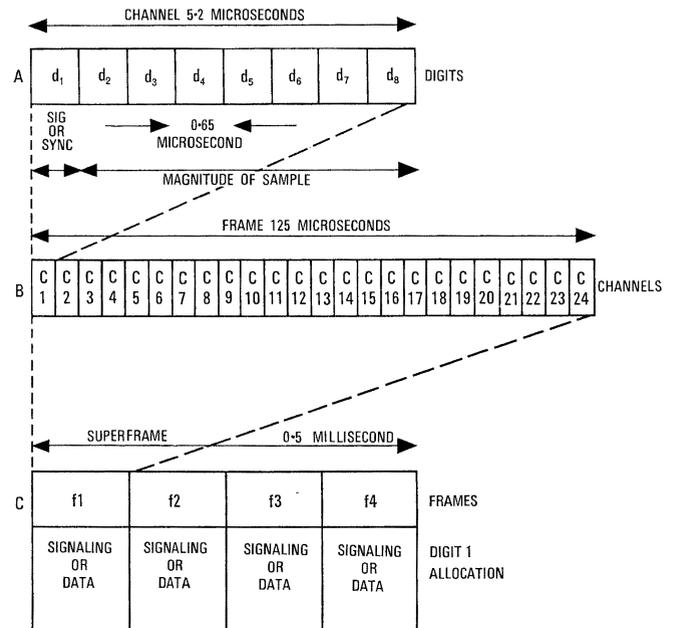


Figure 1 - Basic time division allocation.

speech channel, and also for the important function of end-to-end synchronization. Figure 1 shows the basic time-division allocation of binary digits. This diagram is directly analogous to the frequency plans of frequency-division-multiplex systems.

Each channel occupies 5.2 microseconds every 125 microseconds and the channel state is indicated by 8 binary digits. Digits d_2 to d_8 represent the polarity and magnitude of the signal sample and digit d_1 is used to send data, signaling, or synchronizing information according to the frame allocation shown in Figure 1 C. Four frames are referred to as a superframe [3] and the superframe structure is continuously repeated.

The code generated by the pulse-code-modulation multiplex terminal must be suitable for line transmission, and regeneration at any point through the line. This requires as a minimum:

- that there be sufficient timing content (which depends on the presence of marks or transitions between mark and space) to ensure correct re-timing within the repeater,
- that there be a nil or constant (or nearly constant) direct-current component in the line signal to enable unambiguous recognition of the code characters after they have passed through line transformers.

The symmetrical binary code is not suitable on both counts and it can be improved by two simple code translations.

The first is that alternate digits are complemented. In this system even digits have been chosen, that is to say d_2 , d_4 , d_6 and d_8 . The effect is to improve the density of marks and the density of transitions and hence the tim-

* Coding and decoding equipment.

ing content in the vicinity of zero level code, the most frequently transmitted code. The second translation is to reduce the unipolar binary signal to pseudo-ternary form by reversing the polarity of alternate marks. This technique is now well known as alternate-mark-inversion (AMI).

3. Multiplex Terminal

Shown in block schematic form in Figure 2 the multiplex terminal is similar to the numerous 24-channel systems described in previous literature. The most noteworthy departure from previous practice is the use of a nonlinear codec. Thus the processes of amplitude companding and encoding-decoding previously performed sequentially by separate and identifiable apparatus are now combined in the single operation of analog-to-digital conversion. Although this technique imposes severe design problems, once it is satisfactorily accomplished it gives better accuracy and reproducibility with standard components of modest tolerance.

3.1 Companding Characteristics

Because amplitude companding and encoding-decoding are combined, the companding law [3] has a profound effect on the detailed instrumentation of the codec. Figure 3 shows the positive half of the compression characteristic. It consists of eight straight-line segments, but since segments 7 and 8 are colinear with each other and also with the corresponding segments of the negative

half, the complete characteristic is composed of 13 segments. The expansion characteristic is clearly the complement.

The slope of each segment is in the ratio of 2:1 with the slope of adjacent segments. The use of 13 segments appears to be extravagant but the simple binary relationship between the slopes of the segments permits straightforward and economic instrumentation of the characteristic which approximates closely to a smooth curve. The companding advantage provided by this characteristic is 24:1 decibels which used in association with an overload capacity of + 2 decibels (relative to 1 milliwatt 0 reference level) and 128 quantization steps, gives good speech quality with four tandem connections made at audio frequencies.

3.2 Decoder

The processes of encoding and decoding are complementary. The function of the encoder is to generate a code that will represent the amplitude of the signal sample and the function of the decoder is to reconstruct the signal sample amplitude from the code to a given resolution accuracy for the system.

It is possible to construct an encoder by using a decoder in the feedback mode (feedback comparison encoding). This technique is a good one for nonlinear encoding as the encoding and decoding characteristics will automatically be complementary. The decoder, more con-

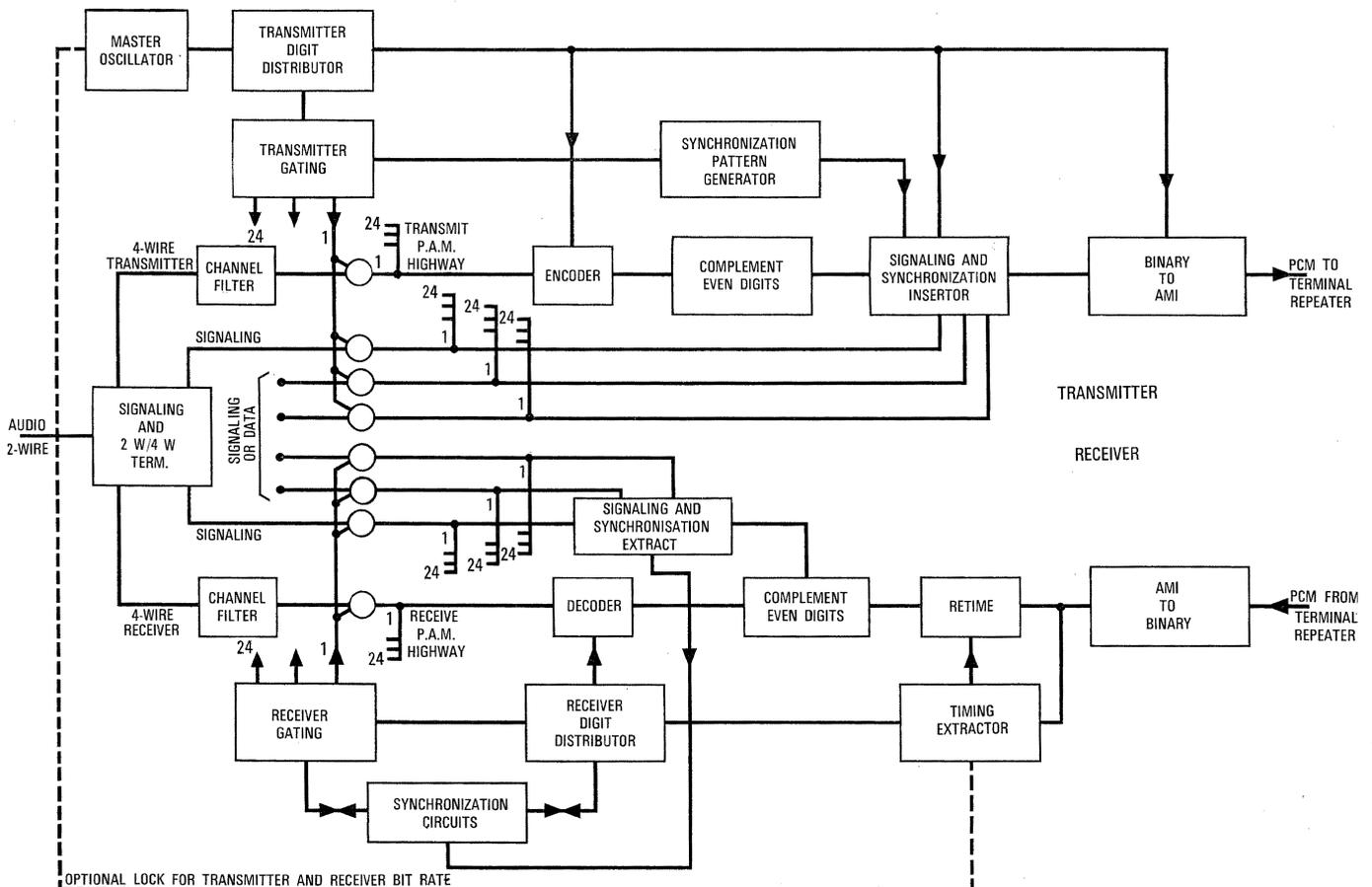


Figure 2 - Multiplex equipment.

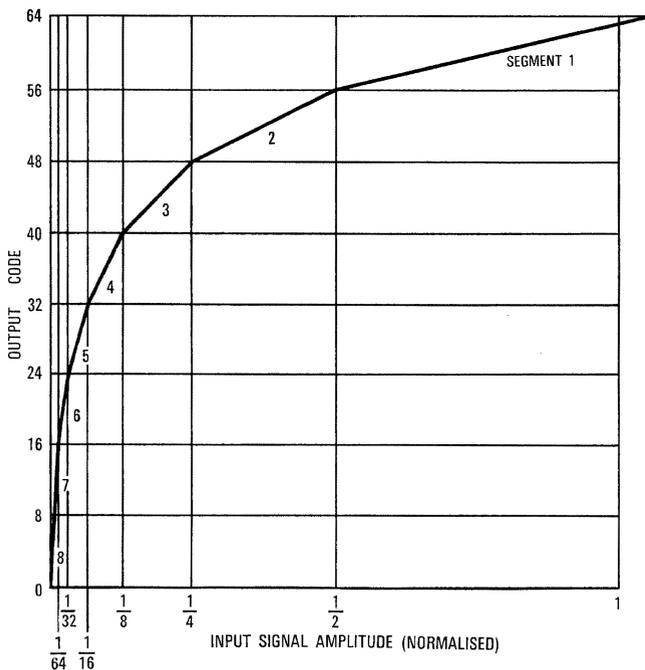


Figure 3 - 13-segment compression characteristic.

veniently described first, consists of a number of logically controlled reference units* which may be switched on, either singly or in combination, to produce an output level corresponding to each code combination. These reference units are assembled so that the transfer characteristic complements the encoder transfer characteristic (Figure 3). The nonlinear decoder is essentially seven lower-order linear decoders, one for each segment of the characteristic.

The relation between the reference units which form the lower-order linear decoder and the segments is shown in Table 1.

Table 1

Segment	Reference Units	Pedestal (Sum of Reference Units)
1	64, 128, 256, 64	512
2	32, 64, 128, 32	256
3	16, 32, 64, 16	128
4	8, 16, 32, 8	64
5	4, 8, 16, 4	32
6	2, 4, 8, 2	16
7		
8	1, 2, 4, 4, 4, 1	0

Each group of reference units relates to the resolution of levels corresponding to its particular segment but is offset from zero by a pedestal which is the sum of all preceding reference units. Thus to generate a level on segment 3 the appropriate combination of reference units 16, 32, 64 and 16 must be added to the pedestal of 128 units.

* Reference units, weight units, and current generators are often used synonymously.

To obtain efficient conversion not only is the accuracy of the reconstructed sample important but also the accuracy of each quantum step. A quantum step is the amplitude difference between reconstructed samples produced by any two adjacent codes. The severity of this tolerance problem is inversely related to the number of reference units used and inevitably results in a compromise. The practical solution adopted has the merit that standard components of modest tolerance and saturating transistor logic can be used resulting in resistor-transistor micrologic being utilized throughout. The entire reference unit assembly is constructed from ten resistor-transistor-logic devices and the switching and control logic from a further eight, all in *TO-5* outline.

3.3 Encoder

A simple assembly of lower-order linear decoders can be used to accurately generate the expansion characteristic. The same assembly used in the feedback loop of the encoder can be made to generate the complementary compression characteristic. Whilst this is conventional the complete encoder characteristic differs from the decoder characteristic (Figure 3) in that the analog-input to output-code is symmetrical about its origin — the output code being directly symmetrical binary.

A functional diagram of the encoder is shown in Figure 4. Two comparators are used and the input-signal sample is phase split. When the signal input is zero the two outputs of the phase splitter are also zero. When the signal input is $+A$ the outputs of the phase splitter are $+A$ and $-A$ and poled as shown in Figure 4. The circuit biasing, when the signal sample and the output of the decoder are both zero, ensures both comparators are balanced. The output of the decoder is restricted to positive values and so a balance for positive signals can be reached only on comparator 1 and for negative signals only on comparator 2.

The decoder has one half the number of quantum steps that the complete encoder is capable of coding. This is because the decoder output is compared only to the deviation of the input signal from zero, zero in this case being taken as the center of the repertoire of codes. For this condition, digit-one of a symmetrical binary code represents the polarity of the input signal and can be determined from the output of one comparator when the decoder output is zero. This resolution of digit-one is also used to operate the switch (Figure 4) which selects the correct comparator for the remainder of the coding process.

If the signal is positive, comparator 1 is used for coding the signal amplitude and if the signal is negative, comparator 2 operates. At the end of each coding operation the switch is returned to comparator 1 so that the first digit (the polarity digit) for the next sample can be determined. The output code for this arrangement is symmetrical binary and so whatever transfer function the decoder possesses the overall transfer function of the encoder will be symmetrical about zero.

To avoid center clipping, any differential offset voltage between the two comparators should be minimal. When

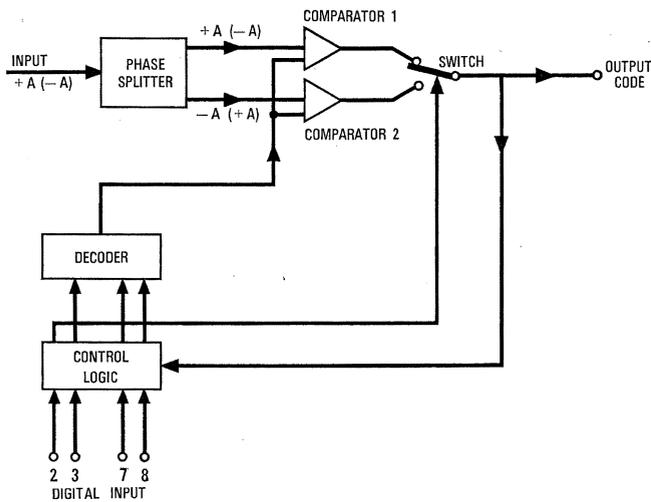


Figure 4 - Symmetrical non-linear encoder.

the output from the decoder is zero the comparator outputs should always be the complement of each other without regard to the input signal amplitude or polarity. If a differential offset voltage existed between the two comparators, say V_{do} , then for all input signal amplitudes of either polarity less than V_{do} volts, the comparator outputs will be of the same sense, namely both marks or both spaces. This indication of a differential offset voltage is used to actuate a feedback mechanism so as to re-establish the balance.

To obtain maximum benefit from the compression characteristic and to avoid asymmetry in the decoded signal sample, it is important that the input signal sample is aligned symmetrically to the nonlinear encoding characteristic. This centering process is obtained automatically by a feedback mechanism. Misalignment is detected by comparing the number of channels that are coded positively with those coded negatively over a fixed group of 23 channels. If 12 or more channels are coded positively, then the sense of the correction is in one direction and if 12 or more are not coded positively the sense of the correction is reversed. The maximum speed of correction in either direction is approximately one quantum step per ten frames. This method of using the polarity of the majority of channels within a group to control the correction, rather than using each individual channel polarity, gives a significant increase in the sensitivity of the detection of misalignment.

3.4 Synchronizing

It is necessary to keep the sampling, decoding, and distributing functions of the receiver in step with the incoming signal with reference to the digit period, the character period, the frame period, and the signaling-frame period. The information required to achieve synchronism is conveyed as a pattern of 10 bits, each bit occupying the digit-one d_1 position of channels 9 to 24 in frame f_4 . Digit-one in channels 1 to 8 is transmitted arbitrarily as a mark, and does not convey meaningful information. The synchronizing pattern is:

11010101010101

To find synchronism the receive terminal examines every eighth bit of the incoming pulse-code-modulation signal starting at an arbitrary position. When two consecutive marks are detected, an examination is made to see if alternations follow. If alternations persist for a count of seven then the synchronizing pattern is assumed to be detected. If any violation of the pattern occurs before the count of seven is reached the mechanism is reset and a search for two consecutive marks continues. This sequence continues for approximately five frames and if no synchronizing pattern gets detected, a slip of one digit occurs and the search continues. When synchronism is established the presence of the synchronizing pattern is confirmed repeatedly. If any errors are detected the search operation is not started again unless three or more consecutive incorrect synchronizing patterns have been received.

The time to achieve synchronism depends upon the number of fortuitous simulations occurring and there is a finite, but very small, probability that this can be indefinitely long. Measurements taken of several hundred events gave an average re-synchronizing time of 2.26 milliseconds, with a maximum time of 4.88 milliseconds.

4. Signaling

4.1 Principles

Pulse-code-modulation junctions replace physical junctions between telephone exchanges and must transmit all the speech and signaling information normally carried by a physical junction. Thus the condition for setting up, holding, fee-metering, releasing, and other operations, must be accepted by the pulse-code-modulation transmission system and faithfully reproduced at the distant end.

The system can provide up to three signaling channels per speech channel; refer again to Figure 1 which shows the basic time-division allocation of these channels. Clearly, if any one of these auxiliary channels is not used for telephone signaling purposes then it may be used for the transmission of other information, of which data may be the most important. Since the sampling frequency of each of these channels is 2 kilohertz, any one of them would have a synchronous signaling (or data) capacity of 2 kilobits per second.

This high bit-rate enables a comma-free code to be used for all telephone signaling information. A comma-free code is one which neither has nor requires any punctuation. The most obvious codes are a series of spaces (0000...) or a series of marks (1111...) and an examination of any single digit will show which code is being transmitted. If the total number of signals to be transmitted is two, then these are the only codes required, if however further codes are needed, the next simplest is a series of alternations (0101...). Now three consecutive digits must be examined to distinguish between the steady-state transmission of a code and a change in signal condition.

Further comma-free codes can be devised if more than three signals are to be transmitted but at the expense of examining more consecutive digits to identify the signaling

condition. Three codes used in association with three signaling channels per speech channel appear to satisfy all signaling needs.

4.2 Signaling Terminations

Many switching systems have been devised and are in use in all parts of the world. Each of these switching systems, whether in-connected by physical conductors or by transmission systems, requires a signaling termination to interface with the transmission path. Although it would appear that the variation in signaling terminations is at least as great as the switching systems which they interconnect a survey of the most commonly encountered systems may reveal some commonality which, whilst acknowledging a diversity of peripheral operations, could enable some standardization in basic functions.

Although many terminations will be needed if pulse-code modulation is used for all suitable diverse applications, present indications are that three basic arrangements appear to satisfy 87 percent of British Post Office requirements. These are:

- conventional E and M signaling termination. This termination is used whenever it is necessary to interwork with an existing electro-mechanical relay set. A variant is to provide three E and M circuits per termination so as to gain access to all three signaling channels,
- outgoing signaling termination, shown functionally in Figure 5. This is used in place of an electromechanical relay set and provides direct connection to the appropriate selector level. The most common variant is to equip the unit for single-fee metering, and
- incoming signaling termination as shown functionally in Figure 6. This is used in association with the outgoing set and again provides direct connection to the appropriate selector level. The most common variant in this case is to provide forward-hold facilities.

These signaling terminations convert the switching system signaling conditions into comma-free codes, and in the case of the British Post Office system, frames f_1 and f_3 are combined to provide serial transmission. The precise relationship between these conditions, the comma-free codes, and the super-frame structure is shown in Figure 7. It should be noted that no correlation between the "called-party-answer" code and frames f_1 and f_3 is implied.

5. Line Equipment

5.1 Environment

The design of line equipment is dependent upon the environment in which it is to be used. The system described has been specifically designed, although no restriction is thereby implied, for use on interexchange junction cables. This class of connection is provided mainly by 0.635-millimeter (0.025-inch), 0.9-millimeter (0.036-inch) and 1.27-millimeter (0.51-inch) cable, typically loaded by 88-millihenry coils installed at 1.83-kilometer (2000-yard) intervals. Installation practice may require cables to be buried directly in the ground or run

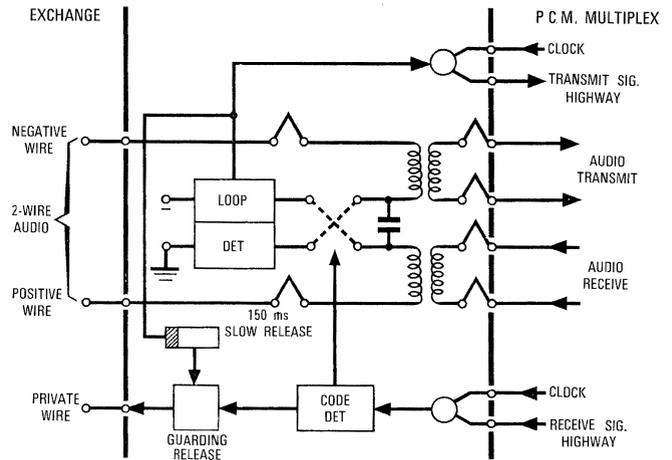


Figure 5 - Outgoing signaling termination.

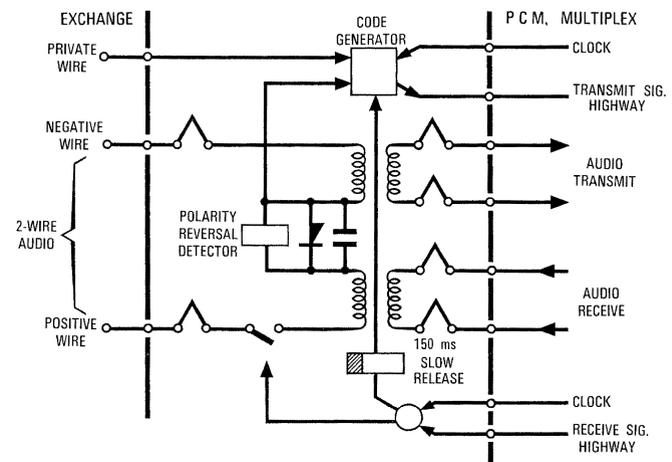


Figure 6 - Incoming signaling termination.

SUPER FRAME		S	S ₁	S ₂	S ₃	S ₄	S _n
		f ₁ f ₂ f ₃ f ₄	f ₁ f ₂ f ₃ f ₄	f ₁ f ₂ f ₃ f ₄	f ₁ f ₂ f ₃ f ₄	f ₁ f ₂ f ₃ f ₄	f ₁ f ₂ f ₃ f ₄
FORWARD SIGNALS	IDLE	0 0	0 0	0 0	0 0	0 0	0 0
	SEIZED	1 1	1 1	1 1	1 1	1 1	1 1
REVERSE SIGNALS	FREE	1 1	1 1	1 1	1 1	1 1	1 1
	BUSY	0 0	0 0	0 0	0 0	0 0	0 0
	CALLED PARTY ANS.	1 0	1 0	1 0	1 0	1 0	1 0

FRAME f₂ UNUSED
 FRAME f₄ USED TO TRANSMIT SYNCHRONISING PATTERN

Figure 7 - Comma-free signaling code allocation.

in cable ducts. Similarly, loading-coil boxes may be buried or housed in a manhole or foot-way box. Cable type and jointing practice will vary from territory to territory; commonly encountered types are star-quad concentrically layed cable, pair-type cable also concentrically layed, and pair-type unit cable. Each of these cable types and jointing practices, although designed to provide adequate crosstalk margins at audio frequencies, will give widely variable results when measured at the frequencies used for pulse-code-modulation transmission such as 768 kilohertz at the half-bit rate.

Identification of this problem has led to the adoption of two modes of operation:

- single-cable working in which go and return paths are in the same cable, and
- double-cable working in which go and return paths are in separate cables.

The limitation in the former mode is generally that of near-end crosstalk (NEXT), and in the latter case of far-end crosstalk (FEXT). Clearly both values are intimately associated with the specific cable type, and also with the jointing policy of the Administration, not only at the time of installation, but also during cable rework over the years following cable installation. In general, single-cable working will be used for installations of modest size, whereas double-cable working will be used for high-density routes. The design and construction of line equipment for single-cable working allows for conversion to double-cable working should the need arise.

5.2 Terminal Repeater

The terminal repeater is essentially the interconnection between the multiplex equipment and the repeated line. To ensure flexibility in patching, the interface between the terminal repeater and the multiplex equipment has been standardised. Thus the transfer of information in both directions is by means of an alternate-mark-inversion pulse train for which the main parameters are specified and closely maintained. These are:

- pulse amplitude 2.37 volts \pm 10 percent,
- pulse width 326 nanoseconds \pm 30 nanoseconds,
- pulse overshoot less than 10 percent,
- interface impedance 75 ohms, return loss 20 decibels minimum.

In the transmit direction the pulse train is launched on the line via a balance-to-unbalance transformer providing voltage transformation so that the transmitted pulse has a nominal amplitude of 3 volts. Because of the attenuation of the preceding section of line it is necessary in the receive direction to use a regenerative amplifier. Except for minor variations, this amplifier is identical with one half of a dependent repeater.

Power for the dependent repeaters is supplied from the terminal repeater equipment, and is fed over the phantom of the four-wire circuit. A constant-current balanced direct-current supply having a maximum value of 50 milliamperes is used. The power-feeding-terminal voltage depends on the number of repeater sections being supplied, but is limited to a maximum of 150 volts between conductors and 75 volts from either conductor to earth.

5.3 Dependent Repeaters

These are designed to be located in manholes or foot-way boxes in place of loading coils. They are mounted in a robust gray-cast-iron housing fitted with a water-tight lid. The housing is polyester-resin impregnated, externally zinc sprayed and epoxy-bitumen painted. Hardware associated with the repeater, and also mounted in the housing, provides for engineering-order-wire access, remote-repeater supervisory, [4, 5] and gas-pressure alarm.

Figure 8 is the block diagram of the dependent repeater which is conventional in concept and incorporates in recognisable entities the main functions common to all types of repeater. Although the line signal is the pseudo-ternary alternate-mark-inversion pulse train the repeater has been designed as a true ternary repeater so that the pos-

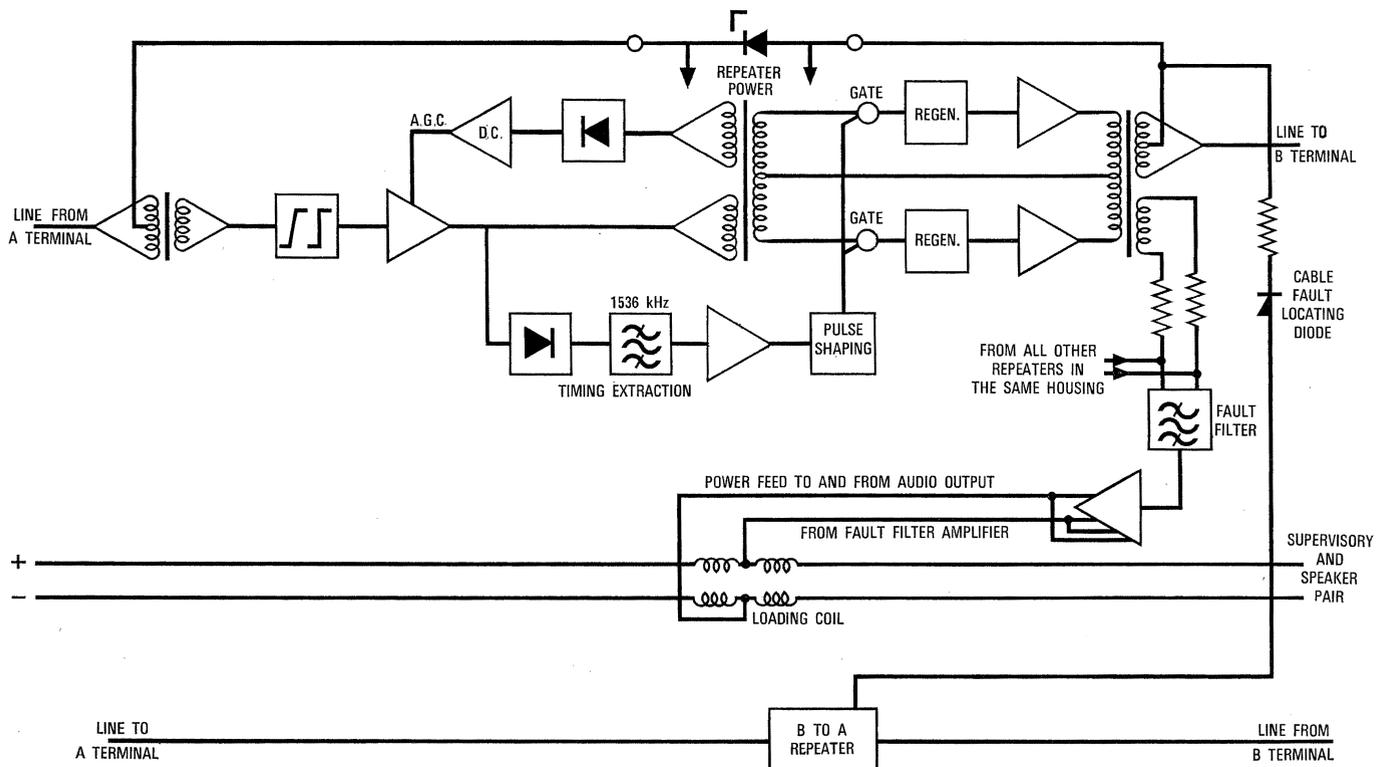


Figure 8 - Dependent repeater.

sibility of increasing the line channel capacity by using a genuine low-disparity ternary signal is preserved.

The design of the timing filter is always a compromise between obtaining a sufficiently narrow pass band, so as to ensure a reasonably unmodulated sine wave from an input signal with virtually random modulation, and controlling the timing-wave phase shift resulting from frequency drift of oscillators or filters. The practical choice has been to use two tuned-circuits connected in tandem providing an overall effective Q of about 100.

Regeneration is provided by two symmetrical blocking-oscillators. Turn-on is controlled by coincidence between the signal and timing wave, and turn-off by the timing wave. Thus individual pulses are standardized with regard to amplitude, shape, and duration, and occupy discrete positions on a regular time scale. Final connection to line is via a pair of buffer amplifiers used to prevent reflections from the line affecting the timing of the regenerative stage.

All 400 repeaters tested to date were capable of rejecting over a range of input levels an interfering signal of 767 kilohertz whose peak-to-peak amplitude was 10 decibels below the signal.

Although the repeater is supplied with a constant current of 50 milliamperes (actually 49 milliamperes ± 1 milliamperes), it will work without degradation with currents down to 42 milliamperes. The voltage dropped by each bidirectional repeater is nominally 11.2 volts, which is sufficiently low to enable a power-feeding section in excess of 30 kilometers (33 000 yards) to be achieved for most gauges of cable in series and with the restrictions defined above.

6. Conclusions

In conclusion a summary of system performance is given. It is necessarily abridged and contains mainly those parameters peculiar to the technique of pulse-code-modulation.

Table 2 – A pulse-code-modulation system performance summary

	Min.	Typ.	Max.
(i) Four-wire to four-wire frequency response: Loss relative to 800 kHz			
300 to 600 Hz	0	0.1	0.2 dB
600 to 2.4 kHz	0	0.1	0.3 dB
2.4 to 3 kHz	0.1	0.3	0.7 dB
3 to 3.4 kHz	–	–	1.6 dB
(ii) Four-wire loss:			
800 Hz & 1.6 kHz	28	–	31 dB
300 Hz & 3.4 kHz	22	–	27 dB

(iii) Quiescent channel noise:	85	70	63	dBm0p
(iv) Quantization distortion margin:				
Energizing signal — gaussian-limited white noise				
– 4 dBm0	25	27	28	dB
– 15 dBm0	29	31	33	dB
– 37 dBm0	25	26.5	28	dB
(v) Interchannel crosstalk:				
Energizing signal level + 2 dBm0	90	75	60	dB
(vi) Intermodulation product margins:				
$f_A = 1300$ Hz, $f_B = 800$ Hz				
2 A \pm B margin at signal levels				
– 6 dBm0	40	45	55	dB
– 15 dBm0	45	45	55	dB
– 40 dBm0	33	38	42	dB
A \pm B margin at signal levels				
– 6 dBm0	33	35	50	dB
– 15 dBm0	35	37	50	dB
– 40 dBm0	35	38	50	dB
(vii) Channel linearity:				
Tracking error over signal range + 2 dBm0 – 44 dBm0				– ± 0.3 ± 0.5 dB

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Mastergroup and Supermastergroup Carrier Multiplex Equipment

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1. Introduction

Telephone carrier systems of 2700-channel capacity are presently being used in increasing quantities on the main arteries that link important traffic centers. The trend exists both on a nation-wide basis and for international connections whether cable or radio links. The higher-order modulation stages of the terminal multiplex system may comprise 300-channel mastergroups and 900-channel supermastergroups: this is in accordance with the frequency translating scheme No 1 recommended by the International Telegraph and Telephone Consultative Committee (CCITT) for 2700-channel systems.

This article describes the design of mastergroup and supermastergroup equipments for a 2700-channel system constructed in conformity with International Standard Equipment Practice (ISEP), which is standard for ITT telecommunication systems. The associated lower-order carrier multiplex with its channeling, group and supergroup translation equipments was described earlier [1, 2].

The equipment specification, which includes the relevant CCITT recommendations, also took into account the requirements of various telephone administrations and operating companies, so as to ensure maximum utilization by a large majority of customers.

2. System Arrangement

The mastergroup translating equipment modulates 3 basic mastergroups (300 channels in the range 812 to 2044 kilohertz) into the basic supermastergroup frequency range 8516 to 12 388 kilohertz. The frequency allocation is shown in Figure 1. The basic mastergroup is obtained in the supergroup translating equipment as a combination of supergroups 4 to 8.

In the 1800-channel system, the supermastergroup translating equipment modulates 2 basic supermastergroups into the frequency range 316 to 8204 kilohertz. For a 2700-channel system, this is further combined with a third basic supermastergroup to produce the line frequencies 316 to 12 388 kilohertz, as shown in Figure 2. Other alternative solutions are possible as, for example, 6-megahertz systems with mastergroups 1 to 4, or 12-megahertz systems in which supermastergroup 1 is replaced by supergroups 2 to 16 (312 to 4028 kilohertz).

The carrier frequencies are derived from a 2480-kilohertz master oscillator with a stability better than 5 parts in 10^8 , as recommended by the CCITT. The carrier generation also supplies frequency comparison pilots and reference pilots for mastergroups and supermastergroups.

Through-mastergroup filter equipment is used for through-routing of mastergroups without further demodulation other than to the basic mastergroup. Through-connection of supermastergroups can be performed with no need for an extra through-filter.

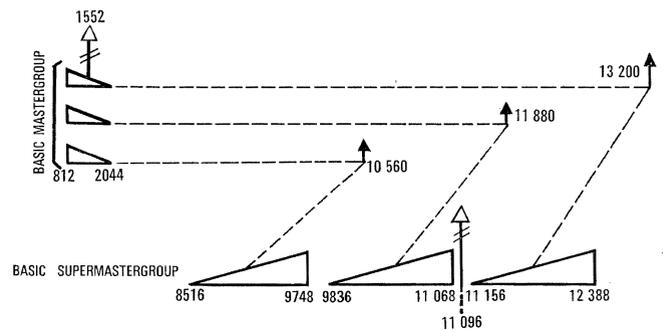


Figure 1 - Mastergroup translation - frequency allocation in kilohertz.

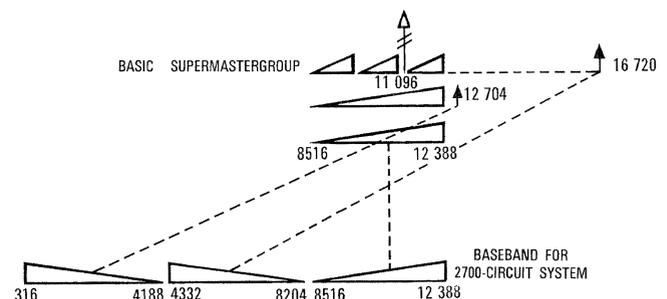


Figure 2 - Supermastergroup translation - frequency allocation in kilohertz.

Automatic gain control or monitoring equipment for mastergroups and supermastergroups is integrated in the respective translating equipments.

3. Electrical Design

Since the CCITT has so far not agreed on specific performance recommendations for supermastergroups, it was decided to interpret the relevant specifications for mastergroups as applicable to the combined arrangement of mastergroup and supermastergroup translations. This gave rise to certain stringent requirements which we met with ample margin.

In order to provide the high performance characteristics specified, the synthesis of the various networks was based on computer-aided design techniques. Special consideration was also given to the reliability aspect: this is reflected in the exclusive selection of modern components with proved reliability and in the system's basic planning scheme.

3.1 Mastergroup Translating Equipment

A functional diagram showing the frequency translation of 3 basic mastergroups in the basic supermastergroup band is given in Figure 3.

At the inputs and outputs of the equipment, equalizers are accommodated to allow for the attenuation of the station cabling in the basic mastergroup and the basic

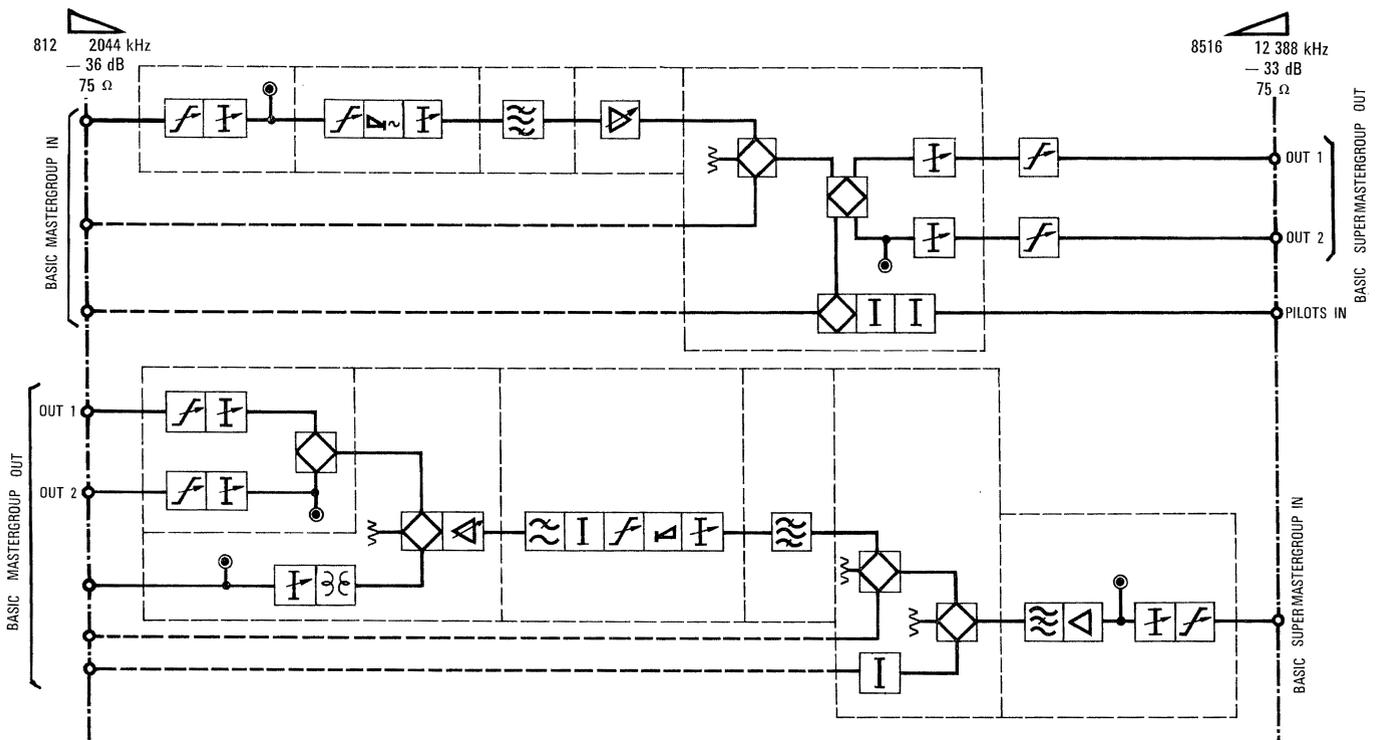


Figure 3 - Block schematic - mastergroup translation.

supermastergroup. Duplicated decoupled outputs at the send and receive side are available for measuring or patching purposes.

In order to equalize the frequency-attenuation characteristics to the tight limits required, equalizing networks are inserted in the receive and transmit directions.

The modulator and demodulator circuits are of the active type making use of planar silicon transistors. This has an advantageous effect on the low basic noise figure of the system, since the lowest relative level can easily be raised to about -40 decibels referred to the level at the 2-wire point of origin by the amplification of the modulating circuits. The amount of feedback applied yields an adequate intermodulation margin and high-stability behavior over a wide range of carrier and supply voltage changes. The modulation circuit requires low carrier input power, since an amplifier stage is inserted in the carrier feeding path; this amplifier also acts as a buffer for certain crosstalk components, reducing them to a negligible level.

The mastergroup band-pass filter selecting the appropriate lower-frequency sidebands is computer-designed on an insertion-loss basis. High-stability ferrite coils and silvered-mica capacitors are used to meet the requirements of temperature changes and long-term performance. A typical frequency-attenuation curve is shown in Figure 4.

The transmit amplifier for each mastergroup is a two-stage common-emitter feedback type with manual gain adjustment.

At the transmit and receive side, the 3 mastergroups are combined by a decoupling hybrid transformer: this arrangement allows for removal or insertion of master-

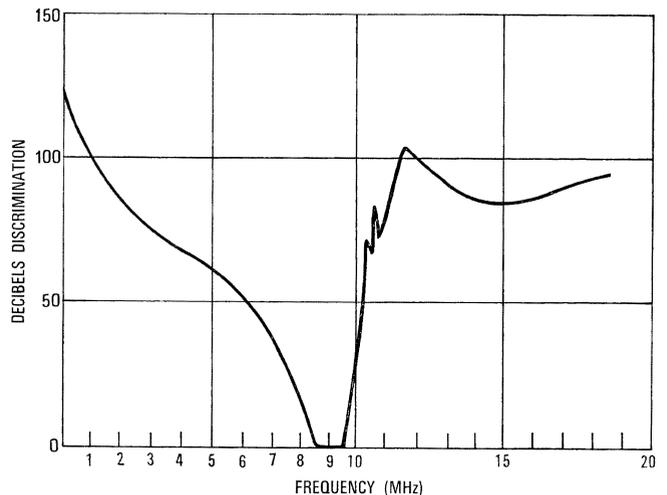


Figure 4 - Typical attenuation curve - mastergroup filter.

groups without interference with through traffic. The combining circuit at the transmit side also includes the injection point of the supermastergroup reference pilot and two decoupled outputs, as shown in Figure 3.

Optionally, a 11096-kilohertz crystal band-stop filter may be inserted in the receive path to eliminate the supermastergroup reference pilot. This is required in cases of through mastergroup connection to prevent possible interference among supermastergroup reference pilots of different links.

The schematic of the receive output-amplifier is similar to that of the transmit amplifier. This amplifier provides manual or automatic gain regulation as required. In the latter case, the heating current of a thermistor inserted in the feedback path is controlled by a separate master-

group reference pilot receiver. The output circuit consists of two identical decoupled outputs and a pick-off point for the mastergroup reference pilot.

3.2 Supermastergroup Translating Equipment

A functional diagram showing the frequency translation of 3 basic supermastergroups into a 2700-channel band is shown in Figure 5.

In this design, particular attention was given to covering a wide range of line-frequency levels to meet the requirements of most likely applications. Many features are similar to those described in the mastergroup translating equipment.

Station cabling equalizers are accommodated at the equipment inputs and outputs. Transmit and receive outputs are duplicated so providing decoupled maintenance points. These facilities allow for measurements without interference with the main transmission path. Additional equalizing is incorporated in the receive and transmit paths to meet the stringent frequency-attenuation characteristics.

Active modulating circuits of the same type as those for mastergroups provide low basic- and intermodulation-noise performance and a high order of gain stability. A buffer amplifier in the carrier feeding path eliminates crosstalk due to coupling via the carrier supply source

and reduces the amount of carrier input power needed.

Supermastergroups 1 and 2 are modulated, and the lower sidebands are selected and combined with supermastergroup 3 allocated in the basic supermastergroup band to form the line spectrum 316 to 12 388 kilohertz for 2700 channels. By including crystal network sections in the supermastergroup band-pass filter, a steep frequency-attenuation curve was obtained in the stop band. In this design, the attenuation characteristics achieved are suitable for through-routing of supermastergroups without need for extra filtering. This cost- and space-saving feature is particularly useful in system branching points, where the majority of the supermastergroups are transferred from one link to another without frequency translation equipment. Figure 6 shows a typical discrimination curve for supermastergroup 2.

The output power at the transmit side is provided by a two-stage common-emitter amplifier followed by a push-pull output. Manual level adjustment is available for maintenance. The maximum transmission capacity of any active element in the system is limited to 900 channels since each supermastergroup has only one transmit or receive amplifier.

The 3 supermastergroups are combined by means of hybrid transformers. Coupled with the mechanical flexi-

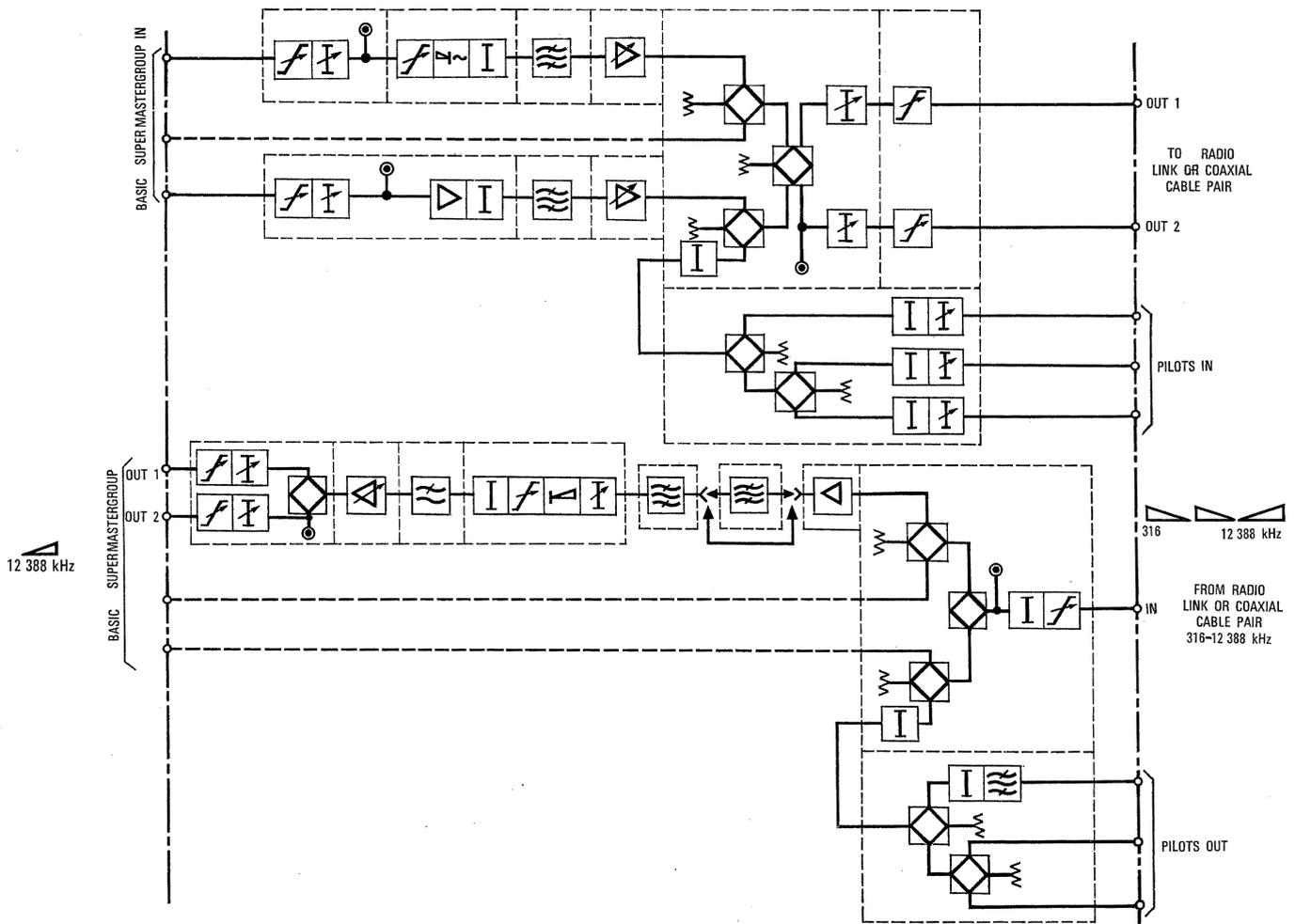


Figure 5 - Block schematic - supermastergroup translation.

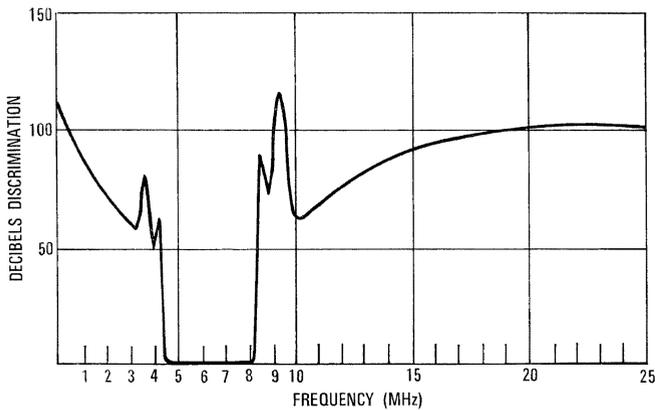


Figure 6 - Typical attenuation curve - supermastergroup filter.

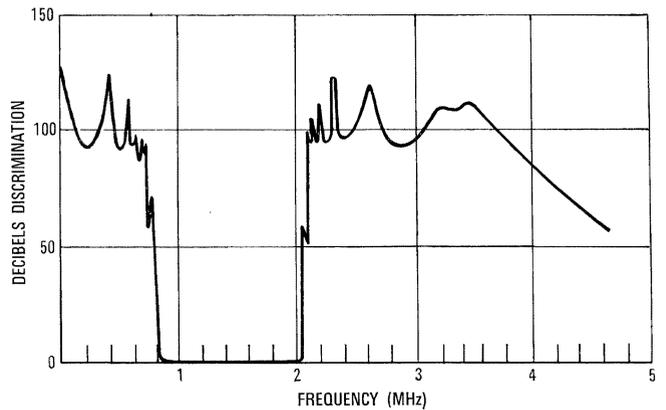


Figure 7 - Typical attenuation curve, through-mastergroup filter.

bility of the design, this scheme allows for ready extension of partially equipped systems. At the transmit side three line or measuring pilots may be injected simultaneously. Similarly, pilot extraction facilities are inbuilt at the combining receive-side including pick-off stop filters for the frequency comparison pilot.

The buffer amplifiers at the receive input-side are necessary to meet the return-loss requirements and to boost the level at the band-pass filter inputs, thus considerably reducing the basic noise contribution.

The four-stage receive output-amplifier incorporates manual or automatic gain control in association with a pilot regulator. Two identical outputs and a pick-off point for the 11 096-kilohertz supermastergroup reference pilot are provided.

In certain applications the lower range of the line frequency spectrum may be occupied by supergroups 2 to 16 with the frequency band 312 to 4028 kilohertz instead of the supermastergroup -1 band, 316 to 4188 kilohertz. In this case, the circuitry for supermastergroup -1 translating is replaced by coupling filters and networks to combine supergroups 2 to 16 with supermastergroups 2 and 3.

3.3 Through-Connecting Filters

It has been pointed out earlier that, for through-routing of supermastergroups, the band-pass filters used for frequency translation could be used without need for additional filtering; therefore, only mastergroup transfer will be discussed here.

The through-mastergroup filter consists of high-pass and low-pass filters connected in tandem with equalizing of the higher and lower limits of the pass band. The filters have been designed according to the insertion-loss method. Coil and capacitor filter sections are employed throughout, since the high-stability coils have sufficient quality factor to match the steep attenuation curve specified at the limits of the pass band. The pass-band attenuation is low enough to permit through connection without additional amplification. A typical characteristic is shown in Figure 7.

3.4 Automatic Gain Regulation

The automatic-gain-regulating system for mastergroups or supermastergroups is part of the translating equipment. It comprises the regulated demodulator receive-amplifier, and a pilot receiver which controls the gain of this amplifier by feeding its output current through the heater of a thermistor incorporated in the feedback path. By this method the output level of the demodulator amplifier is continuously regulated.

The mastergroup pilot receiver operates from the 1552-kilohertz reference pilot. The receiver unit consists of a pilot pick-off crystal filter feeding an alternating-current amplifier followed by a rectifier and a direct-current amplifying circuit in which the rectified pilot signal is compared with a reference voltage. The voltage difference is further amplified and supplies the thermistor heater current. Particular attention was given to stabilizing the reference voltage with respect to temperature and direct-current supply voltage changes. The gain control is slow acting and does not respond to short spurious level variations of the pilot signal, or to short-duration noise. Each automatic-gain-control unit incorporates an alarm relay that operates if the pilot level at the amplifier output deviates beyond certain limits.

The input stage of the supermastergroup regulator comprises a modulation circuit which translates the reference pilot frequency 11 096 kilohertz into 1552 kilohertz by means of a 12 648-kilohertz carrier. The 1552-kilohertz output is then passed to a narrow-band-pass crystal filter and a regulator circuit identical to that used for mastergroup regulation.

3.5 Frequency-Generating Equipment

In general, the frequency generating equipment may be subdivided into two parts, firstly, the master oscillator and basic frequency-generating equipment including certain pilots and, secondly, the various carrier-generating equipments which feed the corresponding translating equipments.

In the design of the first, it was considered important to limit the number of fundamental frequencies to a strict

minimum and to make use of only those frequencies that already existed in previous arrangements. The carrier generating equipments have been closely integrated with their associated translation equipments, thereby forming self-contained units that operate from the supply of one or two basic frequencies.

The application of these electrical and mechanical design concepts ensures compatibility with almost any existing system.

3.5.1 Master Oscillator

The master oscillator generates a frequency of 2480 kilohertz. It uses an AT-cut crystal vibrating on its fifth mechanical overtone and has a stability better than 3 parts in 10^8 per month. (For further details, see [1, 2]).

3.5.2 Generation of Basic Frequencies

The basic frequencies 124 and 440 kilohertz are derived from the 2480-kilohertz master oscillator frequency. Only these two fundamental frequencies are needed to feed the mastergroup and supermastergroup carrier supplies.

As described elsewhere [1, 2] other basic frequencies, 4 kilohertz and 12 kilohertz are produced from 124 kilohertz: they are used to derive the channel and group carrier supplies respectively, whereas the 124-kilohertz signal also feeds the supergroup carrier supply. The frequency comparison pilots 60, 300 and 308 kilohertz are generated by multiplications of 4-kilohertz harmonics.

Subsequent division of 2480 kilohertz by 2, 5, and 2 produces 124 kilohertz. Each division is performed by means of a regenerative modulator. This type of circuit is characterized by a self-starting operation and by a high degree of stabilization, ensuring proper division. It consists of a dual transistor modulator with a positive-feedback loop constructed by means of a suitably inserted amplifier. The wanted sideband is selected by an appropriate narrow-band-pass filter.

For the generation of 440 kilohertz two auxiliary frequencies are first generated, 80 kilohertz by dividing 2480 kilohertz by 31, and 400 kilohertz as the 5th harmonic of 80 kilohertz. The basic frequency 440 kilohertz is selected as the upper sideband after the modulation of 400 kilohertz by 40 kilohertz, which is derived by the division of 80 kilohertz. The regenerative modulator described in the previous section is used throughout as a basic circuit element.

3.6 Generation of Mastergroup and Supermastergroup Carriers

The mastergroups -7, -8, and -9 carriers, 10 560, 11 880 and 13 200 kilohertz, are respectively the 24th, 27th and 30th harmonic of 440 kilohertz. They are filtered out after several stages of multiplication, as shown in the functional diagram in Figure 8.

Each multiplier comprises a transistor switch feeding a selective load. The generation scheme, consisting of successive multiplication stages, makes the filter require-

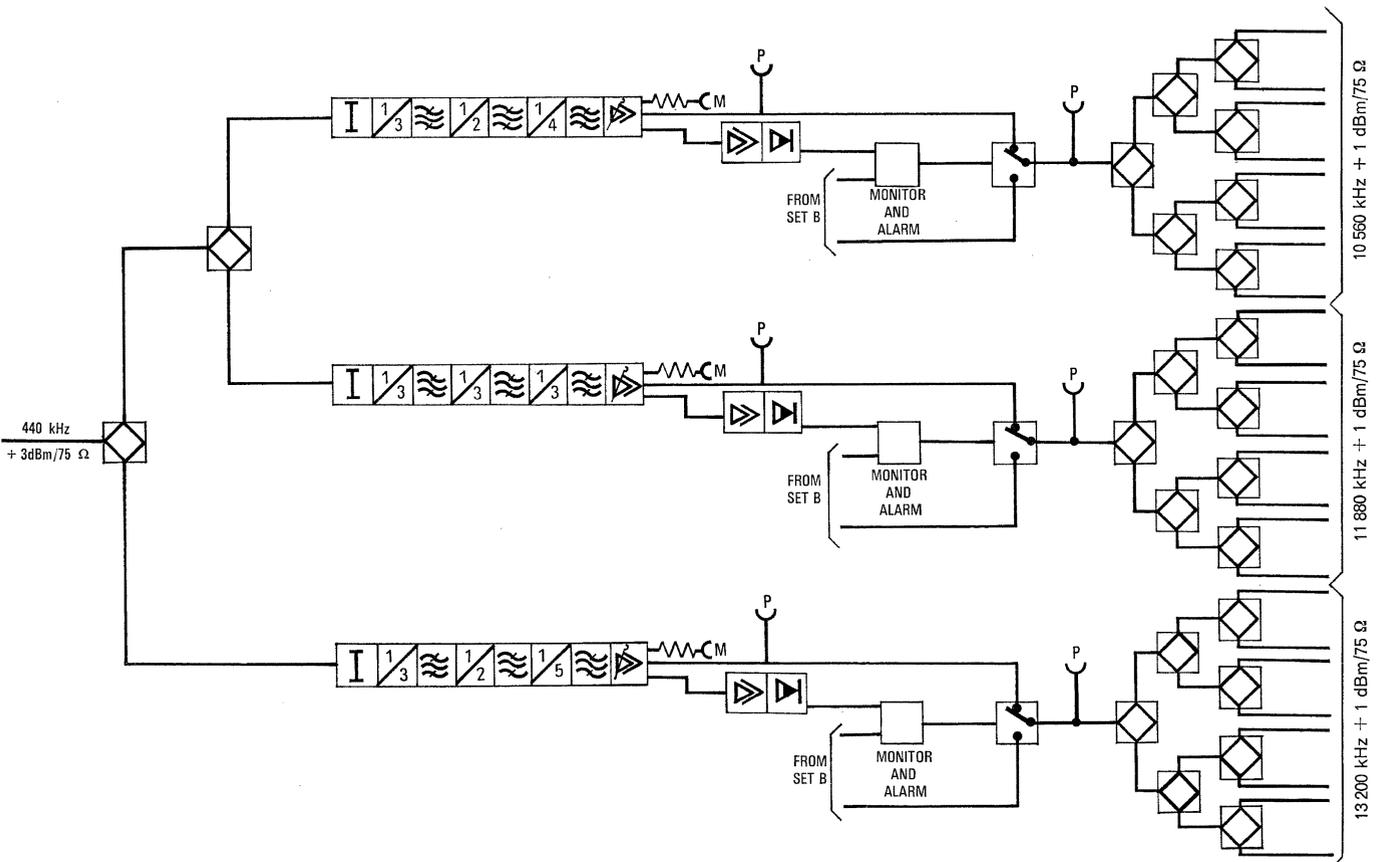


Figure 8 - Mastergroup carrier generation.

ments substantially easier. The carrier output level is practically constant over a wide range of level variations of the 440-kilohertz input signals.

A two-stage feedback amplifier feeds the carrier distribution circuits. The output power is rather low owing to the particular design of the modulating circuits of the translation equipment. To prevent interference between distribution outlets, perhaps due to an accidental short-circuit, effective decoupling is achieved by means of hybrid transformers.

Two supermastergroup carriers are required, 12 704 kilohertz for supermastergroup 1 and 16 720 kilohertz for supermastergroup 2. Their generation is shown in the functional diagram in Figure 9.

The basic design of the multiplier circuits used is identical with that of the mastergroup generation; the same applies to the output amplifier and distribution arrangement.

The supermastergroup - 1 carrier, 12 704 kilohertz is derived from the basic frequencies 124 kilohertz and 440 kilohertz. The carrier 12 704 kilohertz is selected as the upper sideband in the output modulator by mixing 10 560 with 2144 kilohertz.

The frequency 10 560 kilohertz is produced in exactly the same way as the mastergroup - 7 carrier generation, whereas 2144 kilohertz is obtained by frequency addition of the 4th harmonic of 124 kilohertz and the 6th of 440 kilohertz.

The supermastergroup carrier 2 is generated as the 38th harmonic frequency of 440 kilohertz. To ease the filtering requirements, several succeeding modulator and multiplier stages are terminated by simple filters.

3.7 Mastergroup and Supermastergroup Reference Pilot

Figure 10 shows the functional diagram of the 1552-kilohertz generation. The 1552-kilohertz mastergroup reference pilot frequency is filtered out as the lower sideband in a modulator mixing the supergroup - 6 carrier, 1860 kilohertz, with 308 kilohertz obtained at the output

of a harmonic generator driven by 44 kilohertz. The latter frequency is derived from 440 kilohertz in a regenerative modulator dividing by 10. The CCITT recommendation for high-level stability of the pilot generation permits a maximum level change of ± 0.3 decibel during a maintenance period of one month: this is achieved over a wide range of variations of power-supply voltage, temperature, and load conditions by means of an automatic-gain-control circuit which consists of a differential amplifier driven by the rectified pilot output voltage at one input and a stabilized reference voltage at the other. As in the case of carrier amplifiers, a direct-current voltage output is used to monitor the pilot level.

The supermastergroup reference pilot frequency 11 096 kilohertz is produced by modulating the mastergroup reference pilot frequency with 12 648 kilohertz, which is the 6th harmonic of the supergroup - 7 carrier, 2108 kilohertz. This frequency 12 648 kilohertz is also used as a carrier in the supermastergroup automatic-gain regulating equipment. The pilot output level is highly stabilized in a way similar to that of the mastergroup reference pilot.

3.8 Automatic Changeover

All frequency generation equipments are normally duplicated. However, only the carrier and pilot generation equipments are associated with automatic changeover

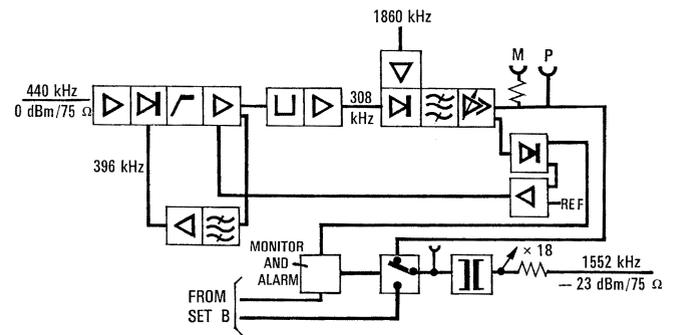


Figure 10 - Mastergroup reference pilot generation.

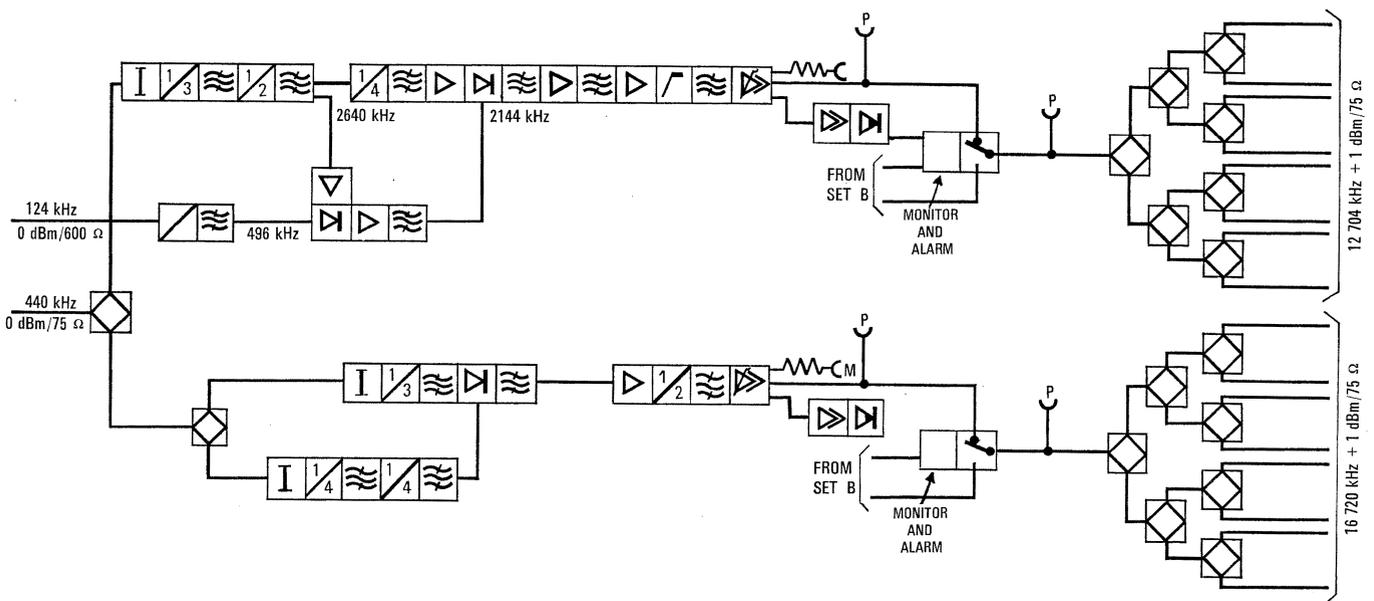


Figure 9 - Supermastergroup carrier generation.

units, one for each frequency. Two sets of distribution facilities are needed for duplicated frequencies without changeover.

When the output level of the working set falls below a preset limit, the load is automatically switched to the standby equipment. Both working and standby sets are continuously monitored.

Duplication of equipments makes sense only if the common part is of a higher order of reliability. For this reason, the switching device was considered the critical element: investigations and tests carried out led to the adoption of a sealed reed relay of proved reliability with a contact-transit time well below 1 millisecond. In this way, adequate decoupling between working and standby sets is also achieved.

Non-urgent and urgent monitoring alarms are operated respectively if one set or two sets are out of order.

4. Equipment Design

The design of this mastergroup and supermastergroup equipment completes the existing ISEP versions of channeling, group, and supergroup equipments. Thus a whole range of transistorized equipments has been designed all in the same standard equipment practice. This range now covers a variety of carrier multiplex systems up to 2700 channels.

The technical data given are valid for operating conditions applicable to all ISEP equipments. The ambient temperature range extends from + 10 to + 40 degrees Celsius, with humidity up to 95 percent, but satisfactory operation with most requirements is obtained over a temperature range from - 20 to + 55 degrees Celsius.

Available standard ISEP components are used where applicable. However, owing to the extended frequency range a need for new components has arisen. Suitable planar silicon transistors have been selected for high-frequency applications; the same reasoning applies for the ferrite-core material and certain types of ceramic capacitors. The choice of the component types was mainly dictated by considerations of reliability.

4.1 Mechanical Construction

4.1.1 General

The detailed characteristics of the ISEP version suitable for transmission equipment (ISEP-T) have already been described [1]. Therefore, in this section, emphasis will be put on the high-frequency applications of ISEP-T. The general principles may be summarized as follows.

- The transmission equipment is mounted in bays, which are built up by means of subracks. Subracks are factory-wired assemblies constituting functional units and accommodating the plug-in units that carry the electrical circuitry. The plug-in units contain the components or circuit elements, which are interconnected by printed wiring to form a complete circuit. Maintenance test points are mounted on the front side of the boards.
- The mechanical design provides for wiring the bays on site without the need of a factory-made cable-

form. The application of this principle strictly limits the amount of equipment to be installed to those system parts required by the customer for his initial needs.

At a later date extensions can easily be carried out.

- To simplify installation work, subracks on the same bay are interconnected by means of plug-ended cables; the station wiring between bays may be of the plug-in or of the soldered type, depending on the equipment. This wiring is located in the bay uprights and is connected to the subracks via a connecting field accessible from the front.
- The bays are of the front-access type and are suitable for back-to-back or wall mounting. Their dimensions are 520-millimeter (20-inch) width, 225-millimeter (9-inch) depth, in accordance with CCITT recommendations. The height may vary depending on equipment applications.

4.1.2 Plug-in Unit

The plug-in unit may consist of one or more printed-wiring boards. The width dimension is governed by the size of the electrical components and is standardized in multiples of 5.08 millimeters (0.2 inch), whereas the height and depth are the same for all plug-in units, 100 millimeters (3.9 inches) and 160 millimeters (6.2 inches), respectively. Measuring points and adjustment facilities are accessible on the front side.

The plug-in units for mastergroup and supermastergroup equipments have been mechanically designed to suit high-frequency applications. Reliability of electrical contacts, electrical screening, and mechanical stability were considered to be the main features required for adequate performance.

To attain the latter objective, the printed-wiring board is mounted in a stiff diecast aluminium frame, which carries the coaxial plug-in connectors and maintenance test points at the rear and front sides respectively.

Two steel plates stamped into a suitable shape click into grooves at the front of the frame and are pressed to each frame side by two clamps mounted on the rear of the frame. These clamps are fixed on pins by means of wire springs. The same pins are also used for guiding the unit to ensure correct plug-in operation for the coaxial connectors. Elastic contact material on the inside of the plates achieves a low contact resistance with the frame metal. Very efficient electrical screening is thus obtained which results in an excellent crosstalk ratio between adjacent units. This construction also provides easy access to the printed wiring or component sides of the boards. Only one height of 30.5 millimeters (1.2 inches) is standardized for all plug-in units of this type.

Gold-plated coaxial plugs and jacks with proved performance ensure a high standard of reliability of the plug-in connections. Figure 11 shows a typical plug-in unit and its screening plate.

4.1.3 Subracks

The subracks are of the same type as that described previously [1], except for the width of 446 millimeters (17.5 inches), which fits into a 520-millimeter (20-inch) rack.

Two sizes of subracks are equipped in the mastergroup and supermastergroup multiplex equipment, not taking into account the incorporated connecting field: 119.4 by 446 by 194 millimeters (4.8 by 17.5 by 7.6 inches) and 342.9 by 446 by 194 millimeters (13.5 by 17.5 by 7.6 inches), respectively, for a 1-shelf and a 3-shelf subrack. All outgoing and incoming cabling to the subrack passes via a connecting field which is mounted as an horizontal terminal strip below the lower shelf. The heights of the connecting fields are multiples of 20.3 millimeters (0.8 inch). All connections to this field are of the plug-in type, bay and station cabling being terminated by coaxial or normal ISEP connectors. This means a substantial simplification and cost reduction of the installation work, since factory-terminated cables achieve plug-in interconnections. Provision for breaking facilities with easy access is another advantage derived from this construction method.

Figure 12 illustrates the bay mounting of subracks by means of plug-in connections.

Owing to the flexibility in equipping ISEP racks, a great variety of bays is possible by combining different types of subracks. In a following section, several combinations of bays for mastergroup and supermastergroup equipments will be discussed.

4.2 Mastergroup Modem Subrack

4.2.1 Description

One complete basic supermastergroup of mastergroup translating equipment is contained within a standard 3-shelf subrack. Each shelf accommodates the plug-in units for translation and automatic gain control of a mastergroup.

The translating equipment of the mastergroup transmit direction is mounted on three boards, a modulator unit, a translating-filter unit, which is identical to that of the receive side, and an amplifier unit, one per mastergroup. The common path consists of the hybrid-coil-type combining unit, including the supermastergroup reference-pilot injection part and duplicated output facilities.

The receive-side translating equipment also occupies three boards per mastergroup, the translating filter, the demodulator, and the receive-amplifier units; the two latter units are interchangeable among the different mastergroups. The basic supermastergroup path comprises

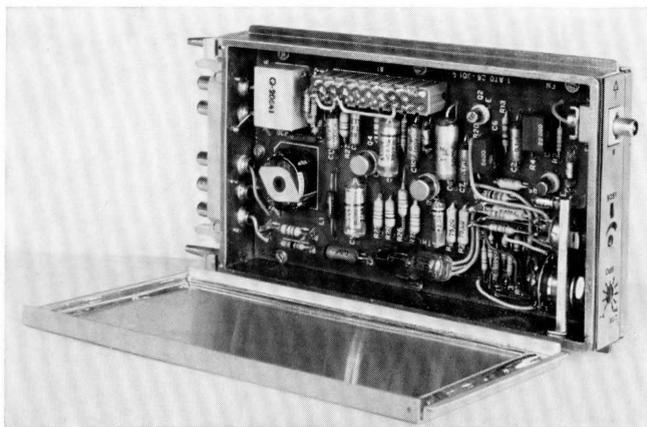


Figure 11 - Plug-in unit and screening plate.

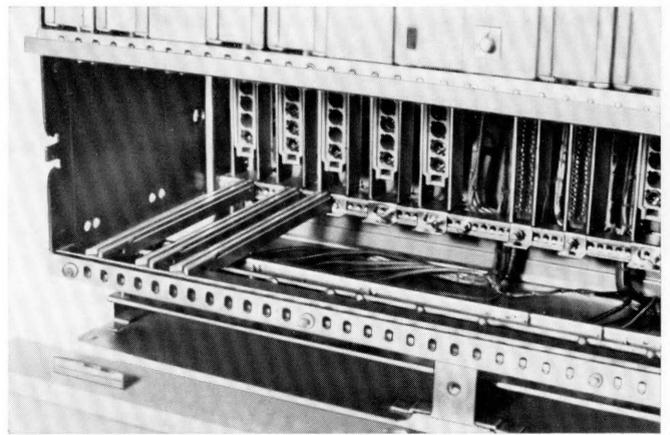


Figure 12 - Detail of bay showing subrack mounting and the plug-in type connection strips.

the receive amplifier, the supermastergroup reference-pilot stop filter, and the mastergroup combining unit. Station-cabling equalizers are mounted on separate units. Three additional plug-in units accommodating the automatic-gain-control equipment may be associated with each mastergroup. Because of the mechanical and electrical flexibility, units for unneeded mastergroups may be omitted. The functional diagram of Figure 3 shows the block schematic of the units. Figure 13 shows a photograph of a complete subrack.

4.2.2 Control and Test Facilities

Facilities are provided at the front of the plug-in units for all routine measurements and adjustments. Coaxial test points allow for access at the basic mastergroup and at the basic supermastergroup inputs, whereas at the outputs duplication with coaxial measuring points is incor-



Figure 13 - Mastergroup translating subrack.

Mastergroup and Supermastergroup Equipment

porated. Hybrid coils are used to prevent interference with traffic while measuring the output levels.

An adjustable resistor permits continuous control of the transmit output level of each mastergroup. According to the operation requirements, the receive output level may be automatically controlled in association with a pilot receiver, or this can be performed manually by means of a switch and a potentiometer for the coarse and fine adjustments respectively.

Coaxial access and breaking facilities for all inputs and outputs, for the pilot injection and pick-off points, and for the carrier feeding inputs are available at the connecting field of the subrack.

4.2.3 Technical Data

— Basic Mastergroup Levels and Impedances (812 to 2044 kilohertz)	
transmit input	minus 39 to minus 36 decibels referred to the level at the 2-wire point of origin, 75-ohms unbalanced
receive output	minus 23 to minus 20 decibels referred to the level at the 2-wire point of origin, 75-ohms unbalanced
— Basic Supermastergroup Levels and Impedances (8516 to 12 388 kilohertz)	
transmit output	minus 33 to minus 30 decibels referred to the level at the 2-wire point of origin, 75-ohms unbalanced
receive input	minus 25 to minus 28 decibels referred to the level at the 2-wire point of origin, 75-ohms unbalanced
— Carrier Supply per Frequency	
nominal level to each mastergroup (modulator or demodulator)	minus 2 to plus 1 decibels referred to 1 milliwatt, 75-ohms unbalanced
— Amplitude-Frequency Response	
each direction of transmission separately	the maximum spread of the attenuation distortion is 0.3 decibel per mastergroup referred to 1552 kilohertz; the maximum spread per supergroup is 0.25 decibel.
— Noise for Looped Equipment	
basic noise	minus 77 decibels relative 1 milliwatt referred to a point of zero relative level, psophometrically-weighted (20 picowatts psophometrically-weighted)
total noise when fully loaded with white noise at equivalent channel level as specified by the CCITT	minus 73 decibels relative 1 milliwatt referred to a point of zero relative level, psophometrically-weighted (50 picowatts psophometrically-weighted)
— Intelligible Crosstalk	
any single path between mastergroups, between transmit and receive directions, or between channels within a mastergroup, crosstalk ratio	90 decibels
— Suppression of upper sideband	
in transmit direction	80 decibels
— Carrier Leak	
per mastergroup	minus 40 decibels relative 1 milliwatt referred to a point of zero relative level
— Modulator Compression Ratio	
transmit or receive directions for carrier levels within plus or minus 3 decibels of nominal	10:1
— Automatic Gain Control	
pilot frequency and level	1552 kilohertz plus or minus 4 hertz at minus 20 decibels relative 1 milliwatt referred to a point of zero relative level
gain control range for a control ratio of at least 10:1	minimum plus or minus 4 decibels

gain stability with nominal pilot frequency, gain changes for a variation in ambient temperature of 10 degrees Celsius plus normal voltage variations. 0.2 decibel

— Power Consumption
per subrack fully equipped with 3 mastergroups, including automatic gain control equipment. 650 milliampères direct current at 20 volts

4.3 Supermastergroup Modem Subrack

4.3.1 Description

Similarly to the mastergroup equipment, all translating and automatic gain control equipment for supermastergroups 1, 2, and 3 forming the line spectrum, is mounted in a 3-shelf subrack. The block schematic of Figure 5 shows the functional units.

In the transmit direction, three boards are needed per supermastergroup: a modulator, a filter, and an output-amplifier unit, except for the supermastergroup - 1 path in which the modulator is replaced by an input amplifier. Two common units are required for combining the supermastergroups and the duplicated outputs and for the coupling and level-adjustment arrangements of three pilots to be injected.

The translating equipment for supermastergroups 1 and 2 at the receive side is distributed on 4 boards to include a line-frequency input-amplifier, a filter, a demodulator and the basic supermastergroup output-amplifier. Supermastergroup 3 is accommodated on three boards since no demodulation is required. Optionally a frequency-comparison-pilot stop filter may be added in the supermastergroup - 1 path. Combining circuits and pilot pick-off points are mounted on two units.

The supermastergroup - 1 shelf may be equipped with send and receive coupling-filters and amplifiers for supergroups 2 to 16, instead of the supermastergroup translating equipment. A total of 8 equalizer units is incorporated to compensate station cabling in the basic supermastergroup and in the line frequency range.

Built-in super mastergroup automatic gain regulation may be equipped on 3 plug-in units.

4.3.2 Control and Test Facilities

Coaxial test points on the front are provided at the inputs and at the outputs of the translating equipment. The latter are decoupled from the main transmission path by hybrid coils. Level-control facilities accessible from the front cater for continuous output adjustment of transmit and receive directions. For the receive direction automatic gain control may be built-in.

Since all the connections to the subrack, whether for bay or station cabling, are of the plug-in type, easy access and breaking facilities for all incoming and outgoing connections are an inherent feature of the subrack design.

4.3.3 Technical Data

— Basic Supermastergroup Levels and Impedances (8516 to 12 388 kilohertz)	
transmit input	minus 36 to minus 33 decibels referred to the level at the 2-wire point of origin, 75-ohms unbalanced
receive output	minus 25 to minus 22 decibels referred to the level at the 2-wire point of origin, 75-ohms unbalanced

— Line-frequency Levels and Impedances (316 to 12 388 kilohertz)	
transmit output	minus 45 to minus 30 decibels referred to the level at the 2-wire point of origin, 75-ohms unbalanced
receive input	minus 36 to minus 25 decibels referred to the level at the 2-wire point of origin, 75-ohms unbalanced
— Carrier Supply per Frequency	
nominal level to each supermastergroup (modulator or demodulator)	minus 2 to plus 1 decibels referred to 1 milliwatt, 75-ohms unbalanced
— Amplitude-Frequency Response	
each direction of transmission separately	The maximum spread of the attenuation distortion is 0.75 decibel per mastergroup referred to 11 096 kilohertz; the maximum spread per mastergroup is 0.5 decibel
— Noise per Looped Equipment	
basic noise	minus 75 decibels relative 1 milliwatt referred to a point of zero relative level, psophometrically-weighted (31.6 picowatts psophometrically-weighted)
total noise when fully loaded with white noise at equivalent channel level as specified by the CCITT	minus 73 decibels relative 1 milliwatt referred to a point of zero relative level psophometrically-weighted (50 picowatts psophometrically-weighted)
— Intelligible Crosstalk	
any single path between supermastergroups, between transmit and receive direction, or between channels within a mastergroup, crosstalk ratio	90 decibels
— Suppression of Upper Sideband	
in transmit direction	90 decibels
— Carrier Leak	
per supermastergroup	minus 50 decibels relative 1 milliwatt referred to a point of zero relative level
— Modulator Compression Ratio	
transmit or receive directions for carrier levels within plus or minus 3 decibels of nominal	10:1
— Automatic Gain Control	
pilot frequency and level	11 096 kilohertz plus or minus 12 hertz at minus 20 decibels relative 1 milliwatt referred to a point of zero relative level
gain control and gain stability	same as Section 4.2.3
— Power Consumption	
per subrack fully equipped with 3 supermastergroups, including automatic-gain-control equipment	780 milliamperes direct current at 20 volts

4.4 Through-Mastergroup Filter

4.4.1 Description

The mastergroup through-connecting equipment is mounted in a 1-shelf subrack. One subrack accommodates six 1-way through equipments.

A through-connection equipment consists of two plug-in units, the through-mastergroup filter, and the adjustable equalizer unit for compensation of the attenuation of station cabling. The outputs are duplicated by means of a decoupling hybrid transformer. A coaxial measuring point allows for maintenance tests without disturbing traffic.

The coaxial plug-in connecting field of the subrack provides facilities for breaking the circuit and making terminated measurements at the input and the output of the equipment.

4.4.2 Technical Data

— Levels and Impedances

input	minus 26 to minus 23 decibels referred to the level at the 2-wire point of origin, 75-ohms unbalanced
output	minus 36 to minus 33 decibels referred to the level at the 2-wire point of origin, 75-ohms unbalanced

— Amplitude-Frequency Response

the maximum attenuation distortion is 0.3 decibel referred to 1552 kilohertz in the frequency range 812 to 2044 kilohertz; the maximum spread per supergroup is 0.3 decibel

— Suppression of Crosstalk

mastergroup-to-mastergroup crosstalk ratio	80 decibels
measuring at single frequencies in a 12-megahertz system using a bandwidth of plus or minus 150 hertz	45 decibels

4.5 Frequency-Generating Equipment

4.5.1 General

The frequency-generating equipment for a complete 2700-channel system is subdivided into two distinct main parts: the master oscillator and generation of basic frequencies including certain pilots, and the various carrier supply equipments.

The first part is mounted on a central bay, and has the capacity for feeding up to eight 2700-channel systems. This bay is shown in Figure 14. The second part comprises the carrier supply equipments for channeling, group, supergroup, mastergroup and supermastergroup translations. They are electrically derived from the basic frequency generation and are decentralized on different bays, which also carry the corresponding translating equipments.

Figure 15 shows the functional layout and station distribution diagram of the frequency-generation equipments for up to 2700-channel systems.

4.5.2 Master Oscillator and Frequency-Generating Bay

This bay may accommodate a total of nine 1-shelf subracks, in addition to the duplicated power supply units, as shown in Figure 14. Each subrack contains all the plug-in boards for a particular frequency-generating set, and includes a suitable connecting field to provide the required number of connections and distribution outlets.

The basic frequency generation equipments (4, 12, 124 and 440 kilohertz) and the 2480-kilohertz master oscillators are duplicated: they do not require any automatic changeover circuits, since the distributions feeding the carrier supply equipments are also duplicated. A third spare oscillator can be added on request: it may be connected manually to either of the basic frequency-supply sources by means of U-links. The output of the latter oscillator is continuously monitored. The pilot equipment consists of the non-duplicated generation of the frequency comparison pilots (60, 300, 308 kilohertz): since the frequency comparison pilots are derived from 4 kilohertz, a U-link arrangement allows for connections with either basic frequency-generation set.

The bay may comprise a subrack for the duplicated generation of the mastergroup reference pilot 1552 kilo-

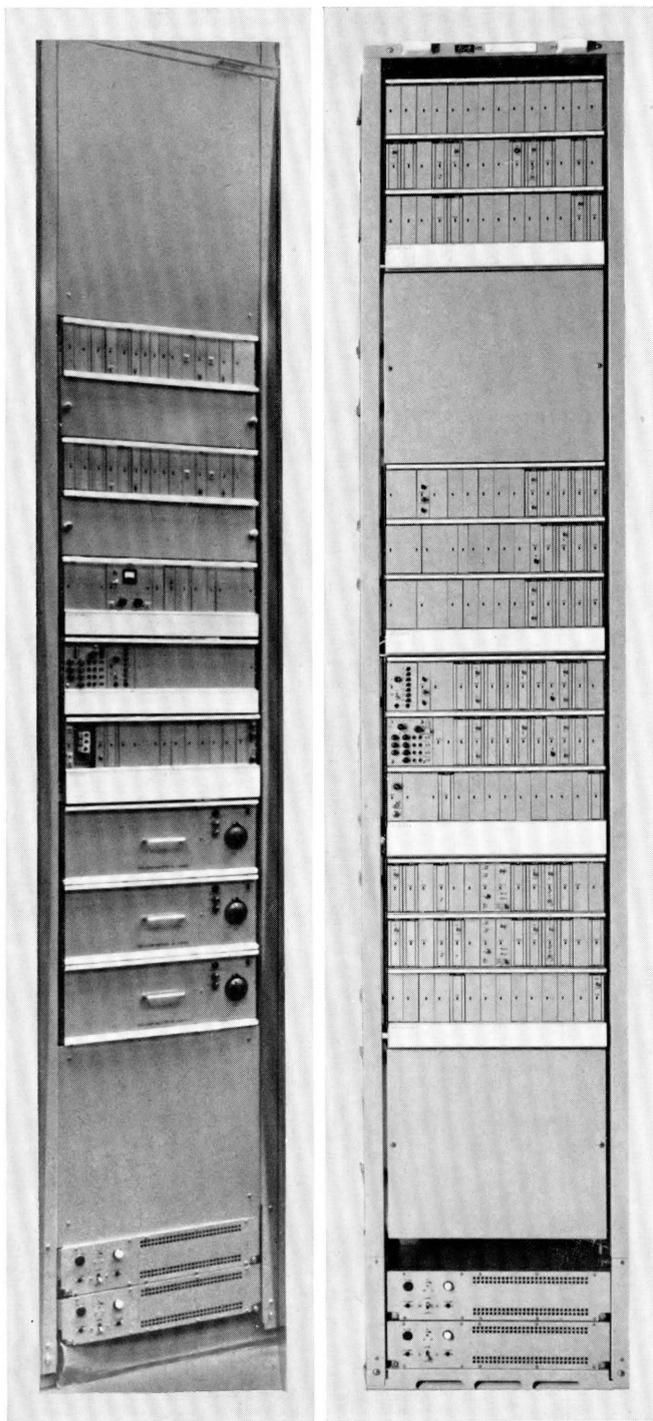


Fig. 14

Fig. 17

Figure 14 - (left) Bay consisting of the master oscillators and generators for the basic frequencies.

Figure 17 - (right) Mastergroup and supermastergroup equipment for a complete 2700-channel system.

hertz, including automatic change-over, and another sub-rack similarly equipped for the generation of the super-mastergroup reference pilot 11 096 kilohertz. The duplicated generation and associated automatic change-over equipment for the auxiliary frequency 12 648 kilohertz is also housed in the latter subrack. This same frequency is further distributed to the automatic-gain-control equipment for supermastergroups.

For each frequency, a coaxial maintenance-test-point is provided on the front side of the plug-in unit containing the output circuit.

A frequency comparator may be equipped to compare the frequency of the incoming 60 kilohertz (or 300 or 308 kilohertz) with the locally generated 60 kilohertz. This apparatus includes a frequency-off alarm device.

Figure 15 shows the number of outlets and the power output per frequency.

It should be noted that the mastergroup and the super-mastergroup reference-pilot generators may be mounted on the corresponding bays carrying the supergroup and mastergroup translating equipments. The power consumption of a fully-equipped bay is 3.5 amperes at 20 volts.

4.5.3 Mastergroup and Supermastergroup Carrier Supply

A 3-shelf subrack contains the duplicated carrier generation with automatic changeover circuits and distribution units for mastergroups or for supermastergroups. The mastergroup carrier generation subrack is shown in Figure 16.

In the mastergroup carrier generation subrack, each shelf accommodates the duplicated generator units and the changeover for one carrier frequency. The distribution outlets are accessible on coaxial plugs, fixed on the connecting field at the bottom of the lower shelf. This type of construction allows for easy plug-in connections of the station cabling, which is terminated by coaxial jacks. The fuse distribution and bay-alarm units, common for the whole bay, are also located in this subrack. Maintenance test points are provided by means of coaxial plugs on the fronts of the output amplifiers.

The plug-in units for the generation of supermastergroup - 1 carrier 12 704 kilohertz are mounted on the left-hand side of the subrack, those for the 16 720-kilohertz carrier on the right-hand side. The carrier-distribution and test-point arrangement is similar to that of the mastergroup carriers.

The carrier generation subracks consume 1 ampere and 0.7 ampere, respectively, for mastergroups and supermastergroups.

4.6 Power Supply and Alarm Circuits

4.6.1 Power Units

Any type of bay can be fed to full capacity by a single power-supply unit. However, the possibility of accommodating two power-supply units is normally provided on each bay as a standard practice. The second unit is equipped only for reliability, both units being connected in parallel via diode circuits. In this way, if one supply fails, the other takes over the complete load without interruption. Both identical units may be mounted at the bottom of the rack.

The output of the power units supplies a 20-volt direct current with positive to ground. Various types of power units are available that allow for operation from alternating-current mains or from station batteries. These external supplies do not need regulation and may vary over a wide voltage range. Protection against overload and overvoltage is incorporated.

Technical Data

- Supply Voltage: 24-volt battery (21.8 to 29 volts)
48-volt and 60-volt battery (minus 12 to plus 22 percent)
220-volt and 110-volt alternating current (minus 15 plus 10 percent, 47 to 63 hertz)
- Output Voltage: 20 plus or minus 0.5 volts for any load conditions, up to a maximum direct current of 6 amperes

4.6.2 Power Distribution, Fuses, and Alarms

Each bay comprises a central subrack, which accommodates the fuse unit and the alarm unit, for example, the mastergroup carrier supply subrack.

The regulated power supply passes through distribution fuses mounted on the fuse unit. The fuses are of the cartridge type, accessible from the front.

The alarm unit provides a station alarm loop and visual indications for the failure of the regulated direct-current supply, fuses, incoming pilots, and frequency generation. Distinction between urgent and non-urgent station alarm is incorporated according to the importance of the failure condition. The individual fuse alarm is located by means of a relay on the fuse unit.

5. Station Planning

It has already been emphasized that the ISEP construction lends itself particularly well to flexibility in the design of bays; this is also reflected in the planning of station layouts.

In the applications of mastergroup and supermastergroup equipment, two standard types of station layouts are to be considered, one for small and one for large stations. In both cases, the master oscillator and basic frequency-generating equipment is mounted on a central bay, whereas the carrier supply equipments are always decentralized.

In small stations, one bay accommodates the translating and carrier supply equipment for the mastergroups and supermastergroups of one 2700-channel system as shown in Figure 17.

Unlike the case of large stations, where supermastergroup distributing frames may be used, the solution for small stations is not satisfactory. The mastergroup and supermastergroup equipments are then separated on two different types of bays, with a respective capacity of 4x3 mastergroups and 4x3 supermastergroups. Figure 18 illustrates the various bays needed for 2700-channel system applications for small and large stations.

6. Acknowledgments

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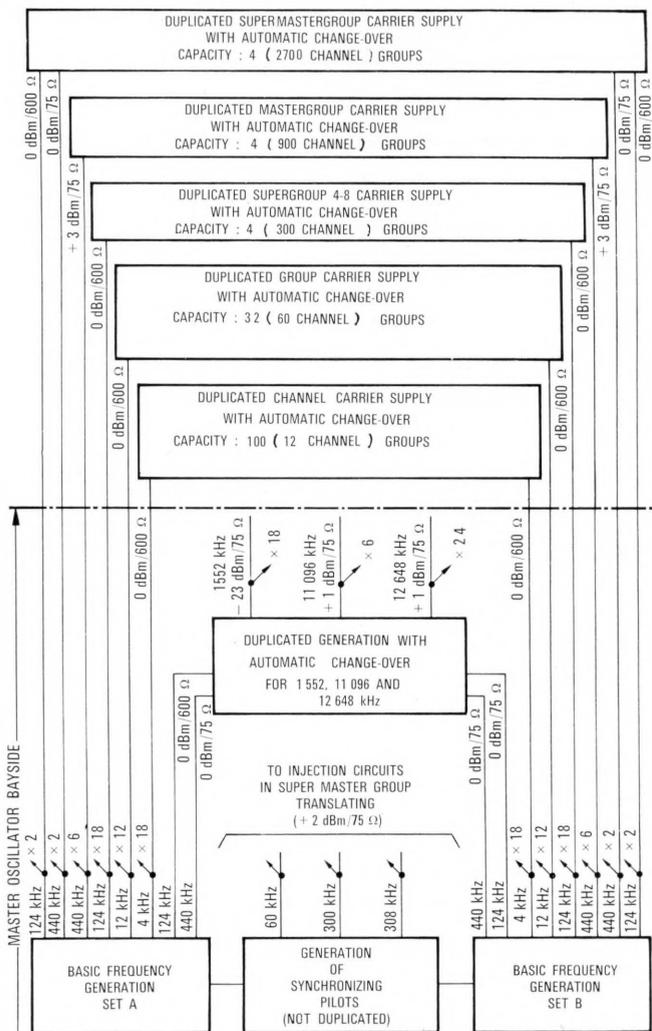


Figure 15 - Station distribution diagram of frequency generation equipments for 2700-channel systems.

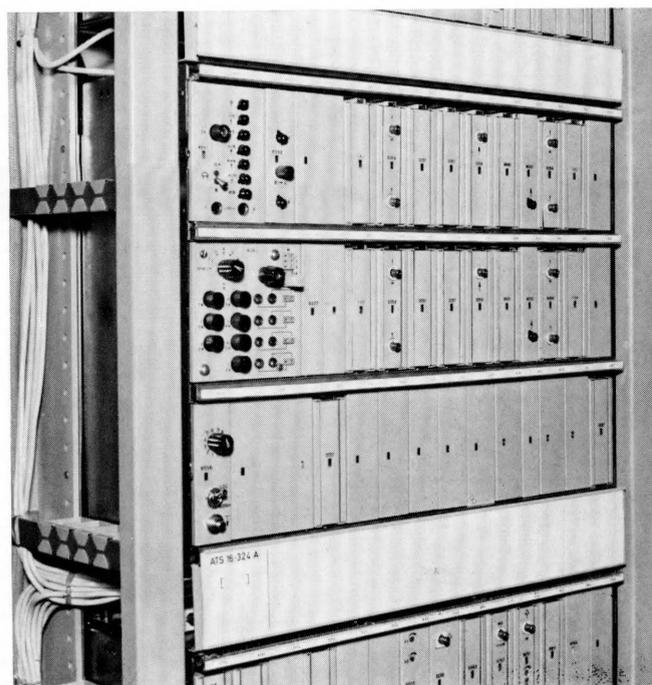


Figure 16 - Mastergroup carrier generation subrack.

Mastergroup and Supermastergroup Equipment

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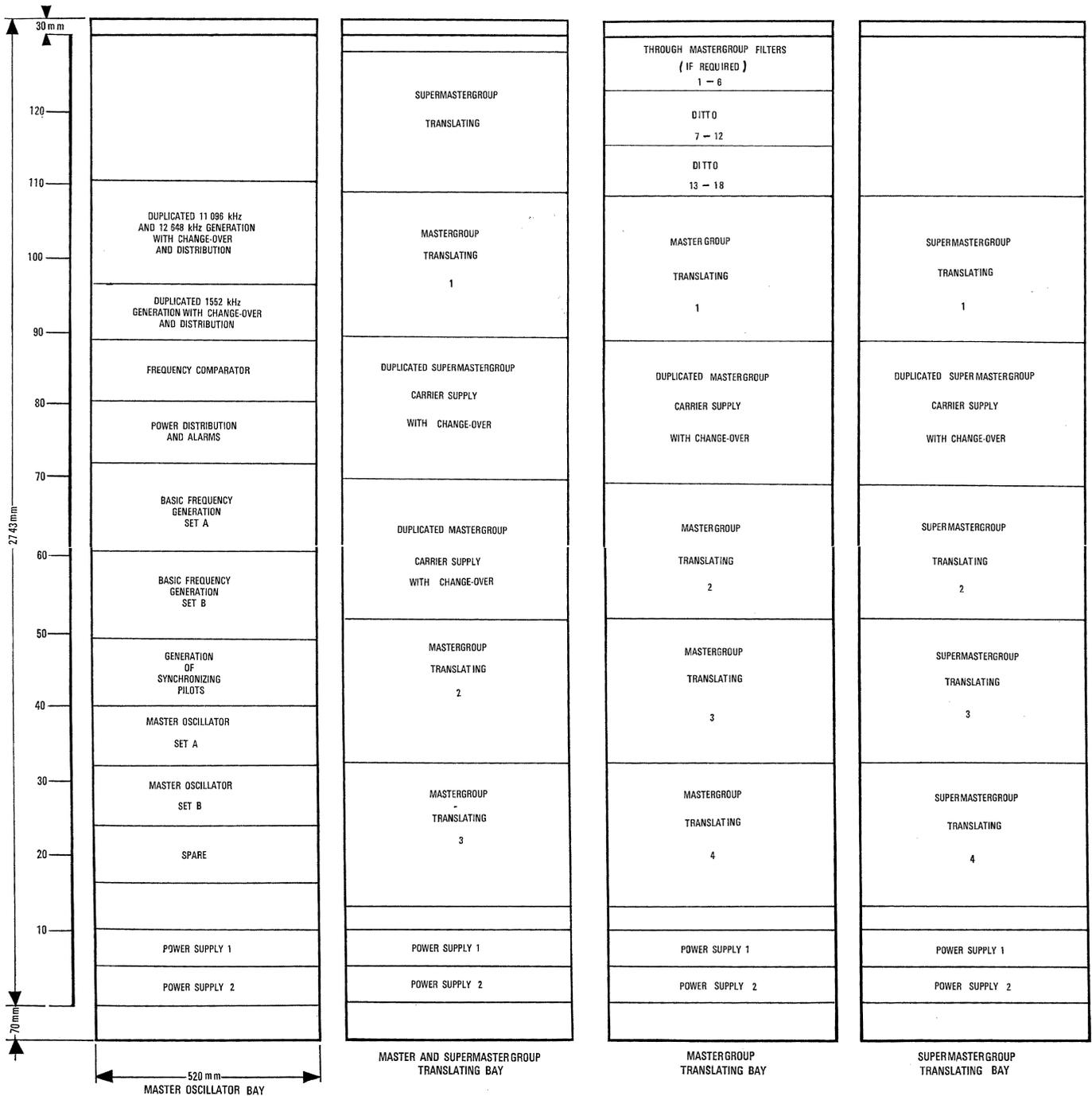


Figure 18 - Typical bay layouts for 2700-channel systems.

Doppler VOR Ground Equipment

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1. Introduction

For short- and medium-range navigation, civil aviation authorities make extensive use of very-high-frequency omnidirectional radio beacons called VOR systems — the term VOR stands for "VHF Omnidirectional". The International Civil Aviation Organisation (ICAO) is the body that recommends performance standards which designers of VOR hope to meet.

Sometimes it has been impossible to meet the accuracy specified by the ICAO because of obstacle reflections which produced bearing inaccuracies. These disturbances occurred if the VOR system was installed, because of airway requirements, in the vicinity of unavoidable obstacles, or if the receiving aircraft had to fly a course over mountainous country.

For many years, therefore, efforts centered around the development of new very-high-frequency omnidirectional navigational aids that would be less subject to the effects of terrain. However, consideration could be given only to those technical solutions that permitted full compatibility with the numerous airborne receivers already in operation and allow them to continue in use without modification. The Doppler VOR development has succeeded in practically eliminating the drawbacks of the conventional VOR system whilst meeting this requirement.

Doppler VOR employs two fundamental principles; a wide-base antenna array for eliminating the effects of multipath propagation and the Doppler principle for the determination of direction.

2. Basic Ideas of the Doppler VOR System

In 1921, Heiligtag proved that bearing errors of varying magnitude are likely to occur when a direct propagation path becomes combined with a second path resulting from reflections.

In a German research institute, investigations by Crone on short-wave direction-finders [1, 2] showed that these bearing errors caused by disturbed propagation conditions can be reduced by extending the antenna aperture or base line so that $D > \lambda$, where D is the distance between a pair of antennas. However, a wide base line will produce ambiguous bearing information in the conventional VOR system. The desirable object is therefore a system with a wide base line that also includes unambiguous bearing information over 360 degrees azimuth.

These considerations suggested the use of the Doppler effect, and Kaule first used the effect with his "frequency direction-finder" in 1941 and that same year, Budenbom applied for a patent [3], in which — as Kaule showed — a pair of rotating dipoles will produce periodic distance variations between the transmitter and receiver that will modulate the incoming carrier in its frequency; the phase of this alternating voltage depending upon the direction of incidence of the wave front.

In 1942, Busignies proposed a directional reception or directional transmission system using the Doppler

effect in which the necessary movement of the antenna was simulated by the cyclical switching of dipoles.

The advantage of these Doppler systems is that the ambiguity of the bearing information is independent of the base line making the system especially suitable for eliminating multi-path propagation since a wide base line can now be used.

In the early fifties, the practical development of Doppler direction finders started in Great Britain [4], Germany, and the United States. The first Doppler VOR was constructed in the United States by the Federal Aviation Administration [5, 6]. The Doppler VOR developed by SEL in Germany uses all the advantages of this system by employing the double-sideband method.

A conventional VOR beacon radiates a direction-dependent variable signal the phase of which varies with the azimuth, and a direction-independent reference signal with a constant phase. The phase difference between both signals is evaluated in the airborne receiver. The resulting angle corresponds to the azimuth referred to the ground station. Conventional VOR beacons generate the direction-dependent signal in a very simple way, by the rotation of a directional antenna pattern.

The conventional VOR supplies the following navigation signals.

- A main carrier (112 to 118 megahertz), amplitude modulated with a subcarrier of 9960 hertz. This subcarrier is frequency modulated with the 30-hertz reference signal at a deviation of ± 480 hertz.
- A portion of the unmodulated carrier that is radiated from a rotating dipole, which, together with the main carrier generates a cardioid pattern rotating at 30 times a second, the phase of which varies with the azimuth. Thus the carrier appears to be amplitude modulated with the variable signal and so two sidebands, equally spaced 30 hertz from the carrier, are generated. The frequency spectrum of these signals when processed by the air-borne receiver is shown in Figure 1.

By employing an entirely different technique using a wide-base antenna array the same signal spectrum is generated by the Doppler VOR. Like the Doppler direction finder, the direction-dependent signal is generated by moving a radiation source on a circle. Because of the Doppler effect, the very-high-frequency signal radiated by the moving antenna appears to be frequency modulated. Hence, unlike the conventional VOR, the azimuth-dependent information of the DVOR is contained in the phase of the frequency-modulated signal. The reference signal is transmitted as an amplitude modulation of the carrier. For the DVOR, the functions of the 30-hertz amplitude-modulated and 30-hertz frequency-modulated signals are exchanged. As long as the data specified for the VOR signals are maintained, this effect has no influence on the VOR airborne receivers.

The specified frequency deviation for the frequency-modulated signal is ± 480 hertz. The time period for

one complete rotation of the radiating source is $1/30$ of a second. Consequently, the resulting diameter of the antenna ring is approximately 5λ , equal to 13.5 meters (45 feet), a very effective antenna baseline which will produce the required reduction of siting errors.

The mechanical rotation of an antenna at a speed corresponding to a frequency of 30 hertz is obviously impractical. In a practical arrangement, the circular motion is electronically simulated by a number of antennas equally spaced around a circle. These antennas are sequentially fed with radio-frequency energy so that a continuous movement of the radiating source is achieved.

To exploit all advantages of the DVOR technique, use is made of both sidebands [7]. The upper sideband (carrier frequency plus 9960 hertz) is radiated from one side of the antenna ring and the lower sideband (carrier frequency minus 9960 hertz) from the opposite side.

Figure 2 shows the calculated maximum bearing errors of a conventional and of a Doppler VOR when propagation is disturbed by a reflecting object. As can be seen, large bearing errors may appear at a reflection factor as low as 10 percent, depending on the angle ϕ formed between the two lines from the aircraft and obstacle to the VOR installation. If an aircraft flies along a radial its course information shows rapid changes between positive and negative maximum errors, depending on the phase differences between the direct and the reflected signals (scallops and bends) [8].

The improvement in course information stability achieved with the Doppler VOR is apparent. In the least favorable case the improvement factor is approximately 10.

Tests of the Doppler VOR at various sites have revealed two essential advantages compared to the conventional VOR; the improvement of absolute accuracy in all types of terrain, and the improved smoothness of the course information, even for flights over mountains.

As a result, the DVOR will permit not only more accurate navigation but also application of the autopilot on routes where it previously could not be used because of the roughness of the course information provided by a conventional VOR under poor siting conditions.

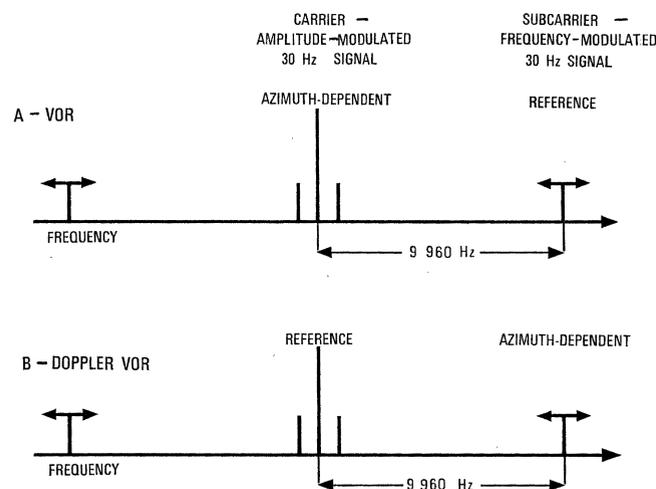


Figure 1 - Comparison of standard VOR and Doppler VOR modulation.

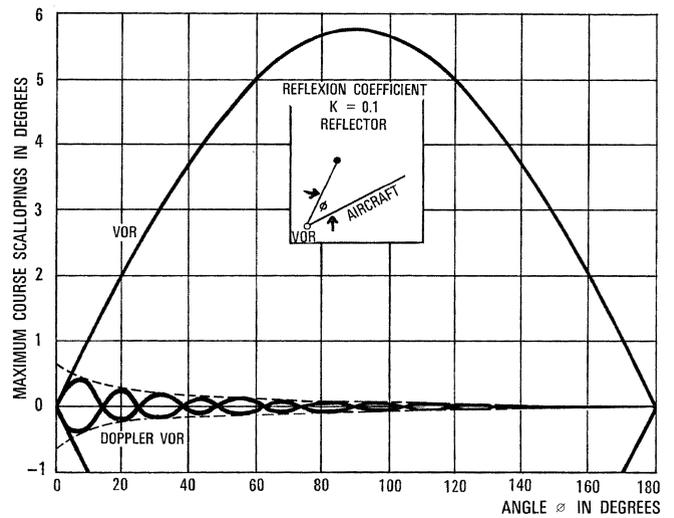


Figure 2 - Improvement in course accuracy of Doppler VOR over standard VOR for an object of reflection coefficient 0.1 near the ground beacon at angles ϕ from the beacon-to-aircraft direct path.

Figure 3 shows the error curves determined by orbiting flights at a height of 1500 feet (450 meters) at a distance of 5 nautical miles (9.25 kilometers) from a conventional VOR and a Doppler VOR. Both facilities were installed at the same poor site near the Rhine River Valley, in the Rheingau Mountains, at Ruedesheim near Frankfurt on the Main, West Germany. In both cases the German Federal Administration of Air Navigation Services (Bundesanstalt für Flugsicherung) used the same airborne receiver for the flight tests. The improvements achieved with the DVOR are shown. It should be noted that en route flights using a conventional VOR at the aforementioned site revealed course-information scalloping and bends which in most cases exceeded ICAO limits, whereas the maximum course deviation of the DVOR was only approximately 0.5 degree.

In autumn 1966, the United States Federal Aviation Administration carried out flight tests with the same DVOR installation, simultaneously using ten different types of VOR airborne receivers to check the compatibility of the DVOR with receivers of different quality. The results of these tests were compiled in Working Paper No. 147 of the last COM/OPS Meeting held in Montreal on October 29, 1966.

A first production model of the DVOR was ordered by the Austrian Administration of Air Navigation Services

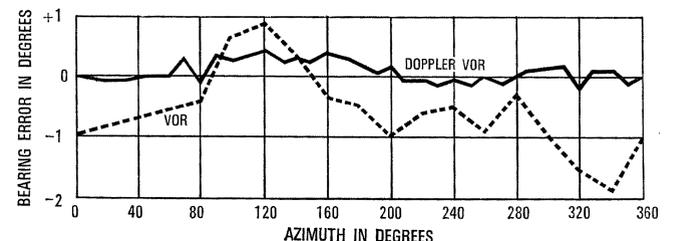


Figure 3 - On tests over mountainous terrain the standard VOR (broken line) had an error spread of ± 1.35 degrees and the Doppler VOR (solid line) was only ± 0.3 degree.

and installed at St. Pantaleon, near Salzburg, for service on the air routes to Vienna, Klagenfurt, and Munich (Bad Toelz). At the same site, a conventional VOR had proved unsatisfactory because it supplied information that was mostly beyond the tolerances not only on account of the poor siting conditions (steeply ascending and descending terrain, large village), but also because of the relatively high mountains nearby. In this case, too, a direct comparison can be made of the data recorded in numerous flight tests by the Austrian authorities using the same airborne receiver.

Figure 4 shows the improvement in course information accuracy of a DVOR during a radial flight across the Alps. The recording shows the course of the conventional VOR with scallops and bends up to ± 3.5 degrees and the smooth, practically undisturbed course of the DVOR.

The technical approach to the Doppler VOR system is described in the following sections.

3. Doppler VOR Equipment

The Doppler VOR consists of a transmitter with modulators, the antenna array with antenna switching unit, and a monitor (see block diagram, Figure 5).

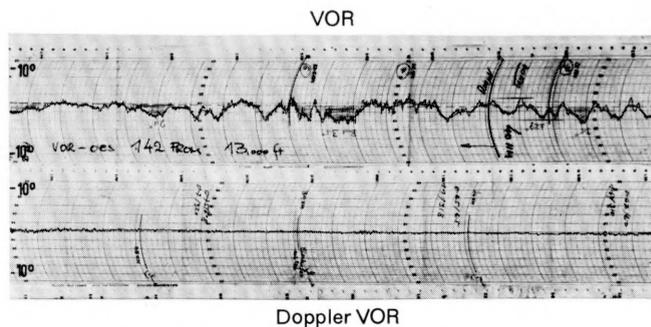


Figure 4 - The upper recording shows disturbances up to 3.5 degrees for standard VOR and the lower record for Doppler VOR is practically undisturbed on a radial flight across the Alps from Salzburg, Austria.

3.1 Transmitter

The transmitter assembly consists of a carrier transmitter and two sideband transmitters offset from the carrier by ± 9960 hertz.

The output power of the carrier transmitter is 200 watts, that of the sideband transmitters approximately 8 watts each for a modulation depth of 30 percent.

Special control circuits are included to ensure an exact frequency spacing as well as correct phase rela-

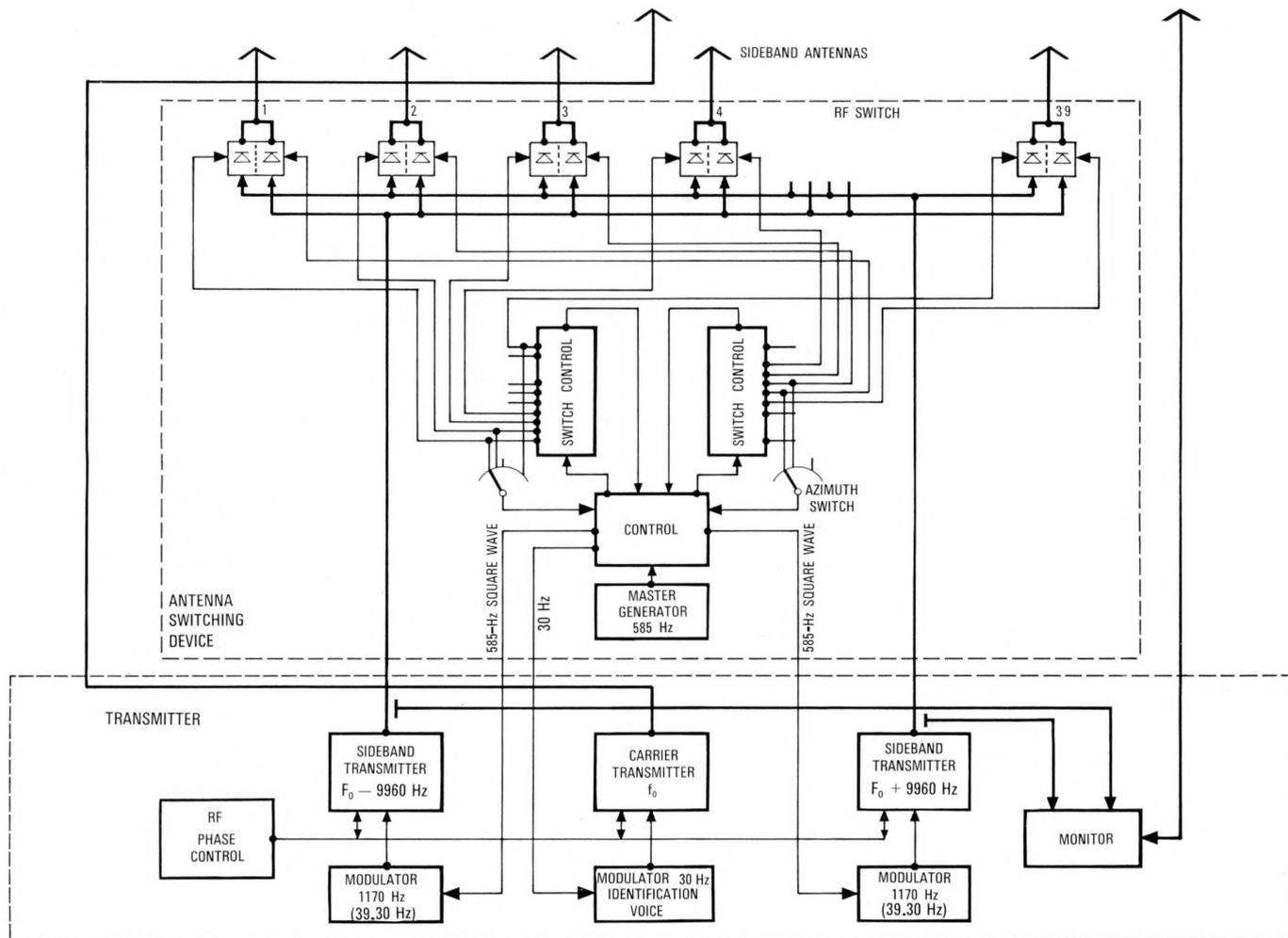


Figure 5 - Block diagram of transmitters, modulators, and antenna switching system.

tionships, so that the radio-frequency signals of the three outputs represent the carrier and the two sidebands of an amplitude-modulated wave.

Only two separate crystal oscillators with frequency multiplication for the carrier and the upper sideband are used to produce the three signals. The lower sideband is obtained by mixing the carrier with the upper sideband. An exact frequency spacing between both sidebands and the carrier is thereby attained. The crystal frequency is $1/12$ of the carrier frequency and is multiplied to the required range of 112 to 118 megahertz.

Figure 6 shows the schematic set-up of the frequency-generating unit. Frequency control is necessary to keep the frequency spacing between carrier and sidebands exactly at the nominal value of 9960 hertz.

To eliminate residual modulation components with a pure amplitude modulation, the resultant of the two sidebands must always be in phase with the carrier vector. Thus the radio-frequency phase of the carrier must be equal to the arithmetical mean of both sideband phases.

The phase-control circuit of the DVOR transmitter is based on the principle that the radio-frequency phase shift between carrier and sidebands is the same as the audio phase difference of the demodulated 9960-hertz signals. These frequencies are obtained by mixing the carrier with the sidebands in the mixers $M-1$ (frequency control) and $M-2$. From the phase difference between the two 9960-hertz signals, a control voltage is derived which varies the radio-frequency phase of the lower sideband by means of an electronic phase shifter until the correct phase relationship between carrier and sidebands is achieved.

3.2 Modulators

3.2.1 Carrier Modulator

The carrier transmitter is amplitude modulated with a 30-hertz reference signal, 1020-hertz tone identification, and speech, if required.

The transmission of the 30-hertz reference signal requires to be extremely stable in phase, since any change of phase immediately results in a bearing error. The phase of the 30-hertz reference signal can be aligned to magnetic north by means of an electronically variable phase shifter.

The carrier transmitter is modulated with the audio signals by means of a transformer that feeds the plate

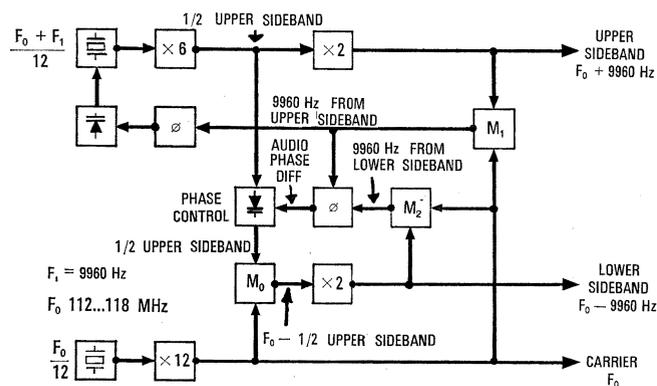


Figure 6 - Generation of carrier and sideband frequencies.

and grid of the transmitter power stages through separate windings.

3.2.2 Sideband Modulators

As mentioned in Section 2, the circular motion of a radiating element is electronically simulated by sequentially feeding modulated sideband energy through an antenna switching unit to 39 antennas equally spaced on a circle. To obtain a good approximation of the mechanical rotation by electronic simulation, the radio-frequency energy fed to the sideband antennas is modulated by half-wave sine or cosine functions.

Figure 7 shows the characteristic timing and distribution of the modulation signals to the upper and lower sidebands. Because of the number of antennas and the rotation speed the modulation frequency equals 39×30 hertz = 1170 hertz.

The modulation signals for the two sideband transmitters are generated separately in the sideband modulator. The antenna switching unit supplies two 585-hertz square-wave signals in quadrature, synchronous with 30 hertz. After generation of the fundamental sine wave by filtering and full-wave rectification, the required half-wave sine or cosine modulation signals with a frequency of 1170 hertz are obtained. The modulation depth of the sideband transmitters is almost 100 percent.

3.3 Antenna Switching Unit

To simulate the antenna movement as described in the preceding sections, electronic circuitry is used which connects the two sideband transmitters with the 39 antennas in the circle.

Because of the disadvantage of a mechanical commutator — wear and maintenance — an electronic device was developed using semiconductors only. Life time could thus be increased considerably and for all practical

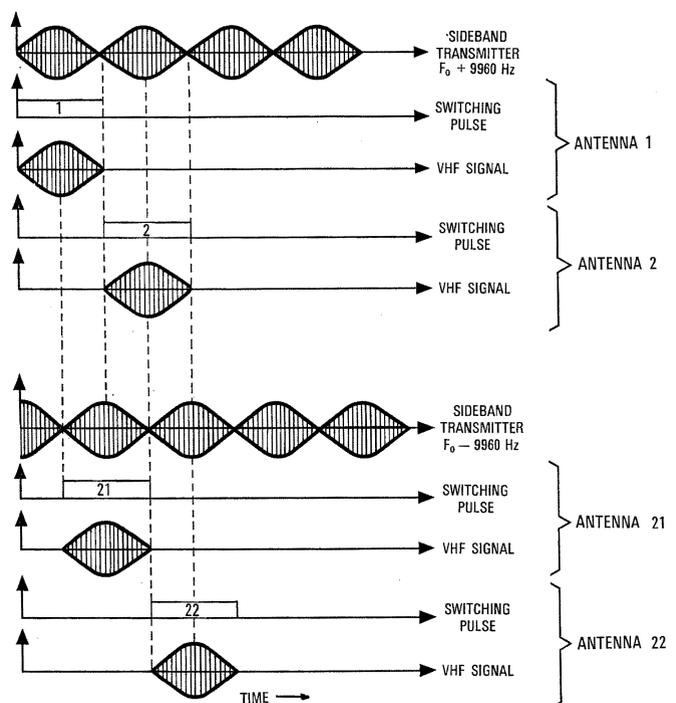


Figure 7 - Antenna switching pattern.

purposes the electronic antenna switching unit operates maintenance-free.

The most essential parts of this unit, the electronic control and radio-frequency gates with energy distributors, are described in the following paragraphs.

3.3.1 Electronic Control

This circuit provides the timing pulses for correct control of the radio-frequency gates and the signals for the carrier and sideband modulators, as shown in Figure 5.

An integral part of this unit is the highly accurate and stable tuning-fork generator which supplies a sinusoidal 585-hertz voltage. Pulse formers convert the resulting signal into 1170-hertz square waves. Logic circuits produce the clock pulses, shifted by 180 degrees, for two binary counters; the modulation signals for the transmitters (twice 585 hertz in phase quadrature and 30-hertz reference signal) as well as other control pulses.

Associated with each sideband is a binary counter which is reset to its initial position after 39 clock pulses. The binary outputs of these counters are decoded by code converters to 39 decimal outputs to control the radio-frequency gates.

The 30-hertz reference signal is also derived from the two counters, the outputs of which are shifted by 180 degrees. By means of the so-called azimuth switches, these outputs can be connected to twice 39 different counter outputs. Thereby, the phase relationship of the 30-hertz reference signal can be varied in steps of $\frac{360}{39}$ degrees = 9.2 degrees in reference to the timing of the radio-frequency gates. This serves for the coarse alignment of the phase of the 30-hertz reference signal to magnetic north corresponding to an azimuth of zero degrees.

A built-in test unit facilitates a quick check of most of the major functions of the electronic circuits during operation.

3.3.2 Radio-Frequency Gates with Energy Distributor

Each antenna is associated with a radio-frequency gate that has one radio-frequency switch for each sideband, connected to a common output. During one rotation, each antenna is supplied alternately with energy from the upper and the lower sideband as required for a double sideband equipment.

A diode in the radio-frequency gate acts in a forward direction biased by a direct current supplied from a transistor switch. The sideband energy from the transmitter is then fed through a distributor to the antenna. When blocked, the diode is reverse-biased by such a high negative unidirectional voltage that even in case of peak power modulation no radio-frequency energy can pass.

Each radio-frequency energy distributor has one input from its corresponding sideband transmitter and 39 outputs for feeding the sideband energy to the radio frequency gates. Another diode is inserted in each of the 39 outputs of the energy distributors to increase the attenuation in the backward direction when the gate is blocked by the electronic control circuit.

In addition to tuning elements for matching each

feed line, the outputs of the radio-frequency gates are provided with tunable resonant circuits to adjust the electrical length of the antenna feed cable, thus optimizing the decoupling of the sideband antennas.

3.4 Antenna System

3.4.1 Antenna Element

To meet international standards, the antennas of the conventional VOR have to be horizontally polarized. For the Doppler VOR, the radiation pattern in the horizontal plane has to meet very stringent requirements as to conformity of amplitude and radio-frequency phase. This is mandatory because all 39 sideband antennas contribute to the navigational information for every azimuth point from 0 to 360 degrees.

The use of Alford loop antennas made it possible to meet these requirements. Measurements with modified antenna elements have demonstrated maximum deviations from the omnidirectional pattern of only ± 0.5 decibel in amplitude and ± 5 degrees in phase.

The antenna element is matched to a 60-ohm feed system by means of a coaxial-balancing and matching transformer.

The photo in Figure 8 shows the radiating element imbedded in Styropor to increase its mechanical stability and to protect it against environmental influence.

3.4.2 Antenna Array

The 39 antennas, that alternately radiate the two 9960-hertz sidebands, are equally spaced on a circle of 13.5 meters (44 feet) diameter. A further antenna of the same type in the center of the antenna ring radiates the modulated carrier.

The antennas are mounted on a counterpoise of 40 meters (123 feet) diameter; height of the radiator elements above counterpoise is 1.3 meters (4 feet). Because

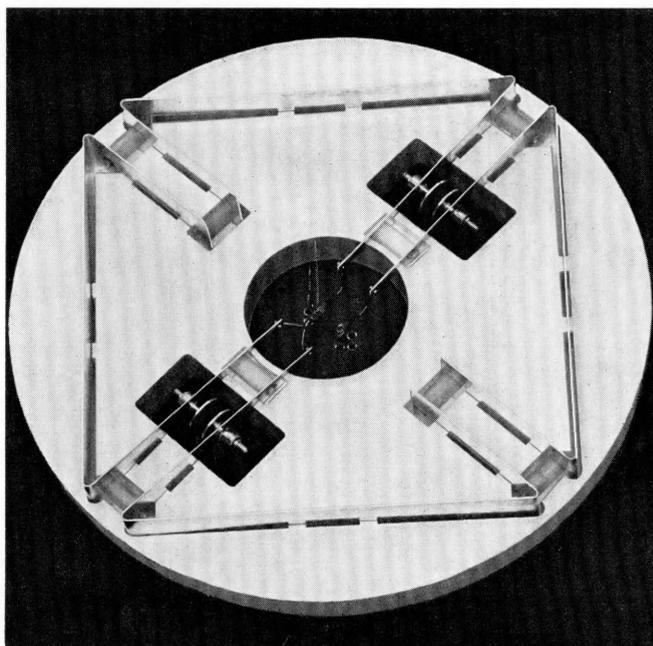


Figure 8 - Radiator element of an antenna mounted on Styropor sheet.

of this arrangement it is possible to install the transmitter in a shelter underneath the counterpoise and the central antenna. The shielding effect of the relatively wide counterpoise reduces siting requirements in the immediate vicinity of the installation.

The counterpoise consists of a stable galvanized steel construction covered with a steel wire grate of 6 millimeters (0.25 inch) thickness.

The photo in Figure 9 shows the antenna system of a DVOR installation in Austria.

3.4.3 Features

Interactions between adjacent elements of the array of 39+1 antennas can seriously affect the quality of the radiated signals. There are two main types of interactions: parasitic radiations from switched-off antennas and coupling of the carrier antenna with the sideband antennas. Through precise alignment procedures these unwanted antenna interactions can be reduced to a negligible minimum.

3.5 Monitor

In accordance with ICAO specifications for VOR facilities the following Doppler VOR signals are monitored at a fixed point in the field: modulation depth of the 30-hertz amplitude-modulated signals (reference signals of the DVOR), modulation depth of the 9960-hertz subcarrier, and the phase between reference and azimuth-dependent signals. The monitor incorporated in the transmitter is similar to an airborne receiver. It is fed by a dipole antenna 150 to 200 meters (490 to 660 feet) distant from the DVOR antenna array.

Compared with a conventional VOR, the functions of the DVOR monitor must be considerably expanded to meet all monitoring requirements due to the complex antenna system. This has led to a technical concept with the following features:

- use of an automatic, gain-controlled radio-frequency amplifier at the monitor input fed by the field dipole via a radio-frequency cable.
- measurement of the phase between reference signal and azimuth-dependent signal on the 9th harmonic of 30 hertz. The arising ambiguity is eliminated by a coarse phase measurement of the 30-hertz signals,
- not only is the level of the 9960-hertz subcarrier monitored, but also the amplitude characteristic is used to check the correct phase relationship between carrier and sidebands,
- monitoring of the return loss of radio-frequency energy from the antenna via the antenna-switching unit back to the transmitter,
- continuous check of the monitor circuits during operation by a test unit that automatically simulates faults and tests the correct operation of all alarm circuits periodically.

Owing to the additional internal monitoring circuits, multiple monitoring in the field under different azimuth angles could be abandoned.

4. Summary and Outlook

Development of the Doppler VOR began several years ago after successful experience with wide-band direction finders had already been gathered. The essential improvements of bearing accuracy and stability achieved dur-



Figure 9 - Doppler very-high-frequency omnidirectional range antenna at the ground station near Salzburg, Austria.

ing practical operation, especially on routes crossing unfavourable terrain, and the less stringent siting requirements needed for Doppler VOR have provided a system which has come up to expectations. This new application of the Doppler technique represents a step forward in the field of radio navigational aids and makes it possible to cope with problems of difficult operational conditions that hitherto could not be solved.

On account of its increased bearing accuracy and in conjunction with a DME (distance measuring equipment) ground station, the Doppler VOR constitutes a highly precise navigational aid for area coverage. By means of a course-line computer in addition to the existing airborne receivers, and assisted by an autopilot, the aircraft can be automatically guided on any air route within the range of such a co-located DVOR-DME installation.

The accuracy of a Doppler VOR could be further increased by transmitting in addition to the 30-hertz amplitude-modulated reference signal, a frequency-modulated reference signal on another subcarrier. However, new airborne receivers, or small additional units to the existing VOR airborne equipment, are required for evaluating the frequency-modulated signal. Assuming an appropriate quality of the airborne receivers, a system accuracy of 0.5 degree can be achieved. For some time ahead, this accuracy is likely to meet all the requirements for a precise medium-range radio navigation aid.

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Wilhelm J. Crone was born in Giengen, West Germany, in 1908. He attended Technical Universities in Stuttgart and Munich, Germany, and received his engineering degree in 1932. He joined the Institute of Physical Technology of the Technical University, Munich, and received the degree of doctor in 1934.

In 1934, he joined the Institut für Elektrophysik der Deutschen Versuchsanstalt für Luftfahrt in Berlin-Adlershof, where from 1939 to 1945 he was head of the Radio Navigation Department. His main activities were concerned with the field of wave propagation. He made investigations on general problems of long-distance radio navigation and was in charge of the development and testing of navigational systems. In 1948, he joined Standard Elektrik Lorenz AG, Stuttgart. He was in charge of development of antennas for television, broadcasting, and radio links. He is now in charge of the development of radio navigational aids.

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In 1951, he received his engineering certificate and in 1955, an engineering degree from the Technische Hochschule, Stuttgart, Western Germany.

In June 1955, he joined the Standard Elektrik Lorenz AG, Stuttgart. He was actively engaged in the development of electronic computers, and since 1963 he has worked on the development of radio navigational aids.

Air-Navigation Horizontal Situation Display

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1. Introduction

Modern flying machines have grown in size, speed, and complexity with the concomitant growth of avionics equipment necessary to fly them and of the pilot's task to evaluate operating conditions from an instrument-cluttered cockpit panel. While flight information offers itself in clusters, the human mind must process it and act on it sequentially. This multiplicity of inputs can make man a limiting factor. Pilots do not return to tell about a fatal hesitancy and confusion under great strain or in an emergency [1].

Airborne computer automation, of course, can, and does, help to narrow the human gap in the open man-machine loop, but it cannot hope to fully close it. Vital decisions that affect the conduct of a mission must often be based on en-route judgment of a situation. They remain with the pilot [2]. An analysis of, and a solution for, the improvement of pilot-aircraft intercommunication follows.

2. Modern Aircraft

Today's and tomorrow's American aircraft [3—7] offer great advances at the expense of greater complexity, which can be summarized as follows.

- With the supersonic transport, commercial traffic is now following military aircraft into supersonic speeds. In fact, all categories show the tendency to go faster and farther with bigger loads, calling for more and faster pilot activities.
- Military aircraft fly fast at low terrain-hugging altitudes to escape radar detection. This requires en-route response to ground details.
- There is a tendency toward improved short-haul aircraft, fast and steep climb or descent, and shorter or improvised runways. Terminal-type activities have thus grown more difficult.
- The increased density of air traffic is complicating air routing and landing activities of the pilot. Chart information requires frequent revisions.
- To protect pilot and craft against adversities, all advanced aircraft carry equipment, such as radar, which must be monitored.

All this adds up to the fact that many aircraft require new and refined controls for their operation. Some of the new functions are automated and can be programmed before take-off; on-board computers have become standard equipment in modern avionics systems. But however automatic, they must be understood and monitored by the pilot, and they do not release him from his responsibility to properly time and conduct navigational and tactical phases of his mission, to decide rapidly between alternative ways and means, and to guard constantly against the unforeseen. Human factors remain the most-decisive element in modern flight.

3. Mission

The requirements of the mission, as distinguished from equipment operation, represent the second area of pilot responsibilities. Mission-to-pilot relation is very different for commercial and military flights. Commercial traffic follows a repetitive and highly regulated pattern (Figure 1). The pilot is interested only in data on his flight corridor or close-up data for the two terminal areas. Such data include take-off regulations, weather, concurrent traffic, en-route range stations, en-route landmarks, terrain hazards, and ground-controlled approach. The military tactician or pilot however must be prepared to operate in any direction and, if base or target are mobile, between any points. The omnidirectionality of his operation requires area knowledge instead of line or point knowledge. It places greater dependence on en-route sensing equipment, such as radar ground mapping, terrain-avoidance radar, search and countermeasures radar, infrared equipment, or low-light-level television. These techniques

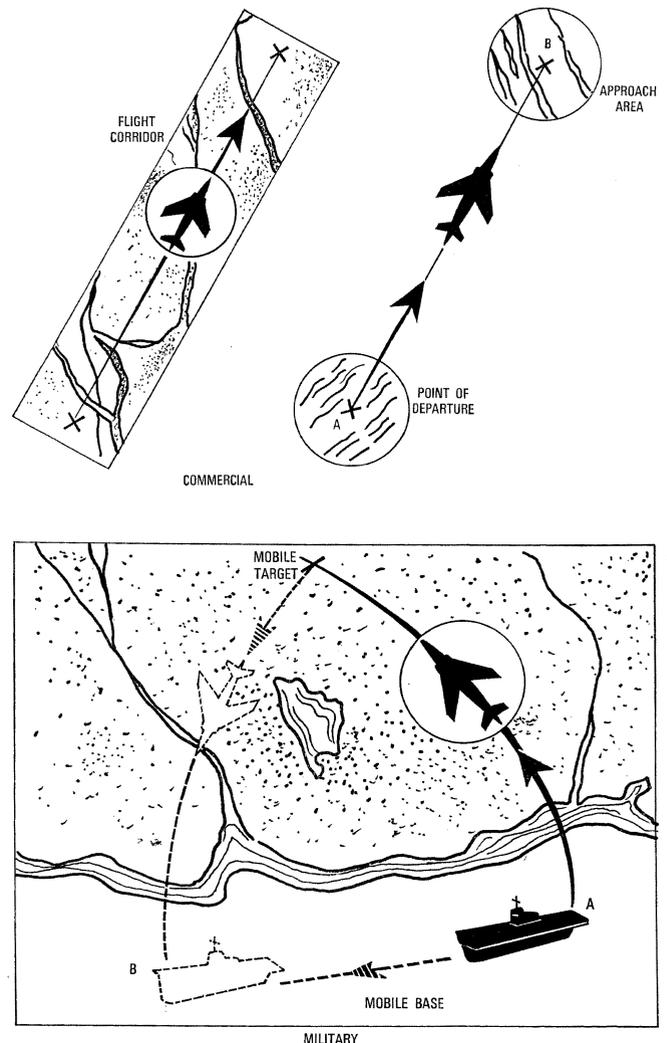


Figure 1 - Commercial traffic follows a repetitive pattern and uses strip charts or terminal charts. Military missions may go in any direction and require area knowledge instead of line or point knowledge.

have made our aircraft and pilots into specialists. At the same time, their faster and more-difficult missions may be flown anywhere, in any climate, among any people, without nearby help.

4. Pilot

The above summaries of aircraft and mission trends indicate the importance of human factors and of the help that improved man-machine communication can provide to the pilot or air crew.

Modern computer automation can help them through the "Integrated Avionics Systems." Listed below with their sponsors are several of these systems [8—13] presently in state of development for a number of major tasks.

- IHAAS — Integrated Helicopter Avionics System (United States Navy)
- ILAAS — Integrated Light Attack Avionics System (United States Navy)
- AAFSS — Advanced Aerial Fire Support System (United States Army)
- F-111/MK II — F-111 Advanced Avionics and Weapons System (United States Air Force)
- A-NEW — Anti-Submarine Warfare System (United States Navy)
- SST-AVIONICS — Integrated System for the Supersonic Transport (Boeing Airplane Company)

Integrated avionics represent a new system concept in which electronic technology is used to receive, process, and control many of the navigational, operational, and weapon details and to coordinate them where possible to form consolidated flight modes, such as take-off, cruising, landing, and attack. The pilot is now the general manager of a system having a higher order of capabilities than previous systems offered.

5. Horizontal Situation Display

The massive and urgent requirement of so many aircraft and mission inputs to the pilot's brain has created the concept of an integrated situation display as a companion to the integrated avionics system. It uses the outputs available from the avionics system, automatically selects the important data for a given situation, integrates these data into single pictorial situation images in the vertical and horizontal reference planes, and displays them to the pilot continuously in all weather.

The ITT horizontal situation display offers an example. It has been developed over the years in conceptual steps toward a complete horizontal image. It started as a present-position geographic display that moved a servo-motored navigation chart behind a fixed aircraft marker, or vice versa. This offered a timesaving substitute for hand-held charts, but without additional functions such as an instrument was hard to justify in the overcrowded cockpit. Therefore, navigational mode-controls, including directional bearing information and finally electronically generated symbology, were added to offer a more useful horizontal situation image.

5.1 AN/APA-115

In its pioneering development of horizontal situation displays, Gilfillan delivered in 1961 to the Air Force the AN/APA-115 (Figure 2, left top), which was essentially a compact microprojector displaying the magnified image of a servo-motored microchart on a rear-projection screen. The aircraft position marker and bearing cursor were the shadow of a reticle which was servo-rotated inside the mechanism to show the present ground track. In an overall size of only 7.5 by 9 by 9.5 inches (190 by 230 by 240 millimeters), the instrument was able to store the jet navigation chart of the entire United States continent and to display any of its points in adequate detail. Modifications of the AN/APA-115 were delivered to the National Aeronautics and Space Agency in 1965 for flight tests of ground approaches of vertical and short take-off aircraft and for a supersonic-transport simulation program. An Air Force study was concerned with its use in manned orbital spacecraft.

5.2 JANAIR Display

In 1963, Gilfillan delivered as a subcontractor to the Bell Helicopter Company the Army-Navy JANAIR display [14]. This, again was an optical microchart projector. Three

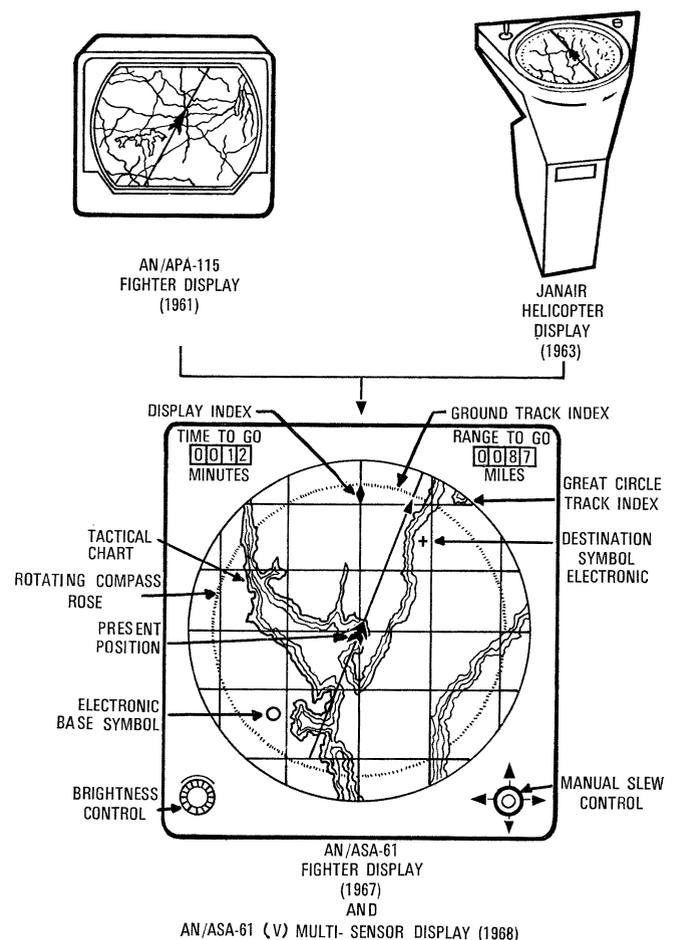


Figure 2 - The AN/ASA-61 or -61 (V) horizontal situation display 1967-68, shown at the bottom, combines two prior developments. The AN/APA-115 (1961) was a computer-operated microprojector, using a chart transparency drum (Figure 4) and the new halogen projection lamp. The JANAIR display (1963) added cathode-ray-tube generated position symbols on the chart and display image rotation in alternative forward or north-up mode.

features were added. The optical system included a servo-rotated Dove prism which rotated the light beam prior to projection and thereby enabled the display to be operated alternately in forward mode (to see landmarks as they appear through the windshield) or in north-oriented mode (to read chart legends and en-route location data in an upright position). The display, furthermore, used several directional reticles, such as a compass rose and separate cursors for ground track and destination headings. Finally, a cathode-ray tube was placed parallel to the optical track. By means of a beam combiner, its image was combined and boresighted with the optical display of a moving chart and of rotating directional signals. In this first version of a combined cathode-ray tube and optical display, the electronically generated symbology was used to display the continuously shrinking operating range of the helicopter by means of a fuel range circle surrounding the aircraft marker.

5.3 AN/ASA-61

In 1967, ITT Gilfillan delivered to the Air Force the AN/ASA-61 [15] which combined the features of the AN/APA-115 and the JANAIR display in a compact package 8 by 8 by 20 inches (200 by 200 by 500 millimeters) that fits the instrument panel of an advanced fighter aircraft. Having completed bench and environmental tests successfully, the instrument was then included in a flight test program of the United States Air Force, during which it was evaluated for use in high-speed low-altitude flights. A more-advanced multisensor version AN/ASA-61 (V) has been delivered recently.

6. Horizontal Data

In presenting in Table 1, a summary of information pertaining to the horizontal situation of a fighter aircraft, it is not suggested that the pilot requires all this information at the same time. Each aircraft, each mission, in fact each flight phase, may require a different combination of inputs. However, to avoid the need for individual dials for each presently used equipment, the unified instrument must be capable of combining different sets of related data into integrated images so that the pilot may select one to suit his immediate requirement.

Table 1 - Information needed in varying combinations to orient a pilot in the horizontal plane

Navigation	Mission	Sensors	Weapons
Present position	Destination or target location	Ground-mapping radar	Weapon radar
Present heading	Destination heading	Radar threats	Infrared weapons
Fuel range	Miles-to-go Time-to-go	Targets	Television guidance
Land marks	Geographic orientation	Search and track radar	Bombs
Terrain radar	Briefing instructions	Low-light-level television	Laser ranging

6.1 Geographic Orientation

The first group of input data refers to the navigational situation [16]. The pilot wants information on where he is and where he is headed. These data may be generated by a variety of navigational computers and are usually numerical counter readouts. In contrast, the horizontal situation display shows an aircraft marker moving over the appropriate chart, which is a multicolor replica of his present surrounding area (Figure 2) selectable in various chart scales. The peripheral compass rose and the rotating marker offer him a pictorial and a numerical reading of his flight heading, alternatively in forward- or north-oriented direction, as preferred. As on the Janair display, the conventional counter readout representing his fuel consumption can be replaced by an electronically generated circle around the aircraft marker which indicates his present operating range in any direction. In low-altitude flight, various radar-sensors, which usually require a separate instrument, can be displayed on the screen by the built-in cathode-ray tube.

The second group of input data refers to the mission. The operating base, destination, target position, and their bearings can be shown on the chart and on the compass rose by electronic symbols and cursors if this information is available from the computer initially. The counters at the top corners of Figure 2 will offer the same information in terms of miles and minutes to go. Desirable details of a mission may be entered on the chart before take-off. The display may also be slewed to special positions in which printed instructions appear on the screen. Such instructions relieve the pilot's memory in respect to take-off and landing regulations, briefing instructions, en-route equipment checkout, or emergency procedures.

6.2 Situation Around the Aircraft

The field of view around the central aircraft markers that the horizontal situation display affords the pilot can be utilized for a third information complex concerning the invisible electronic environment of the aircraft. Radar, infrared, or television sensors that advanced aircraft carry in various assortments, can feed suitably processed information into the cathode-ray tube, from where it is projected on the screen either as a separate image or in overlay to the chart. Ground-mapping radar and charts have unique distortions each of their own and do not naturally register with each other. These, however, become negligible on short ranges. In summary, the display will produce optically and electronically generated ground details, target locations, and signals of radar threats on the pilot's own safety. By the same method, the display can substitute for separate cathode-ray-tube displays that may be needed otherwise for weaponry purposes. This variety of display capabilities is brought about simply by combining and boresighting an optical and a cathode-ray-tube display in a single package.

7. Description

Figure 3 shows the two units making up the AN/ASA-61 in a current modification. Their dimensions are approximately as follows.

	Indicator	Generator
Face	8 by 8 inches (200 by 200 millimeters)	8 by 7.5 inches (200 by 190 millimeters)
Depth	20 inches (500 millimeters)	16 inches (400 millimeters)
Weight	30 pounds (13.6 kilograms)	26 pounds (11.8 kilograms)

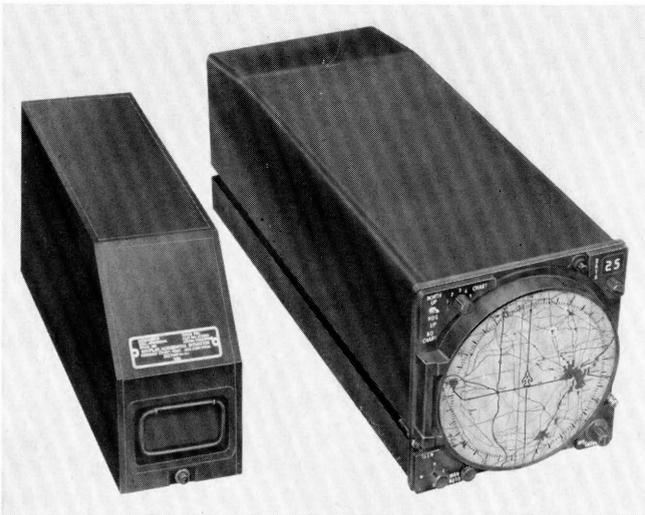


Figure 3 - The horizontal situation display consists of an indicator unit fastened to the aircraft instrument panel, and a generator unit which can be placed in a remote location.

7.1 Display Indicator

As panel space in any aircraft cockpit is at a premium, 60 percent of the total face of the display unit is utilized by the 7-inch display screen, leaving the remaining corners for the controls, such as for brightness, chart scale, north or forward mode, and the joy stick for manual chart slewing. Only those controls unique to the horizontal indicator itself are placed on its face. An integrated avionics system requires that all tactical mode controls not unique to the display be placed on a central control panel, where they are integrated with other concurrent subsystem operations.

The indicator unit consists of two major subassemblies: the outer chart servo assembly, which carries the servo-motored chart photographs, and the inner optical projector assembly, which combines a 10-times chart projector with a cathode-ray tube.

7.1.1 Chart Servo Structure

On AN/APA-115 and AN/ASA-61 alike, a transparent cylindrical drum carries the microchart transparencies (Figure 4). This drum is held on a cradle which is moved back and forth in geographic longitude by a servo-motored ball-return lead screw. The drum is rotated in geographic latitude by an equally large servo-motored bull gear ring. As the drum cradle with its drum and its pick-back latitude servo moves back and forth, it bisects the entire indicator into an inner portion that houses the projection assembly and an outer frame assembly. They are

held together by front and back plates. The hollow frame and its surrounding dustcover represent the structural backbone of the instrument, which is responsible for its stability and the accuracy of latitude-longitude chart positions.

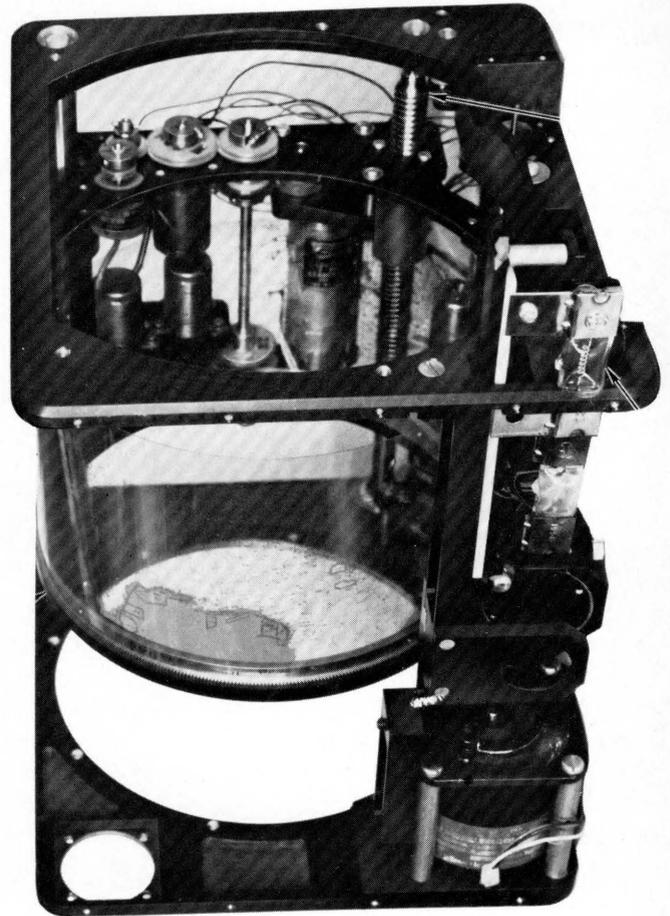


Figure 4 - AN/APA-115 chart drum and servos. The AN/APA-115 and the AN/ASA-61 use a transparent cylindrical drum as microchart carrier. The drum concept permits the use of individual chart chips of areas and area scales to suit each mission.

The four corners of the servo structure contain the chart illuminator, the high-voltage supply for the cathode-ray tube, and the two servo-mechanisms. The illuminator forms a fan-cooled duct of triangular cross-section throughout the indicator (bottom of Figure 5). The chart area that must be displayed represents $\frac{1}{10}$ of the screen diameter, or 0.7 inch (18 millimeters). It is illuminated by a small but very bright halogen lamp, placed between a back reflector and an aspherical condenser. Two of these lamps — one in operation and one spare — form a sealed and explosion-proof cartridge, which is good for 600 hours of flight time. The cartridge is reversed to put the spare in operation or can be replaced en-route through a door on the face of the instrument.

7.1.2 Optical Projector

The optical projection unit occupies the interior of the drum. It consists of a 5-inch cathode-ray tube placed behind the screen. A special lens with a speed equivalent

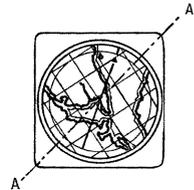
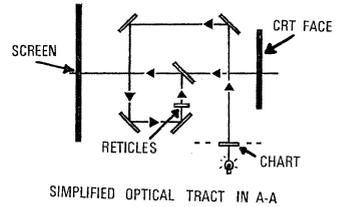
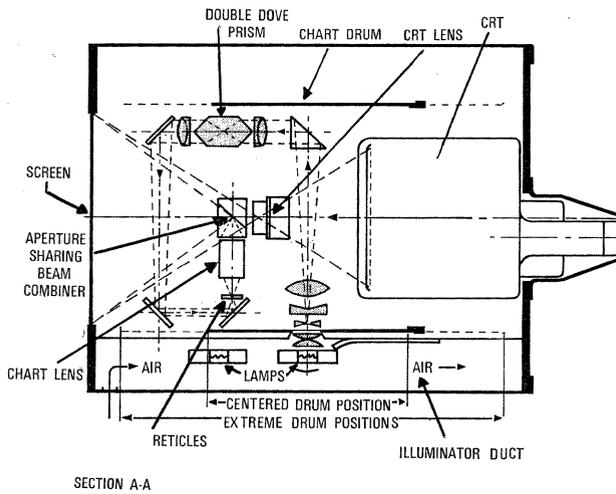


Figure 5 - AN/ASA-61 indicator diagonal cross-section in which the optical system is placed. The two projection systems, one for the chart and the other for the cathode-ray tube, use a beam combiner to produce a common image on the front screen.

to f/1.8 projects its image on the screen with a magnification of 1.75 times. The chart is projected on the screen in two steps by a separate projection system that is folded 5 times by means of corner prisms and elbow mirrors to fit it into small space available (Figure 5, right top). A lens system with unit power first creates an intermediate image of the chart at the point of the reticles. The system serves three purposes:

- it eliminates astigmatic distortion that would be caused by the use of a cylindrical transparency,
- it collimates and then again decollimates the beam of light for the use of a servo-motored double Dove prism, which rotates the screen image in alternate north or forward modes,
- it permits the use of two servo-rotated reticles in an intermediate focal plane. One of these carries the micro image of a peripheral compass rose, while the other carries the micro image of a centered aircraft marker and a diagonal cursor line. The shadows of these reticles appear on the screen in sharp register with the chart and indicate north and ground track directions as well as present position.

The entire optical system is assembled on a casting to permit accurate relative alignment and sealing against moisture. The intermediate image is projected on the screen in 10-times magnification by means of a special projection lens and a novel beam combiner which combines and registers the chart image with the cathode-ray-tube image.

7.2 Optical Performance

The use of an optical projector in an aircraft cockpit would not be practical, and might even be hazardous, without a few of the following state-of-art advancements that we have pioneered.

7.2.1 Halogen Lamp

The new halogen lamp was originally developed as a ruggedized wing-tip signal lamp for jet aircraft. Its small size and high brightness make it ideal for cockpit use. It contains a sturdy tungsten filament, the size of the opti-

cal entrance pupil, in a quartz envelope of only 0.4 inch (10 millimeters) in diameter. Its iodine or bromine atmosphere prevents blackening of the inner wall by tungsten vapor deposits, which would quickly reduce the light output. This results in long life at high and constant brightness with an input of only 75 watts. The lamp, therefore, does not pose the problems of limited lifetime in airborne use. However, its small size makes it extremely hot in operation.

Severe space restrictions in the corner of the illuminator require the lamp to operate within only 1 inch (25 millimeters) of the photographic chart film. Protecting the film from heat, particularly in tropical climates, calls for the use of advanced heat reflective and heat absorptive optical filters to reduce heat irradiation through the condenser and for thermal barriers in the opaque sections to reduce heat convection. Ample air-cooling must be provided through the illuminator duct.

7.2.2 Chart Drum [15]

The conventional use of roll film between two spools as in cameras would appear to offer larger geographic storage capacity than a cylindrical chart drum that holds only 5 individual, though larger, microchart transparencies. The concept of the optical storage drum, however, offers the important advantage of making projection lamps and chart film accessible for convenient exchange through the hinged indicator face without removing the instrument from the panel. The instrument, therefore, is not limited to a load of only 5 chart chips, but offers, in fact, random choice of charts covering any area in any scale, as the tactical mission requires. In contrast, use of a roll of film with 50 to 100 frames offers tactical random choice of charts only at the price of great redundancy and logistic inflexibility. At least in one geographic direction, it requires excessive slewing to adjacent areas. Single chart chips are preferable also because of the detail position accuracies required for a servo-motored micro-chart. In 10-times magnification, a position error of 0.001 inch (0.025 millimeter) means as much as 0.3 nautical miles (0.56 kilometers) on a 1:2,000,000 chart. Present state-of-the-art limitations of photographic materials and repro-

ductive processing present problems that are compounded by the use of roll film with many chart frames on one strip.

Finally, roll film does not easily permit the use of separate tactical overlays as single chart chips do. In the indicator, provisions are made to simultaneously insert, register, and project a second transparency that represents a quick photograph of the mission route. This is a logistic convenience because it separates permanent from perishable data. It leaves accurate photography in quantity production to the professional cartographers and permits tactical last-minute planning at the strike base.

7.2.3 Aperture-Sharing Beam Combiner [15]

Conventionally, two projected beams of light are combined into one screen image either by two side-by-side projection lenses or by a so-called beam combiner, which may be a partial or a dichroic 45-degree mirror. In the first method, the matching of two converging projection beams which do not follow the same center line results in keystone distortions which become prohibitive for short object-to-image projection distances. The second method has the advantage that a partially silvered mirror permits concentric beam combination without keystone distortion. However, there is an unavoidable loss of at least 50 percent of the total light. Dichroic beam combiners, consisting of expensive multilayer coatings, reduce the total light loss to about 25 percent but spectral splitting of the white light of the projection lamp causes discoloration of the white chart background to an undesirable magenta color.

A new aperture-sharing beam combiner that permits concentric beam projection with only trivial light losses is shown in Figure 5. It consists of a combination of two projection lenses at right angles to a transparent glass block containing a small elliptical silver mirror at the 45-degree interface of the two segments making up the block. One of the two lenses has a pupil of large diameter, which on its way to the screen fills the entire block. The small mirror obstruction in the center is not imaged. The other lens has been designed as a wide-angle eyepiece. Its exit pupil has the size of and is at the point of the small mirror. The chart light therefore first converges toward the mirror as a crossover point from which the light beam then spreads at a wide angle toward the screen. If the exit pupil diameter is 0.064 inch (1.63 millimeters) and the beamwidth of the other lens at the combining point is 1.5 inches (38 millimeters), the aperture area ratio is only 0.18 percent. The light loss through beam combination, therefore, is trivial.

7.2.4 Directional Fresnel Screen

Insufficient brightness has been the pitfall of many prior displays for the open cockpit of an aircraft. It is generally agreed that the screen must be bright enough to be clearly readable under a sky of sunlit white clouds (10 000 foot-lamberts). In addition, sunshine may fall directly on the screen and may create a threshold level of ambient illumination that the display light from within cannot overcome.

In this design the images of both the cathode-ray tube and the chart are projected on a special screen having lower reflectance than either the white paper of a viewed chart or the phosphor of a directly viewed cathode-ray tube. Optical rear projection has the further advantages that plastic Fresnel lenses can be used behind the screen and that the screen material itself can have sunlight-deflecting properties (exhibited for instance by the semi-specular screen material of Eastman Kodak). The combination has the advantage of a strong light gain toward the pilot's eyes, while incident ambient light or sunlight is strongly attenuated by deflection in other directions.

7.3 Cathode-Ray Tube

Every airborne sensor operation, whether based on radar, infrared, or television, has heretofore required its own unique cathode-ray-tube display. This has been a contributing factor to overcrowding the instrument panel of advanced aircraft. A truly synergetic multisensor display should make it possible to show any type of radar at low frame rates or any type of television at high frame rates or any type of charts and electronic position symbols brightly enough to outshine the adverse ambient light of the open cockpit. Prior displays have placed a cathode-ray tube directly behind the panel face with the result that brightness-reducing dark screens of ambient-light traps had to be used to make the display readable. The chart image was produced either by costly electronic chart scan, by feeding a full-sized strip chart over the face of a cathode-ray tube with a parallax-eliminating fiber-optics faceplate, or by using a cathode-ray tube with an optical rear window through which the chart could be rear-projected onto the phosphor screen. The latter method encounters problems when radar areas have to be displayed brightly in addition to line-type symbols.

It was decided, therefore, to use specially bright direct-view storage tubes and to project their image from the dark of the indicator interior optically on a low-reflectance rear projection screen. This method has the advantage that the screen can be backed by Fresnel lenses, which prevent wasteful hemispheric light spread from the front screen by forming a concentrated light lobe toward the pilot's eyes with corresponding brightness gain. Direct-view storage tubes require fine-mesh metal screens immediately behind the phosphor screen. This inhibits their use for optical rear-window projection under high ambient light.

The tube is produced by the ITT Electron Tube Division in Roanoke, Virginia. It has electrostatic focus and magnetic deflection. In contrast to projection-type television tubes, its view screen requires only 12 kilovolts with no appreciable corona problems at high flight altitudes. The use of this direct-view storage tube in the described modes is based on such circuit features as fast-erase techniques and accurate deflection control.

7.4 Display Generator

Figure 6 offers a simplified block diagram of the electrical system in which the display generator functions as the interface unit between avionics or sensor inputs and

the display indicator. These inputs consist of navigational data and clock pulses from the digital computer; of radar, television, infrared, and other inputs; and of discrete tactical control signals.

As the data processor, which employs microelectronic circuits, accepts the digital input signals, it performs a parity check on each word, determines the channel address, and selects the buffer register in which raw data are stored. On command from the sequence control circuits, it selects the data for the service channel being sampled, adds any necessary offset to the input data, and compares this position command with respective position feedbacks from the shaft encoders. The error data are applied to the appropriate servo controller module from where they feed into 28-volt direct-current stepper motors.

Sensor data in varying forms are accepted by the multisensor processor, which consists of video and deflection preamplifiers as well as unblinking and erase circuits.

The overall control of this partly digital and partly analog generator circuitry comes from the centrally located tactical control panel from where it feeds into the sequence control of the data processor or into the mode control unit of the multisensor processor. The pilot is able therefore to invoke instantly and automatically any display mode or image combination that his tactical situation calls for.

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Eric S. Guttman was born in Berlin, Germany, on 6 April 1904. He graduated as Doctor of Engineering at the Technical University in Berlin-Charlottenburg. He came to the United States of America in 1938.

His past development activities include electric strain gauges, long-range dial indicators, school microscopes, and a variety of optical inspection instruments.

When he joined ITT Gilfillan in 1953, he soon specialized in avionics displays. He is presently Assistant Division Manager for Electro-Optical Instrumentation.

A preceding article covers his development of the navigational situation display for which he received one of the 1966 awards made by the International Telephone and Telegraph Corporation to employees for outstanding contributions.

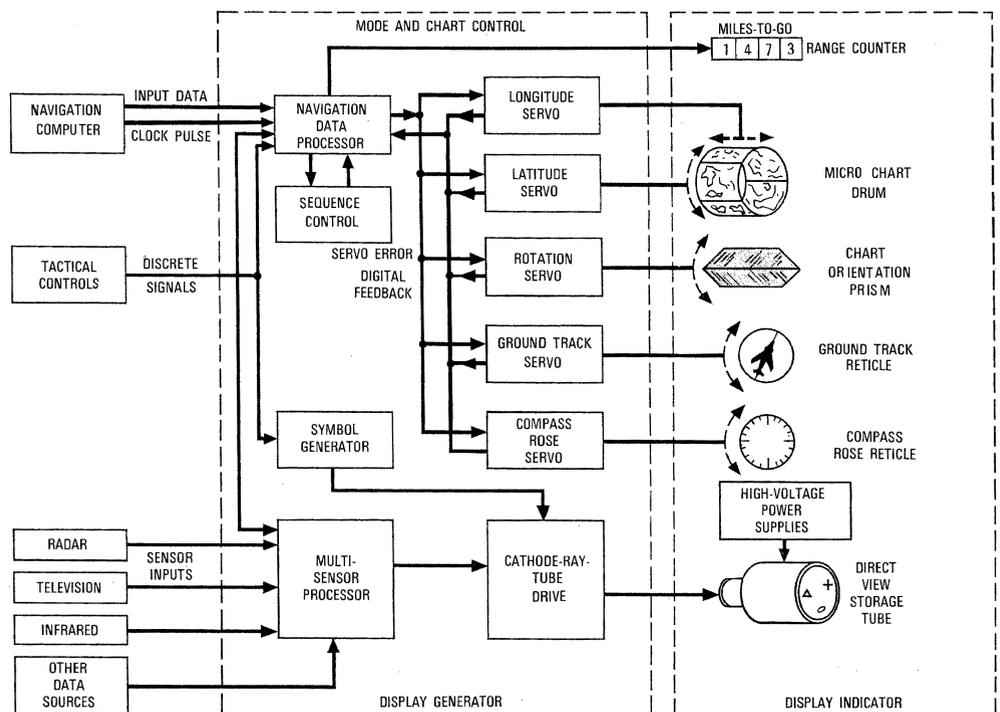


Figure 6 - Electrical system. Navigational and sensor inputs are processed in the display generators, which feed them into either the servo system or the direct-view storage tube.

Computation of Urban Trunking Networks with Alternate Routing by Computer

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1. Introduction

In 1966 a project was set up at the ITT Laboratories of Spain (ITTLS) to develop means and methods of providing computer assistance for telephone network planning. The following advantages of using computers in this field are considerable.

- Gain in time. Through the use of a computer, a planning task can be performed much faster. In a practical case, the time reduction achieved was from three weeks for manual planning to two days by using a computer, only one hour of which was computer time.
- Many alternatives. By using manual methods, one is limited in the number of alternatives one can compute by the available manpower and time. If a computer program is used, many different solutions can be computed based on various hypotheses with the same manpower and in the same amount of time.
- Increased confidence in results. Once the computer program has been completely checked out, no more computing errors can occur. The results obtained are correct. In addition, confidence in a proposal can be substantially increased if it is supported by a comparison with a number of different alternatives.

One of the first problems tackled at ITTLS was headed the "Computation of Urban Trunking Networks with Alternate Routing". This computer program will be described in the following pages.

It must be stressed that the intention is not to use the computer simply as a more powerful desk calculator. Accordingly, effort was directed to the finding of new hypotheses and new algorithms to permit optimization with respect to economical criteria and within the constraints of the practical exploitation of the network.

For a given urban area, the present study makes possible the computation of the number of trunks between the local and tandem exchanges taking into account alternate routing. This implies the computation of first- and second-choice routes. The junctions are assumed to be unidirectional. The rules and methods apply both to extensions of existing networks (final and intermediary steps) and to the computation of an initial network.

2. Method

2.1 Statement of the Problem

The program computes from certain input information the number of trunks between all exchanges, including tandems, for a given grade of service.

The following input data is required.

- Traffic matrix A_{ij} , containing the traffic between all exchanges in the area (in erlangs).
- Cost matrix C_{ij} , containing the cost of a single junction between each pair of exchanges i and j .
- Cost matrices C_{ik} and C_{kj} , containing the cost of a

single junction between each exchange i and tandem k and between each tandem k and exchange j , respectively.

- Routing matrix containing data on the possible routing between two exchanges, i. e. the possibility of a direct route and/or tandem for a second-choice route.
- Minima and maxima. A minimal and maximal number of trunks can be specified for each pair of exchanges. This feature will allow existing plant or any other restrictions to be taken into account.
- Grade of service. The overall grade of service has to be specified.
- Switching cost, i. e. the mean marginal cost of switching one erlang in a tandem exchange.

It is assumed for the moment that traffic has only two alternatives for passing from the originating exchange to its destination: the direct route and a second-choice route going through one tandem (see Figure 1). A generalization of the program will cover the case of two tandems in series (see Figure 2).

The following results will then be computed.

- Number of direct trunks between all exchanges in the area.
- Number of trunks between all local exchanges and tandems, as well as between all tandems and local exchanges.
- The traffic carried on all these routes (mean, variance, degeneration).
- Costs. The results will represent an economical optimum.

2.2 Algorithm for Evaluation of Optimum Network

The algorithm consists of the computation of a basic solution with the following iterations (see Figure 3).

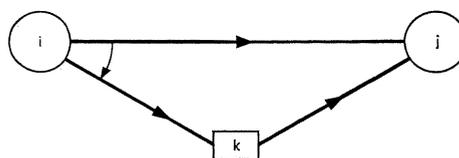


Figure 1 - Alternative routes for traffic: direct route ij and second choice route ikj through one tandem k .

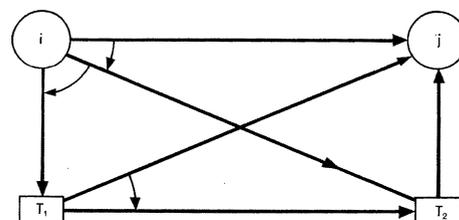


Figure 2 - A generalization of the program to cover the case of two tandems T_1 and T_2 in series.

2.2.1 Computation of Basic Solution

Using an empirical formula (see 3.2), an approximate number of direction junctions is calculated between all exchanges.

Based on these numbers, the overflow traffic is computed for all direct routes and from this the tandem routes, using the inverse of Wilkinson's formula.

The accuracy of the basic solutions affects the speed of convergence on the following iteration.

2.2.2 Iteration

From the basic solution, a series of iterations is calculated, iterations 1, 2, etc. Each iteration step results in a new, more economic network. This procedure continues until no more changes occur in the network between two successive iterations. Normally, between three and five iterations are required.

The new network for each iteration is derived by another series of iterations. Each pair of exchanges is opti-

mized taking into account the previously calculated tandem trunks. When a single pair is considered, it is impossible to isolate the overflow traffic from the traffic passing through a given tandem, and accordingly the calculations (for the pairs) are interdependent because of the tandem routes, which are common for several exchanges. A phenomenon analogous to that produced by mechanical couples can be observed.

2.2.3 Groups and Common Routes

For practical reasons, a "group" concept has been introduced. A group is a set of exchanges having common routes to or from a given tandem (typically, this would be exchanges located in the same building). It is then possible to speak of "originating groups" and "terminating groups".

In addition, the term "common route" has been introduced. A common route is a direct route used jointly by two or more originating exchanges (this decreases the number of junctions: all the traffic is assumed to be concentrated in one of the originating exchanges, while the rest of them are considered to have no traffic at all).

These two concepts are necessary for describing practical cases (see Figure 4).

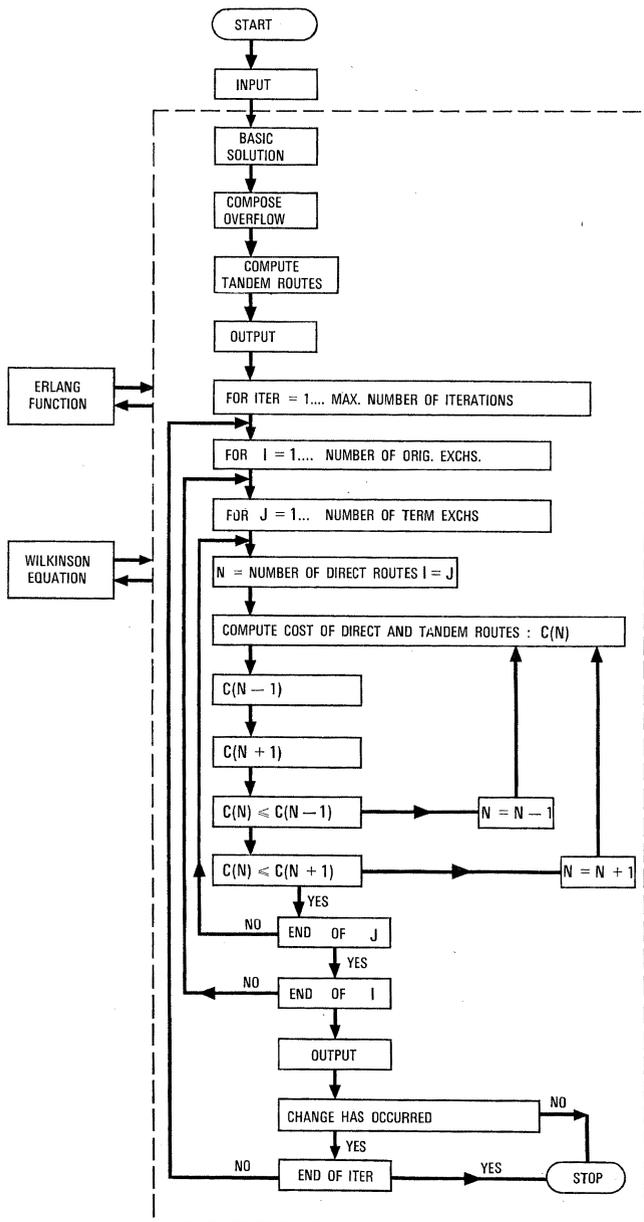


Figure 3 - Flow diagram for the computation.

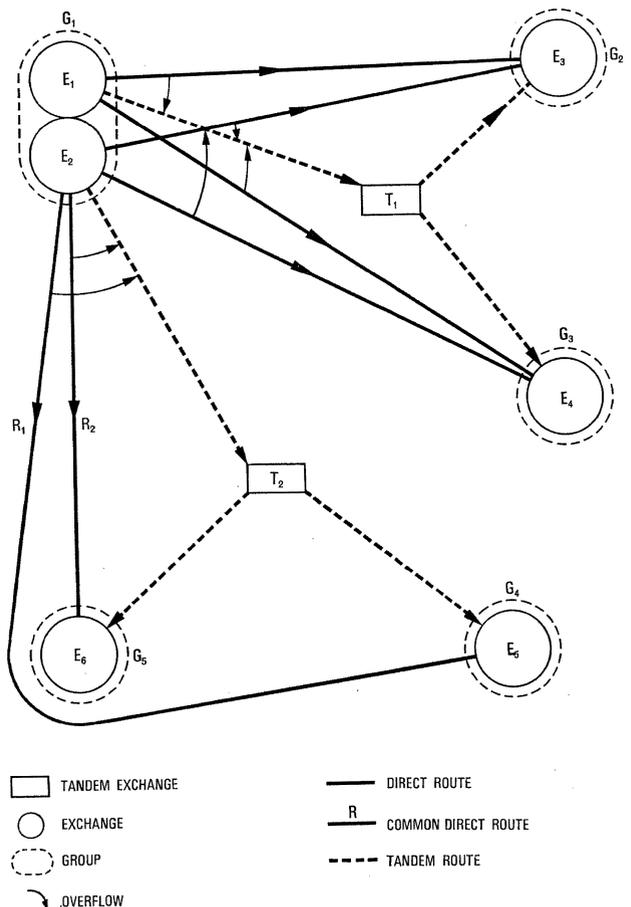


Figure 4 - The "group" and "common route" concepts needed to describe practical cases. In this figure, a small example is shown with six exchanges, two tandems, and five groups. Exchanges 1 and 2 form group 1, while the rest of the groups are formed by single exchanges.

R₁ is a common route from exchanges 1 and 2 to exchange 5; similarly, R₂ is to exchange 6. Only the routes concerning the traffic leaving group 1 have been shown.

3. Equations

3.1 Notations

Originating group	I
Terminating group	J
Originating exchange	i
Terminating exchange	j
Tandem	k
Traffic between i and j	A_{ij}
Cost of junction between i and j	C_{ij}
Cost of junction between i and k	C_{ik}
Cost of junction between k and j	C_{kj}
Ratio of costs of direct route and tandem route	ε
Mean marginal tandem switching cost (per erlang)	SW
Number of trunks between i and j	n_{ij}
Mean and variance of the traffic on tandem route	M_I and V_I
Overflow traffic	M_{ij}
Congestion on direct route	$E(A_{ij}, n_{ij})$
Conditional congestion on tandem route	E_{tl} and E_{tj}
Grade of service	E_{ij}

3.2 Formulas

For the first approximate basic solution, the number of trunks in a direct connection is given by n_{ij} , so that:

$$E(A_{ij}, n_{ij}) - E(A_{ij}, n_{ij} + 1) = \frac{\varepsilon}{A_{ij}} [1 - 0.3(1 - \varepsilon^2)] \quad (1)$$

where $E(A, n)$ represents the Erlang congestion function, and the ratio between the costs of the direct route and the tandem route

$$\varepsilon = \frac{C_{ij}}{C_{Ik} + C_{kJ} + \frac{1}{2} SW} \quad (2)$$

Congestion on the direct routes is evaluated by:

$$E(A_{ij}, n_{ij}) = \frac{A_{ij}^{n_{ij}}}{n_{ij}!} \sum_{P=0}^{n_{ij}} \frac{A_{ij}^P}{P!} \quad (3)$$

The means and variances of the tandem traffics are given by Wilkinson's formulas:

$$M_{ij} = A_{ij} \cdot E(A_{ij}, n_{ij}) \text{ and,} \quad (4)$$

$$V_{ij} = M_{ij} \left[1 - M_{ij} + \frac{A_{ij}}{n_{ij} + 1 + M_{ij} - A_{ij}} \right]$$

total traffic from a group to a tandem by

$$M_I = \sum_j M_{ij} \text{ and } V_I = \sum_j V_{ij} \quad (5)$$

total traffic from a tandem to a "group" by

$$M_J = \sum_i M_{ij} \text{ and } V_J = \sum_i V_{ij} \quad (6)$$

The pure-chance traffic, equivalent to the traffic on the tandem route, is calculated by inversion of Wilkinson's equations:

n_I and A_I are wanted, so that:

$$A_I \cdot E(A_I, n_I) = M_I \text{ and,} \quad (7)$$

$$1 - M_I + \frac{A_I}{M_I + 1 + n_I - A_I} = \frac{V_I}{M_I}$$

This inversion is done by means of the iteration procedure of the second degree, starting from an approximate initial value:

$$(A_I)_0 = V_I + 3 \frac{V_I}{M_I} \left(\frac{V_I}{M_I} - 1 \right) \quad (8)$$

The cost in the case of n direct trunks between i and j is:

$$Cn_{ij} = C_{ij} \cdot n_{ij} + C_{Ik} \cdot m_I + C_{kJ} \cdot m_J + (M_I + M_J) \frac{SW}{2} \quad (9)$$

The number of trunks between group/tandem is m_I , so that:

$$\frac{A_I \cdot E(A_I, n_I + m_I)}{\sum_j A_{ij}} = \frac{\bar{E}}{2} \quad (10)$$

where \bar{E} is the mean grade of service imposed upon the global network. An analog formula is used to calculate m_I (tandem/group).

The conditional congestion in group/tandem is such that:

$$E_{tl} = \frac{E(A_I, n_I + m_I)}{E(A_I, n_I)} = \frac{A_I \cdot E(A_I, n_I + m_I)}{\sum_j M_{ij}} \gg \frac{\bar{E}}{2} \quad (11)$$

The grade of service for each pair is given approximately by:

$$E_{ij} = E(A_{ij}, n_{ij}) \cdot [E_{tl} + E_{tj}] \quad (12)$$

and the mean grade of service by:

$$\sum_i \sum_j A_{ij} \cdot E_{ij} = \bar{E} \sum_i \sum_j A_{ij} \quad (13)$$

The degenerations for the tandem routes are defined as:

$$\frac{V_I}{M_I} \text{ and } \frac{V_J}{M_J} \quad (14)$$

4. Mathematical Tools

4.1 Erlang Function

The equations and formulas necessitate a continuous use of the Erlang congestion function. This function is defined by formula (3) for integer values of n , and by

$$E(A, X) = \frac{A^X \cdot e^{-A}}{\int_A^\infty e^{-t} \cdot t^X \cdot dt} \quad (15)$$

for positive values of X that may or may not be integers.

We will have to use this formula in two different ways: the calculation of the loss $P = E(A, X)$, with P and A being known, and for the computation of the inverse (number of trunks given loss and traffic).

For this, a very good approximation of the function itself is necessary, plus a way to obtain the results with the desired accuracy.

This led us to introduce a new method, based on a continued-fraction expansion of the Erlang function (related to the expansion of the incomplete gamma function). In fact:

$$E(A, X) = \frac{A^X \cdot e^{-A}}{\Gamma(1 + X, A)} \quad (16)$$

with

$$\Gamma(a, u) = \int_u^\infty e^{-t} \cdot t^{a-1} \cdot dt \quad (17)$$

Stieltjes has proved:

$$\Gamma(1-a, u) = e^{-u} \cdot u^{-a} \cdot F(u, a) \quad (18)$$

with

$$F(u, a) = \frac{1}{1} + \frac{a}{u} + \frac{1}{1} + \frac{a+1}{u} + \frac{2}{1} + \frac{a+2}{u} + \dots \quad (19)$$

Thus:

$$E(A, X) = \frac{1}{F(A, -X)} \quad (20)$$

This continued fraction has been studied. It converges rapidly, especially for $X < A$.

Using this form of F , we obtain successive even and odd approximations, which enclose the function by upper and lower bounds. The obtainable accuracy is limited only by the number of approximations computed and the number of digits in the calculation.

This method is especially advantageous in the evaluation of $E(A, X)$ on an electronic computer. The computation is automatically terminated when a certain precision has been obtained. This is of great value in programs of the type described in this paper, because in many cases the function has to be evaluated without knowing the parameters *a priori*. The entire network is computed using an identical precision independent of the number of trunks and the traffic.

In comparing this method with the approximate formulas established by Szybicki and Rapp, the following accuracies can be observed for $X = 1/2$, n is the number of approximations required to obtain the desired precision of 10^{-14} . The formulas of Szybicki and Rapp are not iterative, and their accuracy is variable.

A	Szybicki	Rapp	ITTL
1	$1 \cdot 10^{-2}$	$2 \cdot 10^{-4}$	$1 \cdot 10^{-14} \quad n = 73$
10	$2 \cdot 10^{-5}$	$4 \cdot 10^{-6}$	$1 \cdot 10^{-14} \quad n = 10$
100	$1 \cdot 10^{-9}$	$7 \cdot 10^{-10}$	$1 \cdot 10^{-14} \quad n = 4$

4.2 Inversion of the Erlang Function

Calculation of the number of trunks for a given loss and a given traffic will be made, using Newton's method. Assuming:

$$G = A \cdot E(A, X) - P. \quad (21)$$

We have:

$$\frac{\delta G}{\delta X} \sim -\frac{A}{2} \left[\frac{X}{A \{1 - E(A, X)\}} - \frac{A}{1 + X + A \cdot E(A, X)} \right] \times E(A, X). \quad (22)$$

Starting with an initial value X_0 , iterations are executed using the formulas:

$$(X)_{u+1} = (X)_u - \frac{(G)_u}{\left(\frac{\delta G}{\delta X}\right)_u} \quad (23)$$

where the convergency is good, in spite of the use of an approximate expression for G .

4.3 The Inversion of Wilkinson's Formula

We need this inversion in the calculation of the equivalent trunk group for pure chance traffic (7).

We put

$$H = A_i \cdot E(A_i, n_i) - M_i \quad (24)$$

and

$$B = \frac{\delta H}{\delta A_i} + \frac{1}{Q} \cdot \frac{\delta H}{\delta n_i} \quad (25)$$

with

$$Q = 1 - \frac{1}{M_i + \frac{V_i}{M_i}} \quad (26)$$

$$\frac{\delta H}{\delta A_i} = E(A_i, n_i) \cdot [1 - n_i - A_i + A_i \cdot E(A_i, n_i)]$$

and

$$\frac{\delta H}{\delta n_i} \sim \frac{A}{2} \left[\frac{n}{A_i [1 - E(A_i, n_i)]} - \frac{A}{1 + n_i + A_i \cdot E(A_i, n_i)} \right] \cdot E(A_i, n_i). \quad (27)$$

The starting values are:

$$(A_i)_0 = V_i + 3 \frac{V_i}{M_i} \left(\frac{V_i}{M_i} - 1 \right)$$

and

$$(n_i)_0 = \frac{A_0}{Q} - M_i - 1. \quad (29)$$

The following iterations are given by:

$$(A_i)_{u+1} = (A_i)_u - \frac{(H)_u}{(B)_u}$$

and

$$(n_i)_{u+1} = \frac{(A_i)_u}{Q} - M_i - 1. \quad (28)$$

The procedure is of the second order; there is a rapid convergence. The two procedures, used in the inversion of the formulas of Erlang and Wilkinson, are iterated until the desired precision is reached, for example, for the last case:

$$\left| \frac{(H)_u}{M_i} \right| \leq 5 \cdot 10^{-5}. \quad (30)$$

5. Program

5.1 General Characteristics

The program has been written according to the ITT programming standard concerning both software and hardware. Thus, many ITTE associated companies can execute the program on their own computer without any modification. The program is written in FORTRAN IV (E). It consists of 600 instructions, and requires a memory of approximately 19 000 bytes*.

Results and data require an additional 100 000 bytes of memory. Program and data are stored on disks, and the results are also put on disks during processing. This allows termination of the execution of the program at any chosen moment and continuation at a later point in time. At present, the program can deal with a network containing up to 100 exchanges and 12 tandems. These numbers can easily be increased: on a disk, only 40 out of the 200 available tracks are used.

* This term designates a group of 8 bits.

However, since the method is iterative, and taking into account the effect of all the exchanges upon one another, it is easy to see that the number of exchanges should not be too great; otherwise, the method would decrease in efficiency because of an excessively long calculation time. For a network containing 30 exchanges and 3 tandems, the computer time required to reach the optimum (4 or 5 iterations) is about one hour and a half on a standard computer.

5.2 Input/Output

The necessary input data are:

- the traffic matrix between exchanges,
- the costs of the cables between exchanges and exchanges/tandems.

These costs are indicated by a distance matrix, by a matrix giving the types of cables used, and by the prices of the types of cables,

- matrix of the minimum and maximum numbers fixed for the trunks, in the direct routes,
- the matrix of the tandems that will be used (second choice) on the overflow routes,
- grade of service,
- desired accuracy for the various mathematical procedures,
- marginal mean switching cost per distributed erlang, in a tandem.

The results are presented in tables. There is a table for each originating exchange to the other exchanges, for each exchange to the various tandems, and for each tandem to the exchanges.

After each iteration the following costs are printed out:

- cost of trunks,
- cost of switching,
- overall cost.

The results for the direct routes are:

- number of trunks (as an integer),
 - total traffic offered,
 - congestion,
 - traffic carried by direct route,
 - overflow traffic,
- the total is given for each column.

The results for the tandem routes are:

- number of trunks,
 - traffic: — mean
 - variance
 - degeneration,
 - congestion,
- the total is given for each column.

The results are printed out on a line printer after each iteration. The program is terminated automatically when after one iteration step the variation does not exceed one trunk compared to the previous iteration step.

6. Generalization

The program described above was recently generalized in order to handle more than one alternative route between two exchanges. Up to two tandems can be used in series (see Figure 2).

Traffic from exchange i to exchange j may use any of the following alternatives:

- the direct route $i-j$,
- the tandem routes $i-T_2$ and T_2-j ,
- the tandem routes $i-T_1$ and T_1-j ,
- the tandem routes $i-T_1$, T_1-T_2 , and T_2-j .

The algorithm used in the solution of the problem is similar to that shown above.

7. Criticisms and Limitations of the Method

The program described in this paper should be considered only as a tool for assisting the engineer in his work. We do not claim to give definitive solutions or hypotheses. With this method as a basis, the engineer can decide on the final network for the project by taking into account certain constraints that have not been used in the calculations.

In the present state of the study, we are considering only unidirectional junctions. This is not a serious limitation, and can easily be removed.

The assumption that all exchanges have full availability is more important. In many practical cases, grading has to be taken into account. Certain theoretical problems, permitting the solving of the difficulties caused by grading, have not yet been solved to our satisfaction, for example, in the case of tandems and of second-choice routes.

The cost of a junction is calculated by multiplying the distance between exchanges, and the price per kilometer of the type of cable used. This, however, assumes the availability of cable types providing any number of trunks. In practice, these cables are standardized and contain definite numbers of trunks. A way should be found to take this into account, and even to determine the appropriate cable type in terms of the length, (attenuation, losses . . .) and of the kinds of exchanges and tandems involved (Rotary or Pentaconta systems).

The routing matrix is imposed from the beginning, together with the location of the tandems.

All these points are under study, and we expect before long to be able to introduce a more general method, which will be both more refined and more closely related to a technical reality.

8. Practical Applications

The computer program has been successfully applied to a number of practical cases.

The first application was for Nobelville, an imaginary town with currently 250 000 subscribers (see Figures 5 and 6). Plans were required for an intermediate step and for the final step. The number of subscribers had to be increased immediately by 150 000. The mean traffic per subscriber was 0.1 erlang. Both Rotary and Pentaconta exchanges were used. There were 30 exchanges (24 groups) and three tandems. Computer time was approximately 1.5 hours on a standard computer.

9. Acknowledgments

The authors wish to thank Mr. B. Canceill and Mr. G. Lévy-Soussan for their important contribution to this article.

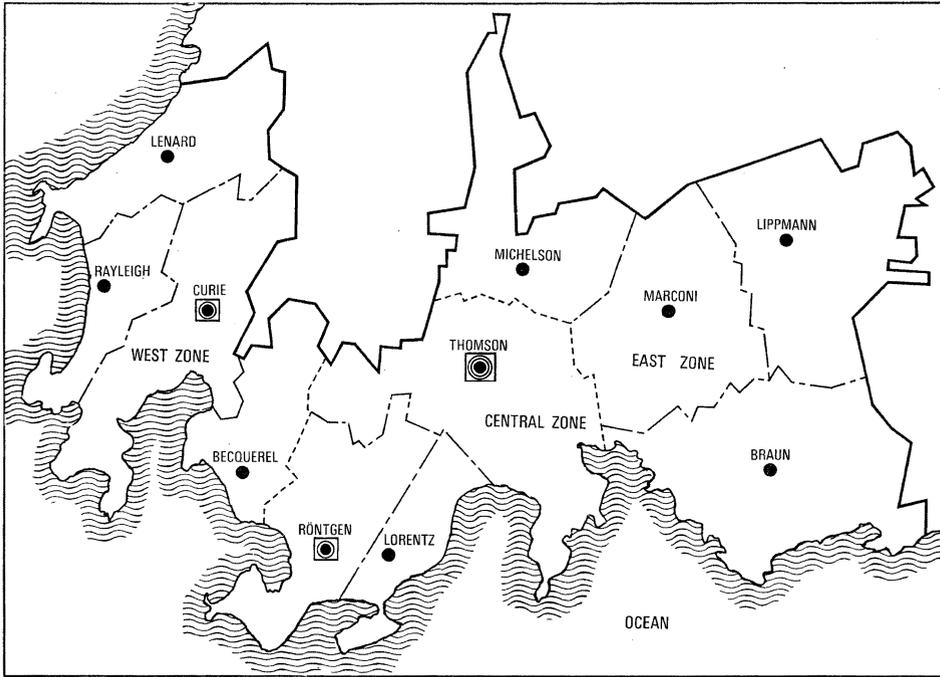


Figure 5 - The map of Nobelville, an imaginary network, where the first practical application of the computer program was made.

	ROTARY	PENTAONTA	
RÖNTGEN	22	10 400	TANDEM 2
	42	10 000	
	32	10 000	
	52	10 000	
	31	4400	
CURIE	26	10 100	TANDEM 1
	46	10 000	
THOMSON	28	10 000	TANDEM 3
	48	10 200	
	94	10 000	
	54	4230	
LORENTZ	23	8760	
	43	10 400	
BRAUN	30	10 200	
		40	
RAYLEIGH	37	10 300	
	56	10 000	
MARCONI		39	10 400
		59	10 400
MICHELSON	38	10 100	
	58	10 000	
LENARD	27	10 000	
	47	10 300	
LIPPMANN	29	8000	
	49	10 200	
BECOUEREL	25	10 200	
	45	10 000	
MICHELSON	62	8320	
MARCONI		69	7280
MICHELSON	68	7280	
	88	5200	
LENARD	67	10 400	
	87	5200	
LIPPMANN			
BECOUEREL	35	10 400	
	55	5200	

Figure 6 - A schedule of exchanges giving number of lines and exchange dialing codes for Nobelville.

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Numerical Evaluation of the Erlang Function through a Continued-Fraction Algorithm

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1. Introduction

The usefulness of the Erlang function is well known in the field of telephone traffic problems. Various approaches exist in the literature for its numerical evaluation; on the one hand for integer values of the number of devices x through a recursive relation, and on the other hand through approximate formulas for non-integer values of x . These approaches have proved valuable in current practice and have made it possible to compute tables. Nevertheless there is a need for improved methods of evaluation for application to specific problems, particularly the optimization of telephone networks, and generally speaking when one has to solve, by successive iterations, a system of implicit equations containing this function $E(A, x)$.

Present methods do not use a continuous algorithm that would make it possible to obtain the requisite accuracy by selecting the appropriate number of terms in, for instance, a convergent series or a continued fraction or even an asymptotic expansion. The usefulness of the continuous algorithm lies in its ready application to computer processing.

The expansion of the Erlang function in a continued fraction from the standpoints of existence, convergence and rate of convergence is therefore the purpose of this paper.

2. Definitions

2.1 Erlang Function

The Erlang function is defined by:

$$E(A, x) = \frac{A^x e^{-A}}{\int_A^\infty e^{-t} t^{x-1} dt} \quad (1)$$

where x and A are real and positive numbers. An equivalent form is

$$E(A, x) = \int_0^\infty \left(1 + \frac{t}{A}\right)^x e^{-t} dt. \quad (2)$$

The function $E(A, x)$ is the probability of loss in a full-availability group of x devices serving on a lost-calls-cleared basis and given an offered traffic of A erlangs.

For integer values of x , this function obeys the recursive equation

$$E(A, x+1) = \frac{A \cdot E(A, x)}{x+1+A \cdot E(A, x)} \quad (3)$$

where

$$E(A, 0) = 1$$

which holds good also for non-integer values of x .

2.2 Continued Fractions

Given the function $f(x_0, x_1, \dots, x_p)$, let us denote the continued fraction expansion by the expression:

$$f(x_0, x_1, \dots, x_p) = F(x_0, x_1, \dots, x_p)$$

$$\equiv b_0 + \frac{a_1}{b_1 + \frac{a_2}{b_2 + \frac{a_3}{b_3 + \frac{a_4}{\dots}}}} \quad (4)$$

where all a_i 's and b_i 's are functions of the real or complex variable x_j . This continued fraction will be expressed more readily under the following symbolism:

$$F(x_i) = b_0 + \frac{a_1|}{|b_1|} + \frac{a_2|}{|b_2|} + \dots + \frac{a_n|}{|b_n|} + \dots \quad (5)$$

When both a_i and b_i sequences have only a finite number of terms the continued fraction is said to be limited; if not, it is said to be unlimited. Only a finite number of a_i and b_i terms may be equal to zero. The following is a limited continued fraction,

$$F_n(x_i) = b_0 + \frac{a_1|}{|b_1|} + \frac{a_2|}{|b_2|} + \dots + \frac{a_n|}{|b_n|} \equiv \frac{A_n|}{|B_n|} \quad (6)$$

where A_n and B_n are polynomials in a_i and b_i , and is called n^{th} approximant of $F(x_i)$. Now A_n and B_n are, respectively, the numerator and the denominator of the n^{th} approximant. They obey the following recursive relations

$$\begin{aligned} A_n &= b_n A_{n-1} + a_n A_{n-2}, \text{ and} \\ B_n &= b_n B_{n-1} + a_n B_{n-2}. \end{aligned} \quad (7)$$

$F_{2n}(x_i)$ is the approximant of even order and $F_{2n+1}(x_i)$ the approximant of odd order. In some cases, it is possible to derive two continued fractions $P(x_i)$ and $I(x_i)$, the successive approximants of which are, respectively, the even and odd approximants of $F(x_i)$.

2.3 Convergence and Rate of Convergence

We say that the continued fraction

$$F(x_i) = b_0 + \frac{a_1|}{|b_1|} + \dots + \frac{a_n|}{|b_n|} + \dots \quad (8)$$

converges towards the function $f(x_i)$, if at the most a finite number of its numerators A_p vanish, if the sequence of its approximants A_n/B_n tends to a finite limit, and if:

$$\lim_{n \rightarrow \infty} (A_n/B_n) \equiv f(x_i)$$

If these conditions are not fulfilled the continued fraction is said to be divergent. A divergent continued fraction is not assigned any value. It may either grow indefinitely or oscillate between two or more values.

A continued fraction converges uniformly in a domain D if it converges for all values of the variables in D and if the sequence of its approximants converges uniformly in D .

We shall call the rate of convergence the ratio of the number of significant digits obtained by the order of the approximant providing these digits. The higher this ratio, the quicker is the convergence.

Evaluation of Erlang Function

The error ε_n with the n^{th} approximant reads:

$$\varepsilon_n(x_i) = f(x_i) - F_n(x_i). \quad (9)$$

Then the number of significant digits α will be:

$$\frac{\varepsilon_n(x_i)}{F_n(x_i)} = \varrho \cdot 10^{-\alpha} \quad (10)$$

where $|\varrho| < 1$ and ϱ is a mantissa

and the rate of convergence r is

$$r = \frac{\alpha}{n} = \log_{10} \left[\frac{\varrho \cdot F_n(x_i)}{f(x_i) - F_n(x_i)} \right] \frac{1}{n}. \quad (11)$$

3. Continued-Fraction Expansion of $E(A, x)$

3.1 General

The incomplete gamma function is defined as

$$\Gamma(a, x) = \int_x^\infty e^{-t} t^{a-1} dt \quad (12)$$

where $\text{Real}(a) > 0$ and $\text{Real}(x) > 0$.

Then $E(A, x)$ can be expressed in terms of $\Gamma(1+x, A)$ as follows

$$E(A, x) = \frac{A^x e^{-A}}{\Gamma(1+x, A)} \quad (13)$$

provided $1+x > 0$ and $A > 0$.

It has been proved (Stieltjes, Wall) that the incomplete gamma function can be expanded into the following convergent continued fraction

$$\Gamma(1-a, x) = e^{-x} x^{-a} \times \left[\frac{1}{1} + \frac{a}{x} + \frac{1}{1} + \frac{a+1}{x} + \frac{2}{1} + \frac{a+2}{x} + \dots \right] \quad (14)$$

of which the general term $\frac{a_i}{b_i}$ is expressed by

$$\begin{aligned} a_{2K} &= a + K - 1 & b_{2K} &= x \\ a_{2K+1} &= K & b_{2K+1} &= 1 \\ a_1 &= 1 & b_0 &= 0. \end{aligned} \quad (15)$$

Let us write

$$F(x, a) = \frac{1}{1} + \frac{a}{x} + \frac{1}{1} + \frac{a+1}{x} + \frac{2}{1} + \dots \quad (16)$$

We thus obtain the continued-fraction expansion of $E(A, x)$:

$$E(A, x) = \frac{1}{F(A, -x)}. \quad (17)$$

3.2 Odd and Even Approximants

Since the continued fraction $F(a, x)$ can be contracted, we can write down the continued fractions $P(A, -x)$ and $I(A, -x)$, giving respectively the even and odd approximants of $E(A, x)$:

$$E(A, x) = \lim_{n \rightarrow \infty} \left[\frac{1}{P_n(A, -x)} \sim \frac{1}{I_n(A, -x)} \right] \quad (18)$$

with

$$\begin{aligned} P_n(A, -x) &= \frac{A}{A-x} + \frac{x}{A-x+2} + \frac{2(x-1)}{A-x+4} \dots \\ &+ \frac{n(x-n+1)}{A-x+2n} \end{aligned} \quad (19)$$

and

$$\begin{aligned} I_n(A, -x) &= 1 + \frac{x}{A+1-x} + \frac{x-1}{A+3-x} \dots \\ &+ \frac{(n-1)(x-n+1)}{A+2n-1-x}. \end{aligned} \quad (20)$$

Simultaneous computation of $P_n(A, -x)$ and $I_n(A, -x)$ is particularly convenient because in the domain of convergence they give both the upper and lower bounds of the required value of the function.

Three cases are to be considered:

- Case 1: $1/P_n(A, -x) < E(A, x) < 1/I_n(A, -x)$
- Case 2: $1/I_n(A, -x) < E(A, x) < 1/P_n(A, -x)$ whatever n may be in both cases.
- Case 3: Cases 1 and 2 occur alternately for increasing n .

An upper and a lower bound of the function are then obtained at each computation step. According to Euler, the odd and even approximants give a better approximation to the function than the corresponding asymptotic expansion.

4. Results

4.1 Computation Algorithms, Cost per Approximant

The computation may proceed from either the end or from the beginning of the continued fraction. If the latter, the recursive relation (7) has to be used but stability is not guaranteed and the result obtained is the ratio of two almost-equal large numbers, which is not acceptable in numerical analysis.

We therefore preferred the algorithm "starting from the end", which yields the readily programmed sequence:

$$\begin{aligned} &b_n \\ &a_n/b_n \\ &b_{n-1} + a_n/b_n \\ &a_{n-1}/(b_{n-1} + a_n/b_n). \end{aligned}$$

The cost of the simultaneous computation of the n^{th} approximants I_n and P_n is

$$n(1 \text{ multiplication} + 2 \text{ divisions}).$$

With the type of computer used, the required time, with double precision, amounts to $1.2 \times n$ milliseconds.

For example the following computation times have been obtained:

$$\begin{aligned} &61.2 \text{ milliseconds for } E(1, 10^{-3}) \\ &12.0 \text{ milliseconds for } E(10, 0.9) \\ &4.8 \text{ milliseconds for } E(100, 0.9). \end{aligned}$$

These times are valid for our computer, using Fortran IV double precision (16 significant figures). The computation sequence is stopped when the first of the following two conditions is met:

$$\begin{aligned} &\text{— if } n = 200 \\ &\text{— if } \left[\frac{I_n - P_n}{I_n} \right] < 10^{-14} \end{aligned}$$

which ensure a minimum of 14 equal significant digits for I_n and P_n .

The printing of results is in floating-point form with 11 significant digits.

The results have been compared with the values given in C. Palm's table.

The range explored was the following:

- for A , between 10^{-2} and 10^6 , for integer powers of 10,
- for x , between 10^{-5} and 10^6 , for integer powers of 10, for the values of 5×10^i , and for some particular integer or fractional values.

4.2 Practical Convergence Domain

We are sure that the continued-fraction expansion of $E(A, x)$ converges for all the positive values of A and x .

Nevertheless, when x is an integer, the continued fraction is a limited one, with $2x$ terms (that is, for I_n and P_n , $n = x$). It can be shown that the formal identity $I_x = P_x$ applies. The maximum obtainable accuracy is then reached; it depends on the rounding errors according to the number of digits with which computation has been performed. These errors may vary with A and x .

The accuracy obtained is in fact rather good in the "practical convergence domain", which is defined below: it amounts to 14 significant figures, for example, in the following cases with n^{th} approximants,

- $x = 5 \quad A = 10 \quad n = 5$
- $x = 10 \quad A = 10 \quad n = 10$
- $x = 10 \quad A = 10^2 \quad n = 5$
- $x = 10^2 \quad A = 10^3 \quad n = 4.$

For $x > A > 1$, convergence becomes very slow, and there are cases for which accuracy cannot be estimated because one of the approximants I_n or P_n is either divergent or convergent to a negative value. The limit for which convergence becomes too slow is not easy to define. The only result we have obtained is that such a limit exists only if $x > A$, and these values of x vary with A .

In order to perform practical applications, we must then define a practical convergence domain, as follows:

- $A > 0$: as large as desired
- $0 < x < A$: x must be bounded by the values of A .

4.3 Rate of Convergence Against A

The convergence rate of I_n or P_n increases rapidly with A . Figure 1 shows on a semi-logarithmic scale the number of true significant digits of I_n , in terms of n , for the following values of A :

- $10^{-1}, 10^0, 10^1, 10^2, 10^3, 10^4.$

The value of x has been kept constant, equal to 0.9. These curves have been drawn from Table 1.

It may be noted that the 4th approximant already gives 14 true significant figures for $A \geq 100$.

For $A \geq 1$, $n = 68$ is enough to insure these 14 significant figures; and for $A \geq 0.1$, only 7 figures are obtained with $n = 200$.

4.4 Rate of Convergence Against x

The rate of convergence of I_n or P_n decreases when x is increased, and we have said that a practical limit was $x \leq A$ when $A > 1$.

Figure 2, drawn from Table 2, shows in terms of n the number of significant figures obtained for I_n , again on semi-logarithmic scales. The values chosen were $A = 10^{-1}$ and:

- $x = 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 1.05.$

We chose the last value $x = 1.05$ in order to avoid the singular value $x = 1$ for which the continued fraction is limited and converges very quickly ($n = 1$ already gives 14 true digits).

The rate of convergence increases when x is decreased, but in a less marked way than its increase with A .

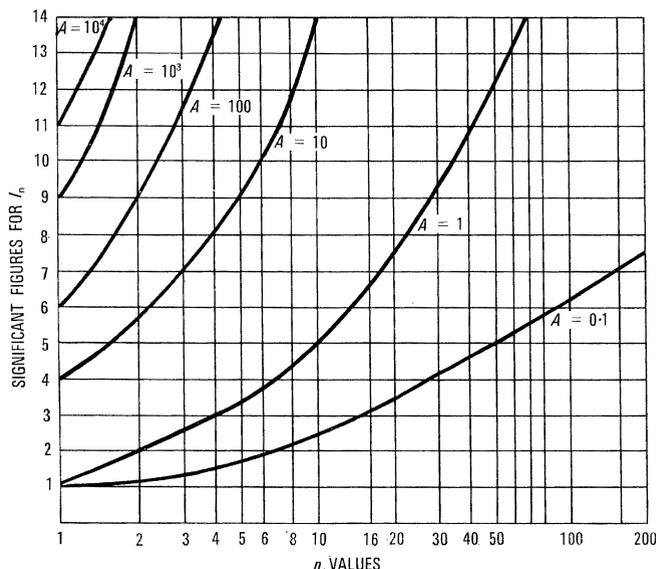


Figure 1 - The true significant digits for I_n for values of A , $x = 0.9$.

Table 1

$x = 0.9, A_1 = 0.1, A_2 = 1, A_3 = 10, A_4 = 10^2, A_5 = 10^3, A_6 = 10^4$

Exact figures	A_1		Exact figures	A_2		Exact figures	A_3	
	$n(I)$	$n(P)$		$n(I)$	$n(P)$		$n(I)$	$n(P)$
1	1	7	1	1	1	1	↓	↓
2	↓	9	2	2	↓	2	↓	↓
3	15	29	3	↓	3	3	↓	↓
4	30	58	4	6	6	4	1	1
5	52	↓	5	11	8	5	↓	↓
6	83	112	6	14	12	6	↓	2
7	151	125	7	15	↓	7	3	3
8	173	178	8	20	↓	8	4	4
9			9	26	29	9	↓	5
10			10	33	34	10	6	↓
11			11	39	44	11	7	6
14			14	68	68	14	10	10

Exact figures	A_4		Exact figures	A_5		Exact figures	A_6	
	$n(I)$	$n(P)$		$n(I)$	$n(P)$		$n(I)$	$n(P)$
1	↓	↓	1	↓	↓	1	↓	↓
2	↓	↓	2	↓	↓	2	↓	↓
3	↓	↓	3	↓	↓	3	↓	↓
4	↓	↓	4	↓	↓	4	↓	↓
5	↓	↓	5	↓	↓	5	↓	↓
6	↓	↓	6	↓	↓	6	↓	↓
7	↓	↓	7	↓	↓	7	↓	↓
8	↓	↓	8	↓	↓	8	↓	↓
9	↓	↓	9	↓	↓	9	↓	↓
10	↓	↓	10	↓	↓	10	↓	↓
11	3	2	11	↓	1	11	1	1
14	4	4	14	2	2	14	2	2

We must go up to $n = 100$ so as to obtain the exact figures 9, 8, 7, 6, 5 and 5 for these values of x . It should be noted that the rate of convergence increases approximately as the logarithm of (A/x) .

Figure 3 and Table 3 are the corresponding ones for $A = 1$ and the same values of x . The general shape is similar but the slope of the curves has increased. The curve $x = 1.05$ is steeper than the others; this is most likely due to its neighborhood to $x = 1$, that is, $x = A$.

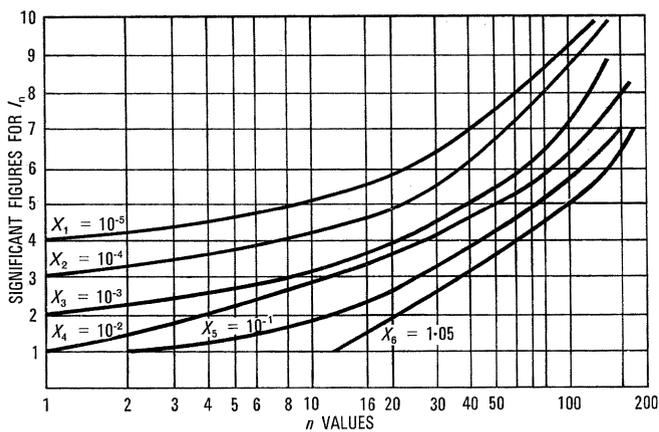


Figure 2 - Significant figures for I_n when $A = 10^{-1}$.

Table 2

$A = 10^{-1}$, $x_1 = 10^{-5}$, $x_2 = 10^{-4}$, $x_3 = 10^{-3}$, $x_4 = 10^{-2}$, $x_5 = 10^{-1}$, $x_6 = 1.05$

Exact figures	x_1		Exact figures	x_2		Exact figures	x_3	
	$n(I)$	$n(P)$		$n(I)$	$n(P)$		$n(I)$	$n(P)$
1			1			1		
2			2			2	1	1
3			3	1	1	3		4
4	1	1	4		4	4	24	13
5		4	5	23	14	5	37	31
6	23	14	6	33	33	6		51
7	33	33	7	57	58	7	95	84
8	54	62	8	80	104	8	114	158
9	97	83	9	153	116	9		
10	121	130	10			10		
11			11			11		
14			14			14		

Exact figures	x_4		Exact figures	x_5		Exact figures	x_6	
	$n(I)$	$n(P)$		$n(I)$	$n(P)$		$n(I)$	$n(P)$
1	1	1	1	2	10	1	11	11
2	2		2	12	17	2	14	
3	10		3	26	43	3	37	41
4	27	46	4	56	55	4	67	62
5	55	59	5	77		5	87	100
6	86	90	6	112	140	6		123
7	130	122	7	198	154	7	175	174
8	165	173	8			8		
9			9			9		
10			10			10		
11			11			11		
14			14			14		

Already for $n = 10$ we have obtained the exact figures 9, 8, 7, 6, 5 and 5.

A comparison of Figures 2 and 3 may also show the rate of convergence against A : the same number of true digits obtained with $n = 10$ for $A = 1$ and $n = 100$ for $A = 0.1$.

4.5 Upper and Lower Bounds Provided by P_n and I_n

In the practical convergence domain, the approximants P_n and I_n always provide an upper and lower bound to $E(A, x)$.

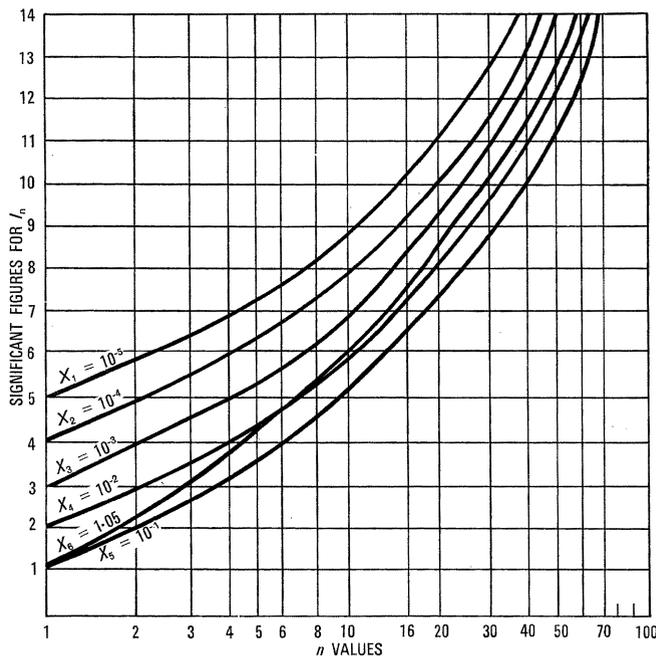


Figure 3 - Significant figures for I_n when $A = 1$.

Table 3

$A = 1$, $x_1 = 10^{-5}$, $x_2 = 10^{-4}$, $x_3 = 10^{-3}$, $x_4 = 10^{-2}$, $x_5 = 10^{-1}$, $x_6 = 1.05$

Exact figures	x_1		Exact figures	x_2		Exact figures	x_3	
	$n(I)$	$n(P)$		$n(I)$	$n(P)$		$n(I)$	$n(P)$
1			1			1		
2			2			2		
3			3			3	1	1
4			4	1	1	4	2	4
5	1	1	5	2	4	5	4	4
6	2		6	4	4	6	8	6
7	4	4	7	7	6	7	10	10
8	7	6	8	10	9	8	13	15
9	10	10	9	13	16	9	18	26
10	14	14	10	19	18	10	23	27
11	17	25	11	24	24	11	30	29
14	37	37	14	43	43	14	51	51

Exact figures	x_4		Exact figures	x_5		Exact figures	x_6	
	$n(I)$	$n(P)$		$n(I)$	$n(P)$		$n(I)$	$n(P)$
1			1	1	1	1	1	
2	1	1	2	2	3	2	2	1
3	2		3	3	4	3	3	3
4	4	4	4	7	6	4	4	5
5	7	7	5	9		5	5	7
6	10	10	6	13	16	6		
7	13		7	18	18	7	15	15
8	17	22	8	22	26	8	20	20
9	23	26	9	28	33	9	26	27
10	28		10	35	39	10	31	32
11	36	36	11	48	41	11	38	38
14	60	60	14	69	69	14	65	68

Case 1: If $2p < x < 2p + 1$, the approximant $I_n(A, -x)$ gives an upper bound and the approximant $P_n(A, -x)$ a lower bound to $1/E(A, x)$.

Case 2: If $2p - 1 < x < 2p$, the roles of I_n and P_n are exchanged.

Case 3: If x is an integer, I_n and P_n provide alternately an upper and a lower bound. The even approximants of $I(A, -x)$ and the odd approximants of $P(A, -x)$ provide upper bounds; the odd ones of I and even ones of P provide lower bounds.

Figure 4 illustrates these 3 cases, showing the peculiar approach occurring in Case 3.

4.6 Computing Method of $E(A, x)$ using Continued Fractions

We have seen that for $x \leq A$, the algorithm of continued fractions could be used in a straightforward manner since the convergence is then regular and fast. But in usual traffic calculations, x is greater than A . Let us then write $x = p + \vartheta$, ϑ being the fractional part of x and $p > A$.

We compute $E(A, A-1 + \vartheta)$ through the algorithm of continued fractions, and we then use the classical recurrence formula to obtain $E(A, x)$. By doing so we keep the advantages of the continued-fraction's controlled accuracy and we can calculate $E(A, x)$, whatever the value

of x . We save $A - n$ steps of the classical recurrence by reference to its application from the beginning, n being the order of the approximant providing the requisite accuracy. This does not always correspond to machine time-saving, because the cost of one recurrence stop is only 1 multiplication + 1 division.

There is indeed an optimum point to be chosen for keeping the application of continued fractions in a domain where they converge fast enough.

5. Comparison with Other Computing Methods

In the determining of a needed value of $E(A, x)$, there is a drawback, that is, no useful series expansion has been yet found for this function. For non-integer values of x , approximate formulas must be used, the accuracy of which may be questioned.

5.1 Classical Recurrence

This can be applied with known accuracy only when the starting value is known, that is, for integer values of x . All existing tables are based on its use. R. Fortet and L. Gautier have shown that rounding-off errors increased only slowly with x . It is also useful once a starting value is known for non-integer values of x .

5.2 Rapp's Method

This is an extension of the classical recurrence starting from $E(A, \vartheta)$ where ϑ is the fractional part of x . Y. Rapp proposed calculations of $E(A, \vartheta)$ through a parabolic approximation formula. He evaluates the accuracy of this approximation by comparison with $E(A, 0.5)$, because it is true that the error should be maximum for $\vartheta = 0.5$. But he already uses an approximation of $E(A, 0.5)$ which can only be said to be good for a large A . The values of the approximation may then be questionable for low values of A .

5.3 Szybicki's Method

It differs from the preceding one only by the type of approximation formula — three points are used, and it is valid up to $x = 2$.

5.4 Comparisons

It may be of interest to calculate with accuracy the error around exact values through Rapp's and Szybicki's methods, and through continued fractions, for $x = 0.5$ and a range of values of A .

In each case, we checked the convergence of the last method referred to.

Table 4 throws some light on these differences:

Table 4

A	Szybicki	Rapp	Lévy-Soussan
0.1	1×10^{-1}	4×10^{-2}	2×10^{-8} $n = 200$
1	1×10^{-2}	2×10^{-4}	1×10^{-14} $n = 73$
10	2×10^{-5}	4×10^{-6}	1×10^{-14} $n = 10$
100	1×10^{-9}	7×10^{-10}	1×10^{-14} $n = 4$

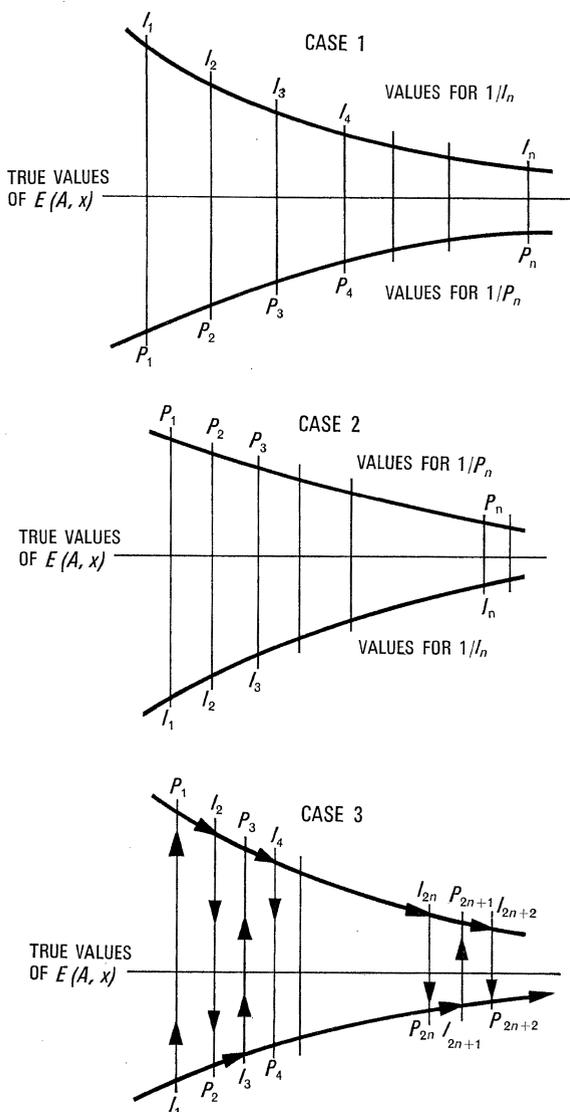


Figure 4 - Upper and lower bounds provided by P_n and I_n .

It is clear that for some values of A , the errors are not of the same magnitude, so that the use of approximate formulas is not desirable when computers are used.

Moreover, an appreciation of the domain in which they can be useful, and outside which they could produce erroneous results, cannot easily be made by the computer. In network optimization, for example, the subroutine "computation of $E(A, x)$ " is frequently used within iterated processes without any knowledge of the intermediate values of A and x . The accuracy obtained in the final result is then unknown, a condition that may not be tolerable in some cases.

Seemingly it would appear more advantageous to use an algorithm with automatic stop when the needed accuracy is reached even if it costs more in machine time.

6. Conclusion

The following advantages of the continued-fractions algorithms for numerical evaluation of $E(A, x)$ may then be summarized:

- it is applicable for all values of x , integer or fractional,
- it can easily be programmed on a computer,
- it guarantees any required accuracy because it provides an upper and lower bound of the function.

We have shown how the use of a continued-fraction expansion could prove useful when no classical means are available for the numerical evaluation of a function.

The algorithm presented above is convenient because it is usable mostly on presently available computers. And even when a series expansion exists and converges it

has been proved (Lévy-Soussan) that both methods were complementary: the rate of convergence of one increases when that of the other decreases.

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Guy Lévy-Soussan was born in 1936 at Casablanca, Morocco. He received his engineer's diploma from the Ecole Nationale Supérieure at Besançon, France, and subsequently his doctor's degree in Applied Mathematics from the Institut de Mathématiques Appliquées, Grenoble, France.

From 1962 to 1966, he worked at the Oceanographic Institute of Monaco under the supervision of Captain Cousteau. Among his various functions, he was notably head of the Data Processing Center for the "Précontinent III" Project.

From 1966 to 1967, he was Project Manager for the CADE group of the ITT Laboratories of Spain, in Madrid.

He is now Technical Advisor for Informatics in Monaco.

European Color Television Standards - The SECAM System

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1. Introduction

The generation of European color television standards from the American NTSC system was outlined in an article published in the preceding issue. Brief descriptions of the NTSC and PAL systems were given explaining their use of subcarrier quadrature amplitude modulation. The present article describes the French SECAM system which unlike the other systems uses frequency modulation for the transmission of chrominance information.

2. Main Features

The SECAM system (in French, *séquentiel à mémoire*) was originally invented by H. de France. It has since benefited from several improvements, which led to the "optimized" version presented at the CCIR session in Vienna in 1966.

Like other systems, the SECAM system transmits one luminance signal E_Y and two chrominance signals E_{DR} and E_{DB} , but it differs fundamentally in that the latter two signals modulate the chrominance subcarriers alternately, one in the course of the duration of one line, the other in the course of the next line, and so on.

At the transmitter (Figure 1), a switch actuated at the line frequency connects the modulator to channels E_{DR} and E_{DB} successively.

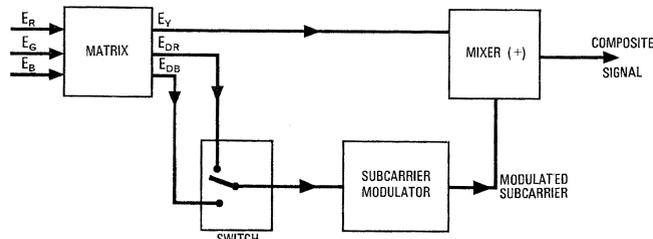


Figure 1 - The SECAM transmitter: part of the simplified diagram.

On the receiver side (Figure 2), the detected composite signal finds two paths, one direct and the other delayed by one line duration through an ultrasonic delay line. A switch then follows directing the two color signals simultaneously to different paths. After demodulation, these signals are fed into a matrix that recovers the three color signals in their original form.

For optimum advantage to be derived from the system and, especially, from the frequency-modulation technique used, the chrominance signals undergo complementary treatments which will be described in the next two sections.

3. Encoder

Figure 3 shows the block diagram of the encoder that converts the three primary signals into composite signals. The former are applied to a matrix that generates, by linearly combining these signals, the luminance signal E_Y which is then passed to the output mixer, and the chrominance signals E_{DR} and E_{DB} for further processing. After passing through a low-pass filter and a pre-emphasis circuit, the latter signals are applied successively to the frequency modulator by a switch operated at the line frequency. Pre-emphasis is used for the same reason as for frequency-modulated sound transmission. This means that the amplitude of higher-frequency modulating signals is increased with respect to the lower-frequency components to improve the signal-to-noise ratio in that part of the spectrum where this is of greatest advantage.

In the modulator, the subcarriers are modulated by the corresponding chrominance signals. Two different subcarrier frequencies are used for red and blue, respectively, in order to achieve the optimum signal-to-noise ratio for the various colors. These frequencies are multiples of the line frequency; this makes it possible, by suitable periodic phase shifts according to the lines concerned, to minimize the interference due to the sub-

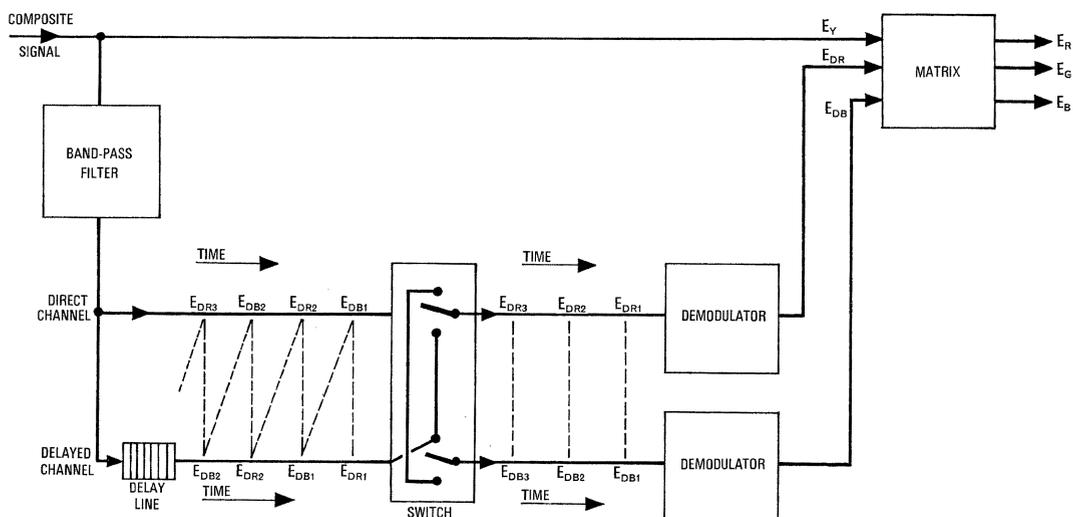


Figure 2 - The SECAM receiver: part of the simplified diagram.

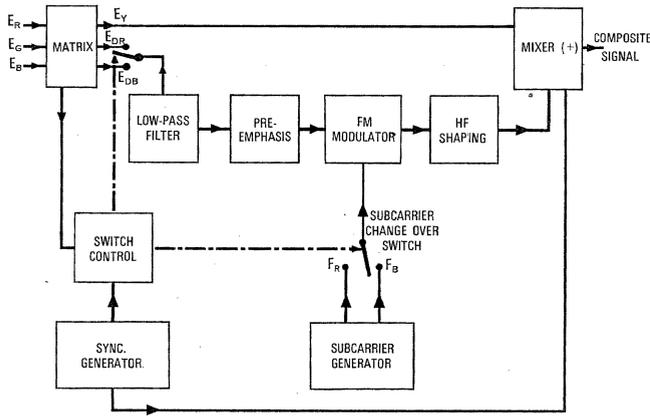


Figure 3 - The SECAM encoder.

carriers affecting the compatible black-and-white picture. In addition, an inverse bell-shaped characteristic at the modulator output reshapes the amplitude of modulated signals and protects against noise and interference despite the low level of the subcarrier.

To ensure that, at the receiver, the signals will be directed to the chrominance channels with the correct phase, line-identification signals are introduced in the intervals between field scans. During these time slots, the subcarrier is transmitted with increasing amplitude in the positive or negative direction, depending on the corresponding chrominance. At the receiver, these signals

appear with a polarity that depends on whether or not the switch operates with the correct phase. Automatic restoration of the correct phase is effected on the basis of this polarity.

4. Decoder

Figure 4 shows the block diagram of the decoder. The composite signal delivered by the detector of the receiver is applied both to a luminance amplifier, the output of which is connected to the cathodes of the picture tube, and to the chrominance channel.

The latter consists, successively, of a band-pass filter, a circuit compensating the amplitude shaping performed at the transmitter, and an amplifier that is blocked when colorless signals are received and activated by the action on a bistable circuit of the identification signals emitted during the flyback of the field scan.

Two outputs from the amplifier go to an electronically-operated changeover switch; one is direct, and the other is delayed by the duration of one line scan by an ultrasonic delay line followed by an amplifier compensating its attenuation. The changeover switch directs the incoming signals to their respective DR and DB paths. Each of these consists, successively, of an amplitude limiter, a frequency demodulator, and a de-emphasis circuit producing on the amplitude of chrominance signals the effects opposite to those of pre-emphasis at the transmitter. The outputs of the two paths are then linearly combined in a matrix to obtain the E_{DG} chrominance signals.

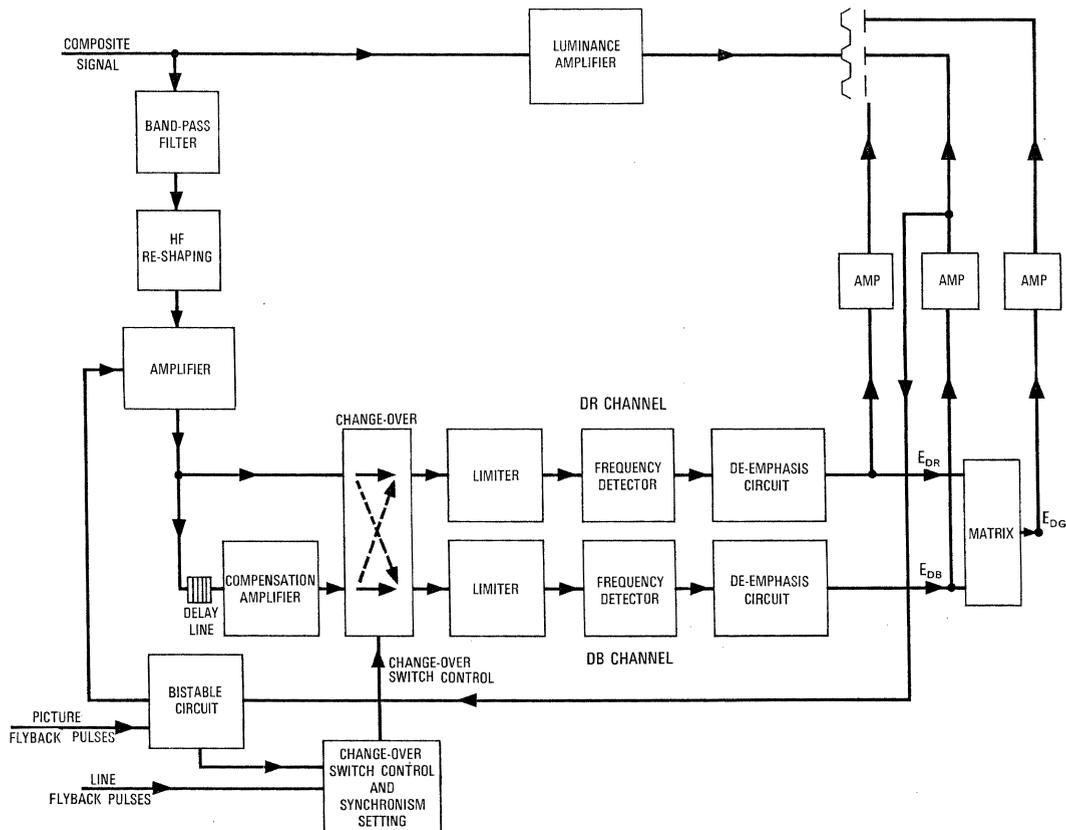


Figure 4 - The SECAM decoder.

The changeover is triggered by the line flyback pulses. In addition, the identification pulses taken from signal E_{DB} control the bistable circuit, which sends, if necessary, an additional pulse to the changeover switch to ensure its correct synchronism resetting.

To sum up, the SECAM decoder incorporates the following functions:

- extraction of the two subcarriers that are frequency modulated by the chrominance signals E_{DR} and E_{DB}
- utilization of a path delayed by an ultrasonic line to obtain both color signals simultaneously
- utilization of a changeover switch operating at line frequency to direct the color signals to their respective paths
- utilization of identification signals to ensure correct synchronism setting of the line changeover switch
- matrix combination of signals E_{DR} and E_{DB} to generate signal E_{DG} .

5. Receiver

Based on these principles Océanic Radio built a SECAM receiver in 1967. Figures 5 and 6 show, respectively, a rear view of the latter and the decoding unit with the delay line.

Certain improvements have been incorporated in this receiver as follows.

- An improvement (Figure 7) in the matrix combination of blue and red chrominance signals to obtain the green one. This protects the process from the effects of supply voltage variations. To achieve this, the chrominance signals E_{DB} and E_{DR} from the frequency demodulators are, firstly, applied to the output stages controlling the control grids of the picture tube; and secondly, after phase inversion and amplitude adjustment, applied to the matrixing stages, the outputs of which are placed in parallel to obtain the green chrominance signal. Such an arrangement makes the ratio of chrominance signals substantially constant despite any variations in the supply voltage applied to the tubes of the receiver stages.
- An improvement in the method of extraction of identification signals for the control of channel changeover. This consists of using a coincidence gate to isolate the identification lines and reject spurious signals. Figure 8 shows the block arrangement of the unit. The gate inputs receive the signal of the blue channel and a rectangular pulsed signal originating from the field scan. The gate thus transmits the identification signals only during the flyback period, and transmits these signals exclusively. The other circuits operating the changeover control circuit are conventional.

Figures 7 and 8 are on page 172 overleaf.

6. Conclusion

In conclusion, it should be recalled that color television transmission techniques follow the principles developed in the United States.

Subsequent improvements essentially concerning the modulation of the chrominance subcarrier have led to the PAL and SECAM systems. Additional developments,

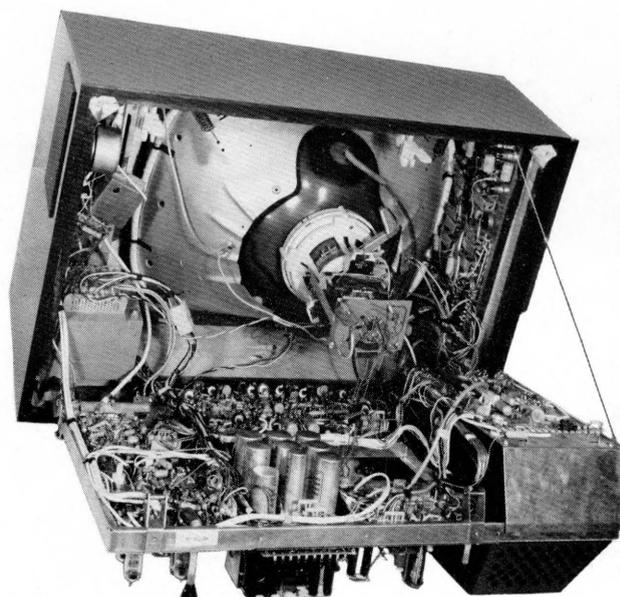


Figure 5 - Rear view of the Océanic-Radio SECAM receiver (open) Inside the cabinet: on the left, the VHF tuner above the transformer, on the right, the dynamic convergence adjusting circuits. On the lowered panel, from left to right: the IF and luminance amplifiers, the HT supply partly hidden by the decoder, the line-deflection oscillator, and the EHT supply.

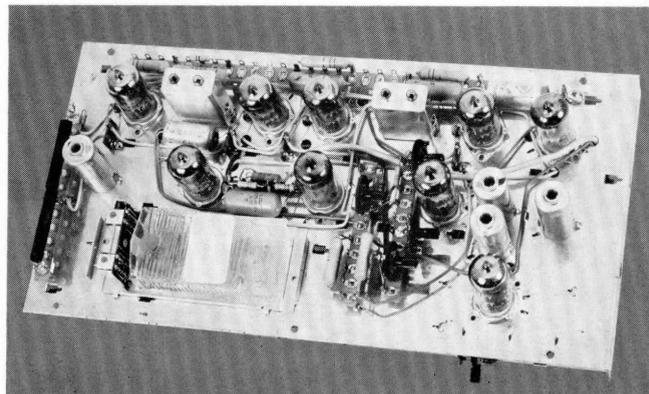


Figure 6 - Chassis of the SECAM decoder: The decoding unit and its delay line (left-hand corner, bottom of the picture).

although in the end not retained for administrative reasons, have provided a valuable contribution by stimulating fruitful discussions.

Although the institution of several different international standards may be considered detrimental from an economic standpoint, it must be observed that our techniques have made such progress in related fields that transcoding from one standard to another raises no problem nowadays.

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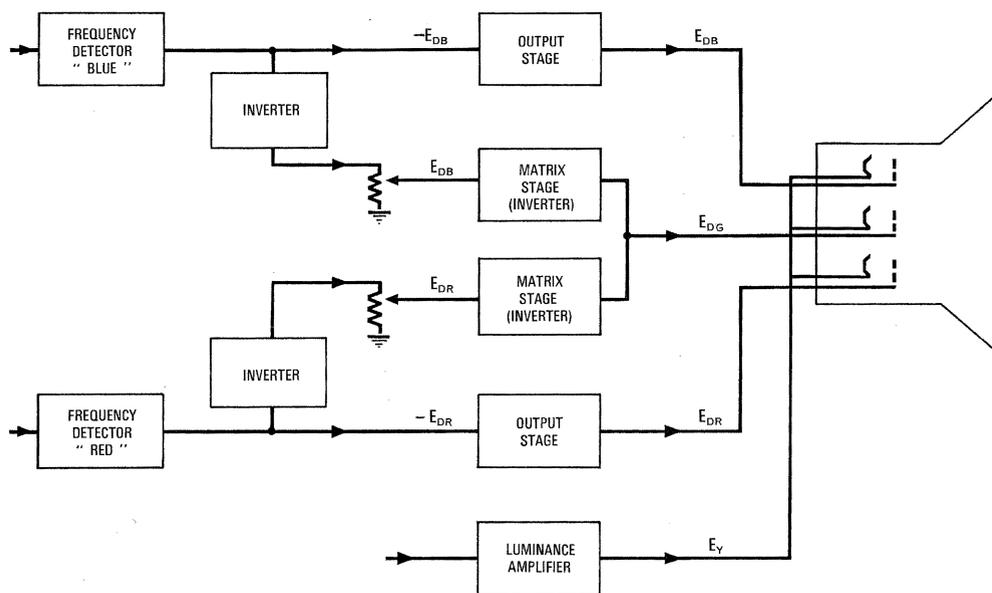
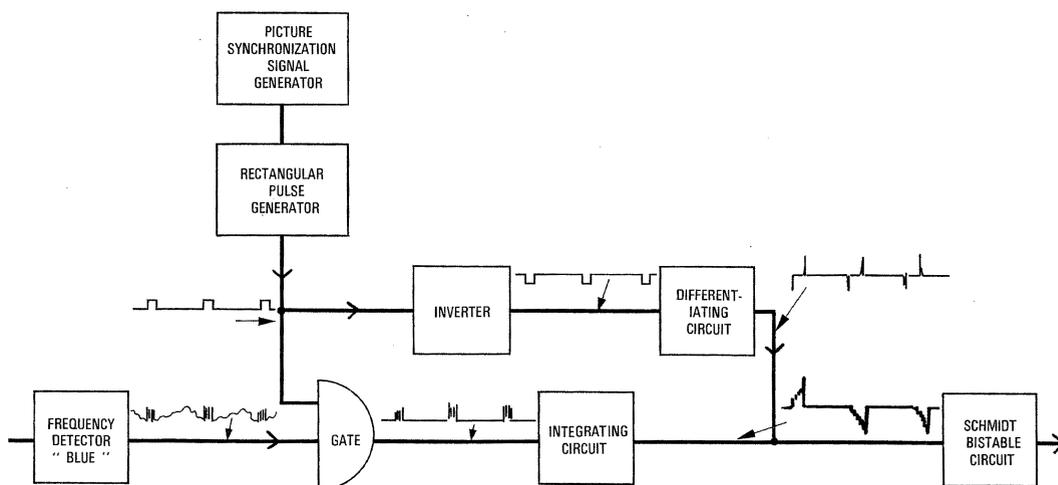


Figure 7 - Matrix combination of chrominance signals.

Figure 8 - Extraction and utilization of identification signals.



Jacques L. Delaitre was born in Paris in 1908. After a college education, he founded a firm, in 1926, under his name, for the design and manufacture of broadcast radio receivers and then of television receivers in 1945.

In 1952, he went into partnership with the company Evernice to form the Burel-Delaitre Company, which was absorbed by Océanic-Radio in 1963. From then onwards he became Technical Director of the Company.

Robert A. Redard was born at Le Raincy, Paris, in 1903. He received his diploma of Engineer from the Ecoles Nationales d'Arts et Métiers and the Ecole Supérieure d'Electricité and joined Le Matériel Téléphonique as an engineer in the department responsible for equipping the first Paris rotary telephone

switching exchanges. Later he became head of the broadcast receiver testing and inspection department.

Transferred to the Laboratoire Central de Télécommunications in 1934, he was responsible for the design and construction of wide band repeaters for transmission on coaxial cables, of terminal equipment for the Eiffel Tower television transmitter in 1936, of the radar receiving station of Port Cros in 1939, and for single-sideband receivers for transoceanic radio communications. Then, as director of the radio department, he took part in the development of modern radar.

Appointed manager of the broadcast radio-television receiver division in 1954, he participated in color television studies which started in 1964.

He left this activity at the end of 1965 to join the technical management of LMT.

Traffic Unbalances in Small Groups of Subscribers

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1. Introduction

A practical problem encountered in traffic engineering lies in the study of small groups of traffic sources; more precisely, in the determination of statistical deviations of the joint traffic of a small group of sources, relative to a given average traffic. Of course, the smaller the size of the group, the more important are the deviations that may be found.

Traffic unbalances can occur in links and devices that handle the traffic originated in a group and consequently they can have a more or less important influence on the overall grade of service of the speech network in which the groups are located.

Knowledge of the magnitude of such deviations permits the avoidance of such unbalances both in the selection of new groups and in the reorganization of existing groups. We are concerned with the general problem of allocating ordinary subscribers and business lines of private branch exchanges to the inlets of a network.

A general study is presented by defining the problem in a rigorous manner and by giving practical solutions to evaluate the traffic deviations in small groups.

2. Theoretical Presentation of the Problem

For clarity, three different cases will be considered.

2.1 Case of Groups from Sources of a Determined Population

Let us assume a large population, composed of M independent traffic sources: normal subscribers, high-average-traffic subscribers, incoming trunks, and private branch exchanges. In this population there is a defined or measured average traffic $\bar{\alpha}$. But, each component S_i of this population has an individual average traffic α_i that can vary with the component S_i between two extreme values Θ_0 and Θ_1 . We have of course

$$\bar{\alpha} = \frac{1}{M} \sum_{i=1}^M \alpha_i.$$

A random variable z can be defined as representing the average traffic of any source among the population. Therefore, z can take every value α_i and its mathematical expectation can be defined as $\bar{\alpha} = E(z)$.

To the random variable z , a distribution function is associated $f(t) = P[z \leq t]$ giving the probability that z may be less than or equal to an assumed value t , t being such that

$$\Theta_0 < t \leq \Theta_1.$$

By considering a large-enough population, we can assume that z takes continuous values and, consequently, its distribution function will also be continuous.

In these conditions, let us choose randomly K sources, $S_{i1}, S_{i2}, \dots, S_{iK}$, to form a group.

We can define a random variable Z that is able to assume all values in the form $(\alpha_{i1} + \alpha_{i2} + \dots + \alpha_{iK})$. Therefore, we will have

$$K \Theta_0 < Z \leq K \Theta_1.$$

The mathematical expectation of Z will be

$$E(Z) = KE(z) = K\bar{\alpha}.$$

The new random variable Z can be considered as the sum of K identical random variables z having the same distribution function and spread over the same interval (in other words, equally normalized).

The distribution function of Z will give us the probability that random variable Z will be less than or equal to any given value. More particularly, we obtain the probability that the joint traffic Z of the considered group will be greater or less than the average traffic $K\bar{\alpha}$ of the group.

In this way, an answer can be given to a first practical problem related to the traffic deviation, around a given average traffic, of the joint traffic of a small group of traffic sources.

2.2 Case of Groups from Sources of Two Different Families

Let us assume two large populations P_1 and P_2 composed of M_1 and M_2 sources, respectively.

As in the preceding case, the following items can be established for both populations.

Populations:	P_1	P_2
Individual average traffic of the source S_i	$\alpha_{1,i}$	$\alpha_{2,i}$
Boundary values of individual average traffic	Θ_0, Θ_1	ψ_0, ψ_1
Average traffic of each population	$\bar{\alpha}_1 = \frac{1}{M_1} \sum_{i=1}^{M_1} \alpha_{1,i}$	$\bar{\alpha}_2 = \frac{1}{M_2} \sum_{i=1}^{M_2} \alpha_{2,i}$
Random variable representing average traffic of any source	Z_1	Z_2
Mathematical expectation	$\bar{\alpha}_1 = E(Z_1)$	$\bar{\alpha}_2 = E(Z_2)$
Distribution function	$f_1(t)$	$f_2(t)$

Let us form a group by picking at random K_1 sources from P_1 , and K_2 sources from P_2 . Having formed the group, we define a random variable Z that takes values as

$$(\alpha_{1,i_1} + \alpha_{1,i_2} + \dots + \alpha_{1,i_{K_1}}) + (\alpha_{2,j_1} + \dots + \alpha_{2,j_{K_2}})$$

with $i = 1, 2, \dots, M_1$ and $j = 1, 2, \dots, M_2$.

Z may be interpreted as the sum $Z = K_1 Z_1 + K_2 Z_2$. This random variable Z will satisfy

$$K_1 \Theta_0 + K_2 \psi_0 < Z \leq K_1 \Theta_1 + K_2 \psi_1$$

and its mathematical expectation will be

$$\begin{aligned} E(Z) &= E(K_1 Z_1 + K_2 Z_2) = K_1 E(Z_1) + K_2 E(Z_2) \\ &= K_1 \bar{\alpha}_1 + K_2 \bar{\alpha}_2. \end{aligned}$$

Two practical problems can be proposed here.

- Influence of sources of the second population in a group mainly formed by P_1 sources.

If $K_2 \ll K_1$ the knowledge of the probability distribution function of Z allows us to determine the influence, in a group mainly composed of P_1 sources, of the presence of some P_2 sources. The effect will be more noticeable when $K_2 = 1$, that is, when only one P_2 source is introduced in a group of P_1 sources. This effect has to be studied by comparing such a mixed group with a pure group of the same size composed of P_1 sources.

- If parameters K_1 and K_2 are unconditional, the mentioned comparison can be made with two groups of $(K_1 + K_2)$ sources of P_1 or P_2 sources, respectively.

2.3 Case of Groups from Sources of Any Number of Different Families

This case is a generalization of the one in Section 2.2.

We must emphasize that if the group is formed by individual sources from different populations, only three characteristics have to be considered; the distribution functions, the ranges of variation for the individual average traffic, and the average traffic of each population.

The solution of the problems related above will be shown in a general manner in Section 4 by obtaining the distribution function of the random variable Z .

3. Historic Viewpoint

The practical problem for the case of a single population was treated for the first time in a paper by Martinez and Rodriguez [1] in which a truncated negative exponential law was taken for the distribution function of individual average traffics that varied between boundary values of 0 and 1.

The first assumption regarding distribution functions was in accordance with measurement results presented by Hayward [2].

This problem was studied later using other mathematical methods and extended to arbitrary boundary values Θ_0 and Θ_1 for individual average traffics in a report by Dartois [3]. This paper also presented a generalization to the case of several populations.

We will present here a summary of all the studies that have been made on this subject.

4. Mathematical Treatment

The mathematical approach used here is classic and is presented in this paper in such a way that each source in a given group can be considered to have one particular distribution of its individual average traffic and one particular range of variation of this traffic. This viewpoint is equivalent to considering a small group of N sources, each source S_j being derived from a population P_j .

With each source S_j , we can associate:

- a random variable z_j derived from population P_j and denoting the individual average traffic,
- the mathematical model of the probability distribution function of the random variable z_j , and
- the corresponding normalization interval $(\Theta_{0j}, \Theta_{1j})$.

Let $f_j(t) = P[z_j \leq t]$, the distribution function of the random variable z_j , defined for $\Theta_{0j} < t \leq \Theta_{1j}$. We suppose, of course, that $f_j(t)$ is a true distribution function, which means in particular that

$$f_j(\Theta_{0j}) = 0 \text{ and } f_j(\Theta_{1j}) = 1.$$

Furthermore, we assume that all random variables z_j , are independent (that is, independence between the considered populations or between the considered sources in the group under study).

Then, we will get Z as a random variable denoting the joint traffic of the group by

$$Z = \sum_{j=1}^N z_j.$$

If we note $\psi_j(x)$ and $\Phi_N(x)$ the characteristic functions of the random variables z_j and Z , we have, under the form of a Stieltjes-Riemann integral,

$$\psi_j(x) = \int_{-\infty}^{\infty} e^{itx} \cdot d[f_j(t)]$$

and, under the assumption of independence

$$\Phi_N(x) = \prod_{j=1}^N \psi_j(x).$$

Finally, assuming that $\Phi_N(x)$ can be integrated in the Lebesgue sense, we get the distribution function of Z by

$$F_N(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-itx} - 1}{-ix} \cdot \Phi_N(x) \cdot dx.$$

Once this distribution function is known, the problem of evaluation of the deviation of the joint traffic of the group around any given value is reduced to a calculation problem. Thus the method described here supplies a powerful tool to undertake the resolution of the small-group problem, provided that different models of distribution and different intervals of variation of the individual average traffic for each component of the group can be combined for the same time.

5. Choice of Mathematical Model

As stated in Section 2, the choice of the mathematical model of the distribution function $f(t)$ has to be made within the scope of a given population.

For a usual subscriber population, some measurements presented in [2] show that the negative exponential model is convenient.

For a given population suppose that we have first the average traffic $\bar{\alpha}$ and second the range (Θ_0, Θ_1) of variation of individual average traffic.

With these data we will derive for the distribution function $f(t)$ of the random variable Z the following model.

$$\begin{aligned} P[z \leq t] = f(t) &= \frac{e^{-\Theta_0/a} - e^{-t/a}}{e^{-\Theta_0/a} - e^{-\Theta_1/a}} \text{ for } \Theta_0 < t \leq \Theta_1 \\ &= 0 \text{ for all } t \leq \Theta_0 \\ &= 1 \text{ for all } t \geq \Theta_1. \end{aligned} \tag{1}$$

With a being a positive parameter that is determined by the relation

$$\begin{aligned} \bar{\alpha} = E(z) &= \int_{-\infty}^{\infty} t [df(t)] \\ &= a + \frac{\Theta_0 e^{-\Theta_0/a} - \Theta_1 e^{-\Theta_1/a}}{e^{-\Theta_0/a} - e^{-\Theta_1/a}}. \end{aligned} \tag{2}$$

But, with this model and these equations, it is easy to show that we cannot have $\bar{\alpha} \geq (\Theta_0 + \Theta_1)/2$. This consideration leads us to extend this model so as to make it possible to consider

$$\bar{\alpha} = \frac{\Theta_0 + \Theta_1}{2} \text{ or } \bar{\alpha} > \frac{\Theta_0 + \Theta_1}{2}.$$

Keeping the general exponential model, which contains the only realistic information in our possession, we will state the following laws.

For $\bar{\alpha} < \frac{\Theta_0 + \Theta_1}{2}$, $f(t) = \frac{e^{-\Theta_0/a} - e^{-t/a}}{e^{-\Theta_0/a} - e^{-\Theta_1/a}}$, $a > 0$. (3)

For $\bar{\alpha} > \frac{\Theta_0 + \Theta_1}{2}$, $f(t) = \frac{e^{\Theta_0/a} - e^{t/a}}{e^{\Theta_0/a} - e^{\Theta_1/a}}$, $a > 0$. (4)

For $\bar{\alpha} = \frac{\Theta_0 + \Theta_1}{2}$, $f(t) = \frac{\Theta_0 - t}{\Theta_0 - \Theta_1}$ (uniform law). (5)

It should be noted that the uniform law is nothing but a particular case of the general exponential law, obtained for the parameter a tending to $\pm \infty$.

6. Negative Exponential Law

Three examples will be presented to illustrate the use of the negative exponential law.

Two kinds of groups, either normal subscribers or private branch exchanges, and combinations of both types of sources will be considered.

Approximate equations will be given and compared to facilitate the use of the general results.

6.1 Case of Groups of Normal Subscribers

In the case of normal subscribers with individual average traffic values in the interval (0, 1), a reasonable assumption would be $\bar{\alpha} < 0.5$.

Then, according to Section 5, we will take

$$f(t) = \frac{1 - e^{-t/a}}{1 - e^{-1/a}}$$

for $0 \leq t \leq 1$ and $a > 0$.

We find, for the joint-traffic distribution function of a small group of N sources for

$$q \leq t \leq q + 1 \text{ and } q \leq N - 1$$

$$F_N(t) = \frac{1}{(1 - e^{-1/a})^N} \sum_{p=0}^q (-1)^p \binom{N}{p} e^{-p/a} \times \left[1 - e^{-\frac{t-p}{a}} \sum_{r=1}^{N-1} \left(\frac{t-p}{a} \right)^r \frac{1}{r!} \right]. \tag{6}$$

A more-convenient expression is found by introducing Poisson's and Erlang's equations in the terms between square brackets, P and E , respectively

$$F_N(t) = \frac{1}{(1 - e^{-1/a})^N} \sum_{p=0}^q (-1)^p \binom{N}{p} e^{-p/a} \times \left[1 - \frac{P_{N-1} \left(\frac{t-p}{a} \right)}{E_{N-1} \left(\frac{t-p}{a} \right)} \right]. \tag{7}$$

A different equation is presented in [1]. It is only another formulation of $F_N(t)$ that can easily be reduced to the present equation.

6.2 Case of Groups of Heavy-Traffic Lines

In the case of heavy-traffic lines or private branch exchanges with individual average traffic in the interval (Θ_0, Θ_1) with $\Theta_0 > 0$ and $\bar{\alpha} < (\Theta_0 + \Theta_1)/2$ we will obtain in the same way

$$F_N(t) = \frac{1}{(e^{-\Theta_0/a} - e^{-\Theta_1/a})^N} \sum_{p=0}^q (-1)^p \binom{N}{p} e^{-n_p/a} \times \left[1 - \frac{P_{N-1} \left(\frac{t-n_p}{a} \right)}{E_{N-1} \left(\frac{t-n_p}{a} \right)} \right], \tag{8}$$

for $n_q \leq t \leq n_{q+1}$ and $q \leq N - 1$, in which $n_p = N\Theta_0 + p(\Theta_1 - \Theta_0)$.

6.3 Case of Groups of Normal Subscribers and Private Branch Exchanges

Let us consider a group from two different populations for which the negative exponential law is assumed.

The discrimination between the two considered populations will be only on the normalization interval. So, we take K sources of population P_1 with normalization interval (Θ_0, Θ_1) and $(N - K)$ of population P_2 with interval (α_0, α_1) .

Then for the first population, we have

$$f_1(t) = \frac{e^{-\Theta_0/a} - e^{-t/a}}{e^{-\Theta_0/a} - e^{-\Theta_1/a}}$$

and for the second one

$$f_2(t) = \frac{e^{-\alpha_0/a} - e^{-t/a}}{e^{-\alpha_0/a} - e^{-\alpha_1/a}}$$

It should be noted that we have taken the same parameter a for $f_1(t)$ and $f_2(t)$. This simplifies calculations and is equivalent to a dependence relation between the 6 parameters m_1 (average traffic in the population P_1), m_2 , Θ_0 , Θ_1 , α_0 , and α_1 : without reducing greatly the scope of the results presented here.

With

$$n_p = K\Theta_0 + p(\Theta_1 - \Theta_0) \text{ for } 0 \leq p \leq K$$

and

$$m_q = (N - K)\alpha_0 + q(\alpha_1 - \alpha_0) \text{ for } 0 \leq q \leq N - K,$$

we get

$$F_N(t) = \frac{1}{(e^{-\Theta_0/a} - e^{-\Theta_1/a})^K (e^{-\alpha_0/a} - e^{-\alpha_1/a})^{N-K}} \tag{9}$$

$$\times \sum (-1)^{p+q} \binom{K}{p} \binom{N-K}{q} e^{-\frac{n_p+m_q}{a}} I_{N,K,n_p+m_q}(t)$$

with $0 \leq p + q \leq N$

and $0 \leq p \leq K$.

$$I_{N,K,A}(t) = \begin{cases} 0 & \text{if } t \leq A \\ 1 - \frac{P_{N-1} \left(\frac{t-A}{a} \right)}{E_{N-1} \left(\frac{t-A}{a} \right)} & \text{if } t \geq A. \end{cases}$$

6.4 Approximation for the Negative Exponential Law

Once the size of the group N and the value of $\bar{\alpha}$ are fixed in a specific case, the first computation difficulty arises in determining the value of a given by (2). There-

fore, to simplify the use of (7) and (8) some approximations are introduced.

The first is derived when $a = \bar{\alpha} - \Theta_0$ is accepted as an approximate solution of (2). Accuracy increases as $\bar{\alpha} - \Theta_0$ becomes smaller.

When $\Theta_0 \leq \bar{\alpha} < (\Theta_0 + \Theta_1)/2$ the resulting equation can be written

$$F_N(t) = \frac{1}{(e^{-\Theta_0/\bar{\alpha} - \Theta_0} - e^{-\Theta_1/\bar{\alpha} - \Theta_0})^N} \times \sum_{p=0}^q (-1)^p \binom{N}{p} e^{-\frac{n_p}{\bar{\alpha} - \Theta_0}} \left[\frac{P_{N-1}\left(\frac{t-n_p}{\bar{\alpha} - \Theta_0}\right)}{E_{N-1}\left(\frac{t-n_p}{\bar{\alpha} - \Theta_0}\right)} \right] \quad (10)$$

for $n_q \leq t \leq n_{q+1}$, $n_p = N\Theta_0 + p(\Theta_1 - \Theta_0)$, and $q \leq N-1$.

For $\alpha \infty (\Theta_0 + \Theta_1)/2$, the last approximation is supplied by the uniform law over (Θ_0, Θ_1) , as shown in Section 5.

To extend the scope of these approximations, we will define the generalized negative exponential law as a law for which the density function, $g(t)$ is given by

$$g(t) = \frac{\beta + 1}{\beta! a} \left[\frac{(\beta + 1)t}{a} \right]^\beta \cdot e^{-(\beta+1)t/a} \quad (11)$$

with $t \geq 0$, in which a second parameter β is included, taking only positive integer values. When $\beta = 0$, $g(t)$ leads to the ordinary negative exponential distribution function

$$f(t) = 1 - e^{-t/a}$$

which gives

$$F_N(t) = 1 - \frac{P_{N-1}(t/a)}{E_{N-1}(t/a)}, \quad (12)$$

often called the Chi-square law. This function multiplied by $(1 - e^{-t/a})^{-N}$ is the first term in the development of (7).

When $\beta \neq 0$, it follows

$$F_N(t) = 1 - \frac{P_{(\beta+1)N-1}\left(\frac{(\beta+1)t}{a}\right)}{E_{(\beta+1)N-1}\left(\frac{(\beta+1)t}{a}\right)}. \quad (13)$$

It is interesting to note that the mathematical expectation of the distribution defined by the density function $g(t)$ (11) gives $\bar{\alpha} = a$. Then, we can deduce the following statement.

"Under the assumption of a density function $g(t)$ (11), the joint distribution function of a small group of $(\beta + 1)N$ sources with average traffic $\bar{\alpha}/(\beta + 1)$ is the same as for one of a group of N sources with an average traffic $\bar{\alpha}$ and can be expressed by the Chi-square law."

In the case of $\beta = 2$, this can be considered in this way: in a group of N sources with average traffic $\bar{\alpha}$, the joint traffic distribution is the same as for a group composed by $N/3$ sources with average traffic $3\bar{\alpha}$. This statement seems to be in agreement with the assumption presented by A. Jensen [4] in the sense that only about a third of the subscribers are active in the busy hour.

Another approximation for (7) is given by the following normal law:

$$F_N(t) = \frac{1}{(2n)^{1/2}} \int_{-\infty}^{t - N\bar{\alpha}/\bar{\alpha}(N)^{1/2}} e^{-u^2/2} du \quad (14)$$

for any value of t .

6.5 Numerical Results

Admitting the difficulty in performing calculations using the laws presented above, we have limited our evaluation to some magnitudes related to small groups to the case of (7).

A tabulation of (7) has been made by means of a 1620 computer assuming groups of $N = 2, 3, \dots, 25$ sources with average traffic in the group $\bar{\alpha} = 0.02, 0.04, \dots, 0.20, 0.25$ and 0.30 .

Main attention has been paid to the $F_N(t)$ queues, that is, to the values of t in the interval $(N\bar{\alpha}, N)$. These results are summarized in a previous paper [1].

For practical purposes, two groups of graphs are included here with the aim of illustrating the way of making use of the equations giving the joint traffic distribution function of the small groups. In fact with these graphs, one can determine the probability P that the average traffic of a group Z will exceed in a certain proportion U percent, the average joint traffic $N\bar{\alpha}$ in the considered group.

The first group of curves (Figures 1, 2, and 3) gives for indicated values of P the variation of $u = U/100$ in terms of $\bar{\alpha}$. (Group sizes $N = 4, 16$, and 25 have been considered).

The second group of curves (Figures 4, 5, and 6) gives the variation of P in terms of $\bar{\alpha}$ for given values of U and for $N = 4, 16$, and 25 . For the sake of brevity, we have not presented here a third group of curves giving the variation of U in terms of P for given values of $\bar{\alpha}$ and for $N = 4, 16$, and 25 . These curves may be found in [5].

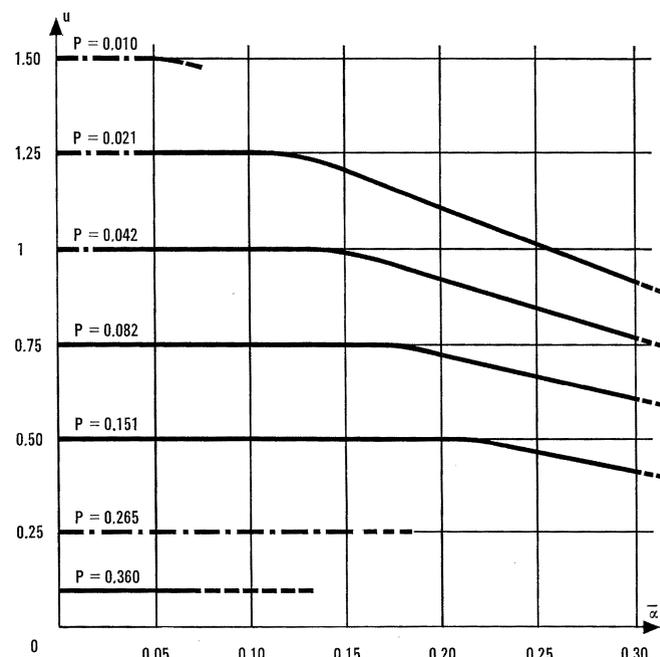


Figure 1 Curves of u as a function of $\bar{\alpha}$ for the indicated values of P and for a number of sources $N = 4$. $100u = U =$ percent deviation from the average joint traffic $N\bar{\alpha}$. $P =$ probability that the average traffic of the group of sources will exceed the average joint traffic $N\bar{\alpha}$.

The study of these sets of curves establishes a very important property from the practical viewpoint of the use of (7).

In the interval $0 \leq \bar{\alpha} < 0.16$, the variation of μ in terms of P depends only on N , not on $\bar{\alpha}$.

Curves in Figure 7 illustrate this property for fixed sizes of groups ($N = 4, 8, 16,$ and 25). They provide a very simple tool for solving the main practical problems encountered with small subscriber groups.

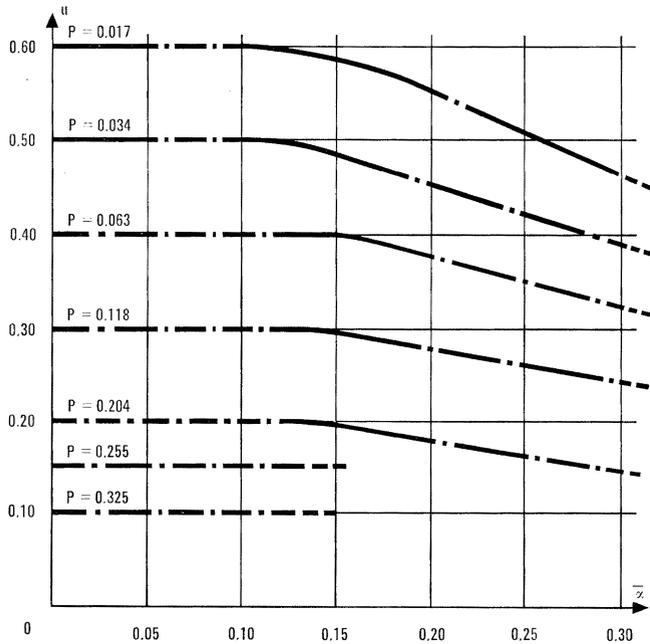


Figure 2 - μ versus $\bar{\alpha}$ for $N = 16$.

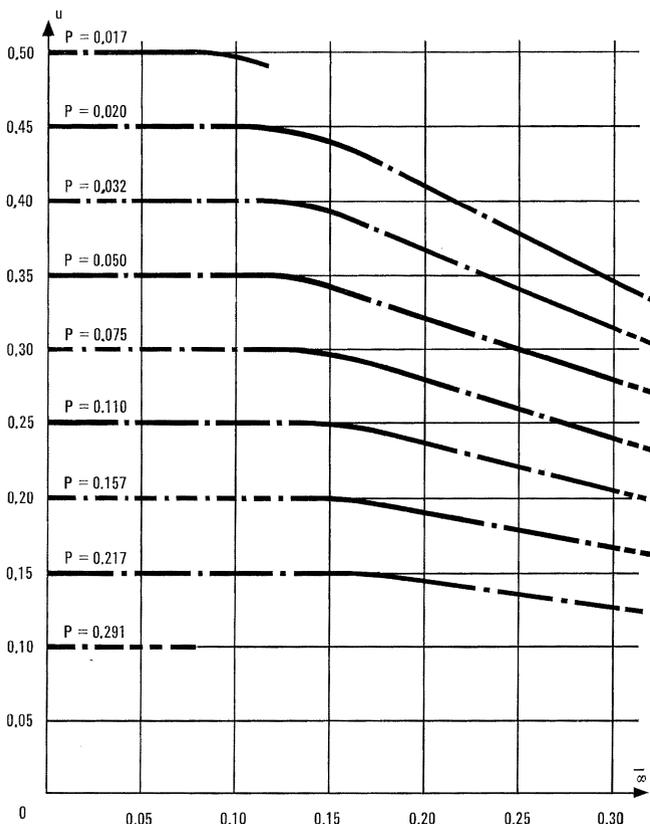


Figure 3 - μ versus $\bar{\alpha}$ for $N = 25$.

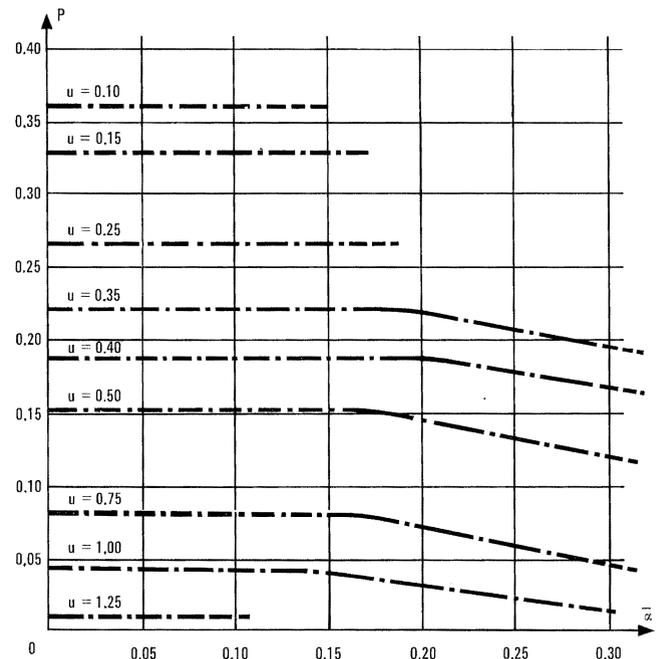


Figure 4 - P versus $\bar{\alpha}$ for $N = 4$.
 $P = 1 - Pr [Z \leq (1 + \mu) N \bar{\alpha}]$.

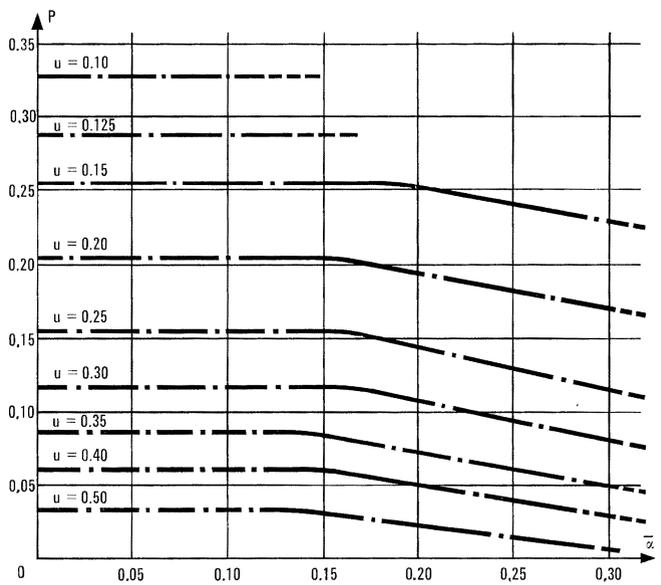


Figure 5 - P versus $\bar{\alpha}$ for $N = 16$.
 $P = 1 - F_N [(1 + \mu) N \bar{\alpha}]$.

6.6 Comparison of Approximate Laws for Light Traffic

Tables 1, 2, and 3 give the probability P that the average joint traffic in a group of N sources will exceed the value $Z = (1 + \mu)N\bar{\alpha}$. This probability has been evaluated for three traffic values using (7), (10), (12), and (13).

The examples here included are meaningful enough to show the positions of each approximate law against the exact law (7).

7. Case of Incoming Trunks: the Uniform Law

In the case of incoming trunks having only small variations of individual average traffic, we can consider the

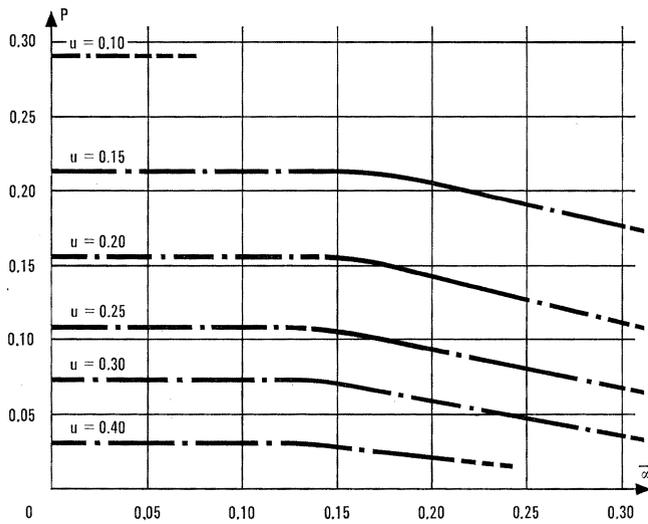


Figure 6 - P versus \bar{x} for $N = 25$.
 $P = 1 - F_N [(1 + u) N \bar{x}]$.

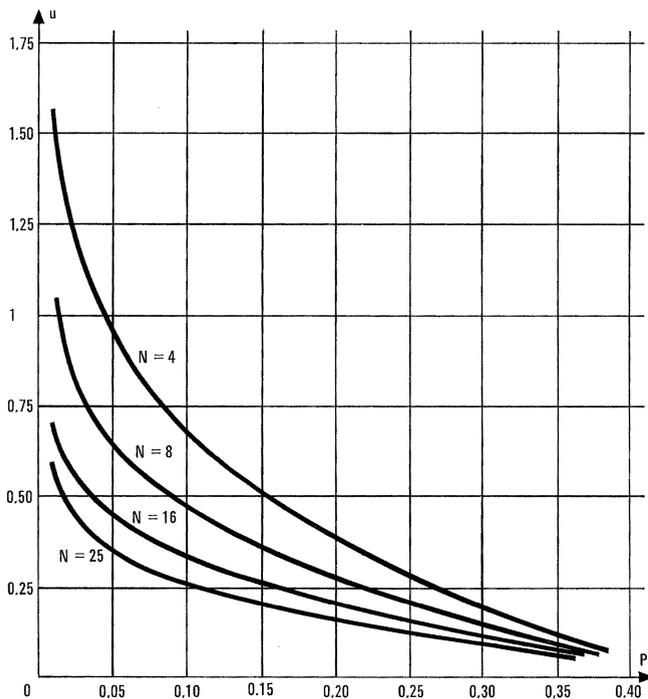


Figure 7 - u as a function of P for the indicated values of N .
 $P = Pr [Z > (1 + u) N \bar{x}]$.

uniform law as defined in Section 5 or even the exponential law (defined by (4)). We will limit the study to the uniform law (5).

$$f(t) = \frac{\theta_0 - t}{\theta_0 - \theta_1} \text{ for } \theta_0 < t \leq \theta_1.$$

For a small group of N incoming trunks, we get the following joint distribution of average traffic for $n_K \leq t \leq n_{K+1}$, $K \leq N-1$, and $n_K = N\theta_0 + K(\theta_1 - \theta_0)$,

$$F_N(t) = \frac{1}{N!} \sum_{p=0}^K (-1)^p \binom{N}{p} \left(\frac{t - n_p}{\theta_1 - \theta_0} \right)^N \quad (15)$$

Table 1

$N = 16 \bar{x} = 0.10$ Normalization over 0.1
 (U = Percentage Deviation from Average $N\bar{x}$)

U in percent = $100 u$	$P = Pr [Z \leq (1 + u) N \bar{x}]$			
	Law (7)	Law (10A)	Chi-Squared Law	Normal Law
—	1	1	1	1
- 78.125		1.0000	1.0000	
- 75.000		1.0000	1.0000	0.9987
- 71.875		1.0000	1.0000	
- 68.750		0.9999	0.9999	
- 65.625		0.9998	0.9998	
- 62.500		0.9995	0.9995	0.9938
- 56.250		0.9976	0.9976	0.9872
- 50.000		0.9912	0.9913	0.9773
- 43.750		0.9779	0.9780	0.9599
- 37.500	0.9514	0.9512	0.9513	0.9332
- 34.375		0.9316	0.9317	
- 31.250		0.9073	0.9074	0.8944
- 28.125		0.8862	0.8863	
- 25.000		0.8442	0.8443	0.8413
- 21.875		0.8059	0.8060	
- 18.750		0.7634	0.7636	0.7734
- 15.625		0.7270	0.7272	
- 12.500		0.6691	0.6694	0.6915
- 9.375		0.6189	0.6192	
- 6.250		0.5678	0.5681	0.5987
- 3.125		0.5045	0.5052	
0		0.4664	0.4668	0.5000
+ 3.125		0.4274	0.4278	
+ 6.250		0.3710	0.3715	0.4013
+ 9.375		0.3271	0.3275	
+ 12.500		0.2862	0.2867	0.3085
+ 15.625		0.2485	0.2490	
+ 18.750		0.2142	0.2148	0.2256
+ 21.875	0.1839	0.1834	0.1840	
+ 24.000	0.1564	0.1559	0.1656	0.1587
+ 30.250	0.1109	0.1105	0.1111	0.1056
+ 36.500	0.0766	0.0764	0.0769	0.0668
+ 42.750	0.0517	0.0515	0.0520	0.0401
+ 50.000	0.0341	0.0340	0.0344	0.0227
+ 56.250	0.0220	0.0218	0.0222	0.0122
+ 62.500		0.0138	0.0142	0.0062
+ 75.000		0.0052	0.0054	0.0013
+ 87.500	0.0018	0.0017	0.0018	0.0002
+ 100.000		0.0006	0.0007	
+ 112.500		0.0002	0.0002	
+ 125.000		0.0001	0.0001	
+ 137.500		0.0000	0.0000	
+ 150.000	0.0000	0.0000	0.0000	0.0000
—	0	0	0	0

Table 2

$N = 16 \bar{x} = 0.25$ Normalization over 0.1

U in percent = $100 u$	$P = Pr [Z \leq (1 + u) N \bar{x}]$			
	Law (7)	Law (10A)	Chi-Squared Law	Normal Law
+ 0.00	0.4804	0.3443	0.4668	0.5000
+ 12.50		0.1713	0.2867	0.3085
+ 18.75	0.1943	0.1100	0.2148	0.2266
+ 25.00	0.1311	0.0686	0.1565	0.1587
+ 31.25	0.0844	0.0406	0.1111	0.1056

Table 3

$N = 4 \bar{\alpha} = 0.25$ Normalization over 0.1

U in percent = 100 μ	$P = Pr [Z \leq (1 + \mu) N\bar{\alpha}]$			
	Law (7)	Law (10 A)	Chi-Squared Law	Normal Law
0.0	0.4596	0.3890	0.4335	0.5000
+ 25.0		0.2073	0.2650	0.3085
+ 50.0	0.1350	0.0947	0.1512	0.1587
+ 75.0	0.0593	0.0375	0.0818	0.0668
+ 100.0	0.0225	0.0189	0.0424	0.0227

This law also may be considered as an approximation of the exponential one in the case where we know that the average traffic $\bar{\alpha}$ of the population under study is near the half sum $(\theta_0 + \theta_1)/2$ of the boundary values of the individual average traffic.

The following particular case is presented as an example of this law. We take a group of $N = 4$ incoming trunks, chosen at random among a large population with average traffic $\bar{\alpha} = 0.8$ erlang and boundary values $\theta_0 = 0.6$ and $\theta_1 = 1.0$ for the individual average traffic.

Results are shown in Figure 8, which give the variations of μ in terms of P (same notations as for preceding curves).

8. Case of Groups of Normal Subscribers and Private Branch Exchanges

This case demonstrates a method of evaluating the influence of combining private branch exchange lines and normal subscribers in the same group.

At the same time, we will give an example of combining two different distribution functions normalized over different intervals.

Let us assume a group of N sources made up of K subscribers belonging to a population defined by a negative exponential law normalized over (θ_0, θ_1) and of $(N - K)$ private branch exchange lines taken from a population for which the uniform law normalized over (α_0, α_1) is considered to be the mathematical model. The individual average traffic distribution functions will be respectively defined by (3) and (5). Therefore, keeping the same notations as for (9), we find

$$F_N(t) = \frac{1}{2} \frac{1}{(e^{-\theta_0/a} - e^{-\theta_1/a})} K \left(\frac{-a}{\alpha_1 - \alpha_0} \right)^{N-K} \times \sum (-1)^{p+q} \binom{K}{p} \binom{N-K}{q} e^{-n_p/a} J_{N, K, n_p+m_q}(t) \quad (16)$$

with $0 \leq p + q \leq N$ and $0 \leq p \leq K$.

$$J_{N, K, A}(t) = \frac{\left(\frac{A}{a}\right)^{N-K} - \left(\frac{A-t}{a}\right)^{N-K}}{N-K!} + \sum_{r=1}^{N-K} \frac{\left(\frac{A}{a}\right)^{N-K-r} - \left(\frac{A-t}{a}\right)^{N-K-r}}{(N-K-r)!} \times \frac{K(K+1)\dots(K+r-1)}{r!}$$

if $t \leq A$.

$$J_{N, K, A}(t) = \frac{\left(\frac{A}{a}\right)^{N-K} + \left(\frac{A-t}{a}\right)^{N-K}}{(N-K)!} + \sum_{r=1}^{N-K-1} \frac{\left(\frac{A}{a}\right)^{N-K-r} + \left(\frac{A-t}{a}\right)^{N-K-r}}{(N-K-r)!} \times \frac{K(K+1)\dots(K+r-1)}{r!} - 2e^{-(t-A)/a} \times \sum_{r=0}^{N-1} \left(\frac{t-A}{a}\right)^r \frac{(N-K+1), \dots, (N-r-1)}{r!(K-r-1)!}$$

if $t \geq A$.

As a particular case and to indicate the influence in a group of homogeneous sources of the inclusion of sources from another population, we will consider the case of a group formed by $N-1$ subscribers and a single private branch exchange. To facilitate the calculation, we assume an equal amplitude for both normalization intervals.

$$\theta_1 - \theta_0 = \alpha_1 - \alpha_0 = \tau.$$

Therefore, (16) becomes for

$$M_s \leq t \leq M_{s+1} \text{ and } s \leq N-1$$

and with M_s defined by

$$M_r = \alpha_0 + (N-1)\theta_0 + r\tau$$

for $0 \leq r \leq N$

$$P_N(t) = \frac{1}{(e^{-\theta_0/a} - e^{-\theta_1/a})^{N-1}} \cdot \frac{a}{2\tau} \left\{ (-1)^{s+1} \binom{N-1}{s} \times e^{-n_s/a} \left[J_{N, N-1, M_s}(t) - \frac{t}{a} \right] + \sum_{p=0}^{s-1} (-1)^p \binom{N-1}{p} \right\}$$

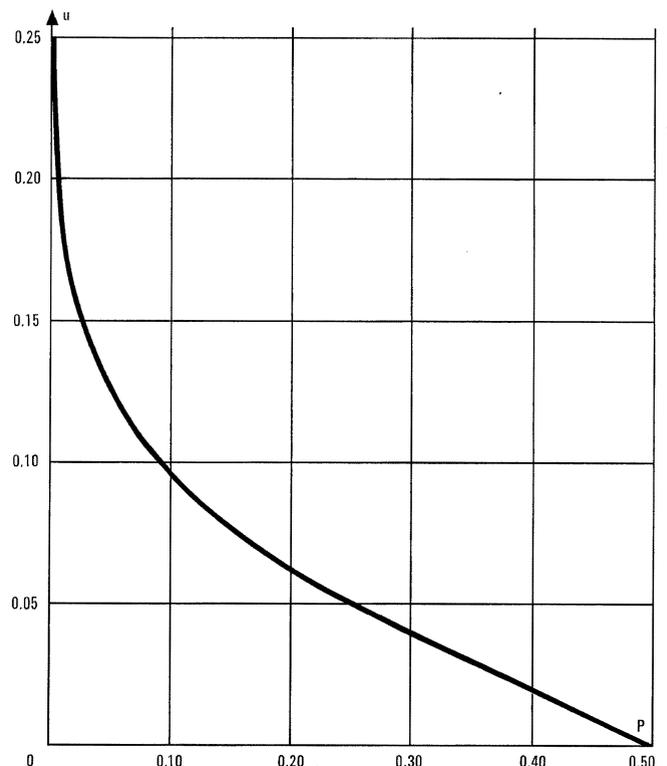


Figure 8 - Uniform law for $\theta_0 = 0.6$, $\theta_1 = 1.0$, $\bar{\alpha} = 0.8$, and $N = 4$. $P = Pr [Z > (1 + \mu) N\bar{\alpha}]$.

$$\times e^{-n_p/a} \left[J_{N, N-1, M_p+1}(t) - J_{N, N-1, M_p}(t) \right]$$

with

$$J_{N, N-1, A}(t) = \frac{t}{a}, \text{ if } t \leq A$$

$$= \frac{2A-t}{a} - 2e^{-(t-A)/a} \sum_{r=0}^{N-2} \left(\frac{t-A}{a} \right)^r$$

$$\times \binom{N-r-1}{r!}, \text{ if } t \geq A. \quad (17)$$

9. Conclusions

By way of conclusions, we will note five main points.

- Although only the negative exponential law and its limit, the uniform law, have been studied in a particular manner in this paper, we do not find any denial of the usefulness of the law (4) to cover the complete variation of the average traffic $\bar{\alpha}$ with respect to the value $(\Theta_0 + \Theta_1)/2$.
- The general method explained in Section 4 can be applied to this law and the corresponding results obtained.
- Among the different approximate equations given in Section 6.4 for the negative exponential law, we want to point out the ease with which the Chi-square law can be applied in the case of small groups of subscriber lines and mostly in the case of light traffic values.
- Once again for small groups of normal subscribers, we want to emphasize the value of the data plotted in Figure 7, in the sense that for $0 \leq \bar{\alpha} < 0.16$, the traffic deviations depend only on the size of the group, assuming the negative exponential law.
- For practical traffic engineering purposes, (9), (16), and (17) merit tabulation provided that actual groups of inlets in a network are composed of light- and heavy-traffic lines.

— The general method proposed in this paper may be used with other different distributions. Measurements and statistics will in the future give more light in the field of distribution functions and normalization intervals, as well as in the assumption of stochastic independence between sources.

10. Acknowledgments

Thanks are due to all members of the traffic engineering group for their help and stimulating discussions.

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Delay Problems Relating to an Invariable Number of Connections Per Hour Having Invariable Holding Time and Carried by a Single Outlet

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1. Introduction

In the literature dealing with the application of the probability theory to switching problems, two studies by Erlang and Fry are available and relate to the single outlet operated on a delay basis.

Both studies assume a theoretical traffic offered to the outlet comprising an average number of n connections per hour, each having the same invariable holding time.

Erlang's study contains the exact solution of the variation of the delay period [1].

Fry develops the related space distribution function, which expresses the probability of finding a specific number of connections simultaneously in progress and calls possibly in abeyance [2].

The following sections deal with the same problem but are based on the assumption that the number of connections is invariable, i.e. every hour is considered to contain exactly n calls and connections.

By the term "call" we mean exclusively the moment the request for a "connection" is made. The duration of a connection equals h . By the term "delay period", we mean the interval during which the call remains ineffective owing to the single outlet being busy.

The solution of the present problem provides explicit expressions for:

- the space distribution function,
- the average total delay per hour,
- the average number of delayed calls per hour,
- the delay distribution function.

In Sections 11 and 12, the concordance between the present study and those mentioned above is demonstrated.

Section 13 contains a number of lemmas that become apparent from the various operations applied. These are in addition to those already stated in a previous paper [3].

2. Theoretical Telephone Traffic

Exactly n calls arrive during each of an infinite number of hours, and this flow is partly described by the following family of $(n + 1)$ equations:

$$B_c(n, t) = \binom{n}{c} t^c (1-t)^{n-c}.$$

This time distribution function indicates the probability that exactly c calls arrive during some interval t , which is smaller than the hour, that is, c may assume any value ranging from 0 to n inclusive.

With every call a connection with a duration h is associated, either directly, if the outlet happens to be free, or indirectly, after the insertion of a delay period.

This theoretical traffic suffers a deformation caused by the hunting discipline imposed and the structure of the network.

3. Hunting Discipline — Switching Network

The network consists of two distinct parts: a single outlet that carries the effective traffic, and $n-1$ circuits that store the delayed calls. The total of n circuits ensures that no calls can be "lost" on account of a shortage of circuits.

An arriving call immediately seizes the outlet, provided the latter is idle. If, however, the outlet happens to be engaged by some other connection, the call seizes one of the delayed call circuits. Here it waits until the outlet is liberated. The effective holding time begins at this moment.

One can imagine a wide variety of methods whereby the delayed calls might be served. In this paper, only two such methods are considered, the delayed calls are served in the order of their arrival, or in a random order.

Of the four expressions that contain the solution of the present problem, the delay distribution function is the only one that is affected by the order in which the delayed calls are served.

4. Calls per Hour

In connection with space distribution functions that relate to a type of theoretical traffic characterized by an invariable number of calls per hour, combined with invariable holding time, problems arise with respect, first, to the period that is situated between the beginning of the hour and the moment the first call arrives, and, second, to the amount of traffic that flows over at the end of the hour. Unless some additional assumption is made, there is no guarantee that the hour will contain exactly nh erlang, which is a *sine qua non*.

To remedy this situation, it may be suggested that the imaginary infinite number of hours that contribute to the solution of some congestion problem are linked end to end, so that the traffic that leaves one hour re-enters into the next hour. Such a suggestion, however, fails to satisfy the stated condition, since the traffic that stems from the preceding hour need not necessarily be equal to the amount of traffic that flows over into the next hour.

The only assumption that fulfills all the *desiderata* is that the traffic that leaves some theoretical hour at the end, re-enters the same hour at its beginning. Only then is there the guarantee that the two amounts of traffic are identical under all circumstances. This means that the beginning and the end of every hour are joined together so as to form a closed loop. Any point of this loop may be chosen for the beginning and, at the same time, for the termination of one of an infinite number of hours. This assumption would appear to be obligatory.

From the assumption that the hour forms a closed loop, that any point may serve as an observation moment, and

that we are at liberty to let this moment coincide with the beginning and the end of the hour, there follows the conclusion that the expectation of the arrival of one or more calls during some interval coming after the moment of observation depends only on the difference between n , the invariable number of calls per hour, and the number of connections in progress at the moment of observation. This conclusion forms the basis of Section 3.1 of a previous paper published in this magazine [3].

5. Traffic Patterns and Chains of Connections

Owing to the hunting discipline imposed on the arriving calls, the traffic carried by the single outlet will form traffic patterns that comprise groups of connections joined end to end. The denomination "chains" will be used for these groups, since the term "queue" is generally used to indicate a group of waiting calls.

As an illustration, let us use the case in which four calls are offered to a single outlet. We observe the following different traffic patterns. The symbols employed are self-explanatory.

Type	Symbol
One 4-connection chain	4
One 3-connection and one 1-connection chain	3/1
Two 2-connection chains	2/2
One 1-connection and one 3-connection chain	1/3
One 2-connection and two 1-connection chains	2/1/1
One 1-connection, one 2-connection and one 1-connection chain	1/2/1
Two 1-connection and one 2-connection chain	1/1/2
Four 1-connection chains	1/1/1/1

The general traffic pattern comprises K chains of connections. If we denote the numbers of these connections in the order of the chains by a, b, c etc., we have:

$$a + b + c + \dots = n.$$

The probability that exactly this pattern will occur equals:

$$p = \frac{n!}{a! b! c! \dots} (ab)^{a-1} (bb)^{b-1} (cb)^{c-1} \dots \frac{1}{K!} (1-nb)^{K-1}. \quad (1)$$

If we wish to disregard the order in which the chains appear, this expression has to be multiplied by

$$\frac{K!}{r_a! r_b! r_c! \dots}$$

where r_a, r_b, r_c, \dots represent the frequency with which the numbers a, b, c, \dots are repeated. There is the relation:

$$ar_a + br_b + cr_c + \dots = n.$$

In this instance, therefore, a, b, c, \dots represent different numbers. By the combination of the above results we obtain for the probability that a pattern comprises r_a chains with a connections, r_b chains with b connections, etc., in any wanted order:

$$p = \frac{n!}{r_a! r_b! r_c! \dots} \left[\frac{(ab)^{a-1}}{a!} \right]^{r_a} \left[\frac{(bb)^{b-1}}{b!} \right]^{r_b} \times \left[\frac{(cb)^{c-1}}{c!} \right]^{r_c} \dots (1-nb)^{K-1}. \quad (2)$$

These equations were derived by progressively building the various traffic patterns for ascending numbers of n .

When $n = 1$, there is only one pattern with the probability 1. The second call may arrive at any moment with respect to the first call and the resulting chains are investigated. It should be observed that a slight duration b precedes the first connection. A call that arrives during this period or during the first connection will lead to a 2-connection chain. In all other instances, a traffic pattern comprising two 1-connection chains will ensue. The former type of pattern will have the probability $2b$ and the latter $1-2b$.

With $n = 3$, the 3-connection chain will have the probability $(3b)^2$ and the three 1-connection chains $(1-3b)^2$. In consequence, the probability associated with the patterns comprising one 2-connection chain and one 1-connection chain will amount to the remainder, that is, $3(2b)(1-3b)$.

In addition, it will be remembered that there is the relation:

$$\binom{n}{a} \binom{n-a}{b} \binom{n-a-b}{c} \binom{n-a-b-c}{d} \dots = \frac{n!}{a! b! c! \dots}$$

Equation(1) or Equation (2) alternatively form the basis of the calculation of the expressions that form the solution of the delay problem under discussion. It is facilitated by the consideration that at the beginning and the end of every chain of connections all delayed calls have been cleared, and that between successive chains no calls arrive.

This permits, for every chain length, the calculation of the latter's contribution to the desired expressions. Subsequently, these contributions are multiplied by the average number of times the chains appear in the total number of n -call patterns. The total of these products provides the final result.

In Section 6, we therefore first calculate the average numbers of the appearance of the various chains.

6. Average Appearance Number of m -Connection Chain

The simplest way of finding the general expression for this average number is to calculate by means of Equation (2) the sequences of probabilities for ascending values of n . They read:

n	Symbols	Probabilities
3	3	$(3b)^2$
	2/1	$3(2b)(1-3b)$
	1/1/1	$(1-3b)^2$
4	4	$(4b)^3$
	3/1	$4(3b)^2(1-4b)$
	2/2	$3(2b)^2(1-4b)$
	2/1/1	$6(2b)(1-4b)^2$
	1/1/1/1	$(1-4b)^3$
5	5	$(5b)^4$
	4/1	$5(4b)^3(1-5b)$
	3/2	$10(3b)^2(2b)(1-5b)$
	3/1/1	$10(3b)^2(1-5b)^2$
	2/2/1	$15(2b)^2(1-5b)^2$
	2/1/1/1	$10(2b)(1-5b)^3$
	1/1/1/1/1	$(1-5b)^4$
etc.		

When the number of calls per hour amounts to four, the average number of times the 2-connection chain will appear is equal to:

$$2 \times 3 (2b)^2 (1-4b) + 6 (2b) (1-4b)^2 = \binom{4}{2} (2b (1-2b) (1-4b)).$$

Continuing in this manner, the following table is obtained.

<i>n</i>	Average number of chains	Connections per chain
3	$\binom{3}{3} (3b)^2 (1-3b)^{-1} (1-3b)$	3
	$\binom{3}{2} (2b) (1-2b)^0 (1-3b)$	2
	$\binom{3}{1} (b)^0 (1-b) (1-3b)$	1
4	$\binom{4}{4} (4b)^3 (1-4b)^{-1} (1-4b)$	4
	$\binom{4}{3} (3b)^2 (1-3b)^0 (1-4b)$	3
	$\binom{4}{2} (2b) (1-2b) (1-4b)$	2
	$\binom{4}{1} (b)^0 (1-b)^2 (1-4b)$	1
5	$\binom{5}{5} (5b)^4 (1-5b)^{-1} (1-5b)$	5
	$\binom{5}{4} (4b)^3 (1-4b)^0 (1-5b)$	4
	$\binom{5}{3} (3b)^2 (1-3b) (1-5b)$	3
	$\binom{5}{2} (2b) (1-2b)^2 (1-5b)$	2
	$\binom{5}{1} (b)^0 (1-b)^3 (1-5b)$	1
etc.		

In consequence of the above, the general expression for the average number of *m*-connection chains per hour amounts to:

$$\binom{n}{m} (mb)^{m-1} (1-mb)^{n-m-1} (1-nb). \tag{3}$$

By the addition of the *n* expressions we obtain, for the average of the total number of chains per hour, the expression:

$$n [1 - (n-1)b]. \tag{4}$$

7. Average Number of Delayed Calls Per Hour

Of the *m* connections of a chain, *m*-1 calls have been delayed. In conjunction with Equation (3), the average number of delayed calls per hour amounts to:

$$\sum_{m=2}^n (m-1) \binom{n}{m} (mb)^{m-1} (1-mb)^{n-m-1} (1-nb) = \left[\frac{1-b}{b} \sum_{m=2}^n B_m(n, mb) / (1-mb) - \sum_{m=2}^n B_m(n, mb) / mb \right] (1-nb).$$

After introduction of lemmas B and C of Section 13, the above converts to:

$$\left[\frac{1-b}{b} \frac{nb}{1-nb} - n \right] (1-nb) = n(n-1)b. \tag{5}$$

The same result also follows directly from Equation (4).

8. Formation of Chains of Connections — Space Distribution Function

Chains of connections can be formed in a number of ways. The following table has been prepared for purposes of illustration, and it shows the five different ways in which a 4-connection chain can be created. The symbols used are self-explanatory.

In addition, the right-hand column shows the proportions between the occurrence of a specific "formation" of the chain and the occurrence of the chain. They are in no way dependent on the total number of calls that arrive during the hour.

Number of calls arriving	During connection	Formation symbol	Proportion
3	1	3-0-0-0	$\binom{3}{3} / 4^2 = 1/16$
2	1	2-1-0-0	$\binom{3}{2} / 4^2 = 3/16$
1	2	2-0-1-0	$\binom{3}{2} / 4^2 = 3/16$
2	1	2-0-1-0	$\binom{3}{2} / 4^2 = 3/16$
1	3	2-0-1-0	$\binom{3}{2} / 4^2 = 3/16$
1	1	1-2-0-0	$\binom{3}{2} / 4^2 = 3/16$
2	2	1-2-0-0	$\binom{3}{2} / 4^2 = 3/16$
1	1	1-1-1-0	$\binom{3}{1} \binom{2}{1} / 4^2 = 6/16$
1	2	1-1-1-0	$\binom{3}{1} \binom{2}{1} / 4^2 = 6/16$
1	3	1-1-1-0	$\binom{3}{1} \binom{2}{1} / 4^2 = 6/16$

The formation 2-1-0-0 when it appears in a *n*-connection pattern occurs a number of times that is equal to:

$$\frac{3}{16} \binom{n}{4} (4b)^3 (1-4b)^{n-5} (1-nb).$$

Reference is made to Equation (3).

Each chain contributes its specific average amount to the total probabilities *X*₁, *X*₂, *X*₃, . . . etc. These probabilities represent the wanted space distribution function indicating the occurrence of 0, 1, 2, etc., calls in abeyance in addition to the connection that occupies the outlet.

Having determined the appearance of all chain formations, we shall now consider the contributions to the space distribution function of these formations. As a typical example, the 4-connection chain with the formation 3-0-0-0 will be considered (Figure 1).

The three calls that arrive during the first connection have been placed in their average position. The order in which these calls are served is irrelevant to the wanted probabilities. We note that the period of the first connection contributes on the average equal parts to the probabilities *X*₁, *X*₂, *X*₃, and *X*₄. The complete duration of the second, third, and fourth connections contributes to the probabilities *X*₃, *X*₂, and *X*₁. Altogether, therefore,

the chain formation contributes $5b/4$ to each of the probabilities X_1, X_2 and X_3 and $b/4$ to X_4 .

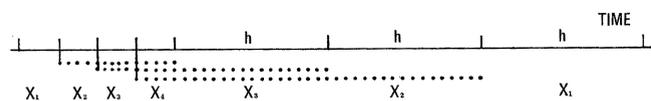


Figure 1 - Contributions to the space probabilities (Chain formation 3-0-0-0).

Application of this method to the various chain formations leads to the following summary, which can be extended at will.

m	Formation	Proportion	Contribution to probability			
			X ₁	X ₂	X ₃	X ₄
2	1-0	1	3h/2	h/2	-	-
3	2-0-0	1/3	4h/3	4h/3	h/3	-
	1-1-0	2/3	2h	h	-	-
4	3-0-0-0	1/16	5h/4	5h/4	5h/4	h/4
	2-1-0-0	3/16	4h/3	11h/6	5h/6	-
	2-0-1-0	3/16	11h/6	11h/6	h/3	-
	1-2-0-0	3/16	11h/6	11h/6	h/3	-
	1-1-1-0	6/16	5h/2	3h/2	-	-
etc.						

Multiplication of the above contributions by their respective proportions and, after summation of these products, the total contributions per chain of connections are arrived at as in Table 1.

Finally, the expressions for the space distribution functions are obtained by the combination of the above results per chain with Equation (3), which states the average number of times the chains appear in the traffic patterns of the outlet.

A few examples shown in Tables 2 and 3 will explain the procedure.

Table 1

m	Contributions per type of chain to the probabilities						
	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇
2	$\frac{3}{2} b$	$\frac{1}{2} b$	-	-	-	-	-
3	$\frac{16}{9} b$	$\frac{10}{9} b$	$\frac{1}{9} b$	-	-	-	-
4	$\frac{125}{64} b$	$\frac{107}{64} b$	$\frac{23}{64} b$	$\frac{1}{64} b$	-	-	-
5	$\frac{1296}{625} b$	$\frac{1346}{625} b$	$\frac{436}{625} b$	$\frac{46}{625} b$	$\frac{1}{625} b$	-	-
6	$\frac{16807}{7776} b$	$\frac{19917}{7776} b$	$\frac{8402}{7776} b$	$\frac{1442}{7776} b$	$\frac{87}{7776} b$	$\frac{1}{7776} b$	-
7	$\frac{262144}{117649} b$	$\frac{341986}{117649} b$	$\frac{173860}{117649} b$	$\frac{41070}{117649} b$	$\frac{4320}{117649} b$	$\frac{162}{117649} b$	$\frac{1}{117649} b$
etc.							

In this manner the following groups of space probabilities were obtained.

$$\begin{aligned}
 X_0(1, b, 1) &= 1-b \\
 X_1 &= b \\
 \\
 X_0(2, b, 1) &= 1-2b \\
 X_1 &= 2b - b^2 \\
 X_2 &= b^2 \\
 \\
 X_0(3, b, 1) &= 1-3b \\
 X_1 &= 3b - 3b^2 - 2b^3 \\
 X_2 &= 3b^2 + b^3 \\
 X_3 &= b^3 \\
 \\
 X_0(4, b, 1) &= 1-4b \\
 X_1 &= 4b - 6b^2 - 8b^3 - 3b^4 \\
 X_2 &= 6b^2 + 4b^3 + 5b^4 \\
 X_3 &= 4b^3 + 7b^4 \\
 X_4 &= b^4 \\
 \\
 X_0(5, b, 1) &= 1-5b \\
 X_1 &= 5b - 10b^2 - 20b^3 - 15b^4 - 4b^5 \\
 X_2 &= 10b^2 + 10b^3 - 25b^4 - 29b^5 \\
 X_3 &= 10b^3 + 35b^4 + 11b^5 \\
 X_4 &= 5b^4 + 21b^5 \\
 X_5 &= b^5 \\
 \\
 X_0(6, b, 1) &= 1-6b \\
 X_1 &= 6b - 15b^2 - 40b^3 - 45b^4 - 24b^5 - 5b^6 \\
 X_2 &= 15b^2 + 20b^3 - 75b^4 - 174b^5 - 99b^6 \\
 X_3 &= 20b^3 + 105b^4 + 66b^5 - 94b^6 \\
 X_4 &= 15b^4 + 126b^5 + 146b^6 \\
 X_5 &= 6b^5 + 51b^6 \\
 X_6 &= b^6 \\
 \\
 X_0(7, b, 1) &= 1-7b \\
 X_1 &= 7b - 21b^2 - 70b^3 - 105b^4 - 84b^5 - 35b^6 - 6b^7 \\
 X_2 &= 21b^2 + 35b^3 - 175b^4 - 609b^5 - 693b^6 - 279b^7 \\
 X_3 &= 35b^3 + 245b^4 + 231b^5 - 658b^6 - 923b^7 \\
 X_4 &= 35b^4 + 441b^5 + 1022b^6 + 302b^7 \\
 X_5 &= 21b^5 + 357b^6 + 729b^7 \\
 X_6 &= 7b^6 + 113b^7 \\
 X_7 &= b^7
 \end{aligned}$$

The above number of groups of probabilities is sufficient for the derivation of the general space distribution func-

Table 2 $n = 2$

Chain-		Contribution per chain			Total contribution of all chains		
Connections	Quantity	X_1	X_2	X_3	X_1	X_2	X_3
$2b$ b	$2b$ $2(1-2b)$	$3b/2$ b	$b/2$ -	- -	$3b^2$ $2b-4b^2$	b^2 -	- -
Total space probabilities:					$2b - b^2$	b^2	-

Table 3 $n = 3$

$3b$ $2b$ b	$9b^2$ $6b(1-3b)$ $3b(1-b)(1-3b)$	$16b/9$ $3b/2$ b	$10b/9$ $b/2$ -	$b/9$ - -	$16b^3$ $9b^2-27b^3$ $3b-12b^2+9b^3$	$10b^3$ $3b^2-9b^3$ -	b^3 - -
Total space probabilities:					$3b - 3b^2 - 2b^3$	$3b^2 + b^3$	b^3

tion. The method that was followed is not reproduced here, since it contains no points of general interest. The method, however, is entirely logical and the reader may verify the following general equation against any of the sets of equations shown above.

It reads as follows:

when $c = 0$, we have

$$X_0(n, b, 1) = 1 - nb,$$

When c differs from 0, we have

$$X_c(n, b, 1) = \sum_{i=0}^c \{B_i[n, -(c-i)b] - nb B_i[(n-1), -(c-i)b]\} - \sum_{i=0}^{c-1} \{B_i[n, -(c-i-1)b] - nb B_i[(n-1), -(c-i-1)b]\} \quad (6)$$

When these equations are applied to specific cases of n , it should be borne in mind that the general expression $B_i(n, 0)$ is equal to 1 when $i = 0$ and equal to 0 for all other values of i .

It will be observed that in these equations the traffic is always negative, since the terms that stand for the "holding time" can assume negative values only.

Every probability comprises two summations, and it is of interest to note that the second summation of X_c is always equal to the first summation of X_{c-1} .

The Bernoulli expressions appearing in the above Equation (6) have, in general, the following characteristic features; $B_i[n, (i-c)b]$ is:

- positive when i is even and smaller than c ,
- negative when i is odd and smaller than c ,
- null when $i = c$,
- positive when $i > c$.

The alternation between positive and negative terms within the range indicated by Equation (6) is the cause of increased complication of the evaluation of the probabilities. This, however, can be obviated by the introduction of lemma (E), which leads to the following equation:

$$\sum_{i=0}^c B_i[n, (i-c)b] = \sum_{j=0}^n \frac{n!}{(n-j)!} b^j - \sum_{i=c+1}^n B_i[n, (c-i)b]$$

which leads to a single equation, as an alternative to the two Equations (6) and (6a):

$$X_c(n, b, 1) = \sum_{i=c}^n B_i[n, (i-c+1)b]$$

$$- nb \sum_{i=c}^{n-1} B_i[(n-1), (i-c+1)b] \quad (6a)$$

$$- \sum_{i=c+1}^n B_i[n, (i-c)b] + nb \sum_{i=c+1}^{n-1} B_i[(n-1), (i-c)b]$$

($c = 0, 1, \dots, n$).

In this equation, the traffic values and all Bernoulli expressions are positive.

9. Average Value of The Total Delay Per Hour

For this value we may write the expressions:

$$D = \sum_{i=2}^n (i-1) X_i(n, b, 1).$$

After introduction of the detailed probabilities stated in the above, we obtain for the values of n ranging from 2 to 7, the expressions:

n	Average value of the total delay per hour
2	b^2
3	$3b^2(1+b)$
4	$6b^2(1+2b+2b^2)$
5	$10b^2(1+3b+6b^2+6b^3)$
6	$15b^2(1+4b+12b^2+24b^3+24b^4)$
7	$21b^2(1+5b+20b^2+60b^3+120b^4+120b^5)$
etc.	

In consequence, the general expression for the average total delay per hour is equal to:

$$D = \sum_{i=2}^n \frac{n!}{2(n-i)!} b^i \quad (\text{where } nb < 1). \quad (7)$$

In conjunction with Equation (4), the average delay per delayed call amounts to:

$$\sum_{i=2}^n \frac{(n-2)!}{2(n-i)!} b^{i-1} \quad (\text{where } nb < 1). \quad (8)$$

10. Delay Distribution Function

This function expresses the variation of the period a delayed call has to wait until it is served. Owing to the premise that the holding time of the connections is invariable, this variation of the delay period cannot be described by a single equation. In conformity therewith,

the curve representing this variation is not a smooth one. A typical delay distribution function for invariable n and invariable holding time b is shown in Figure 2. It is valid for $n = 8$ and $b = 0.1$. It appears to consist of seven smooth curves, one for every section situated between the limits $t/b = 0$ to 1; 1 to 2, etc. There is a sudden change in the value of the slope at the points of juncture. The minimum delay amounts to zero and the maximum to $(n-1)b$; the latter delay may occur when all calls arrive at the same moment. These two limits do not depend on the order in which the delayed calls are served.

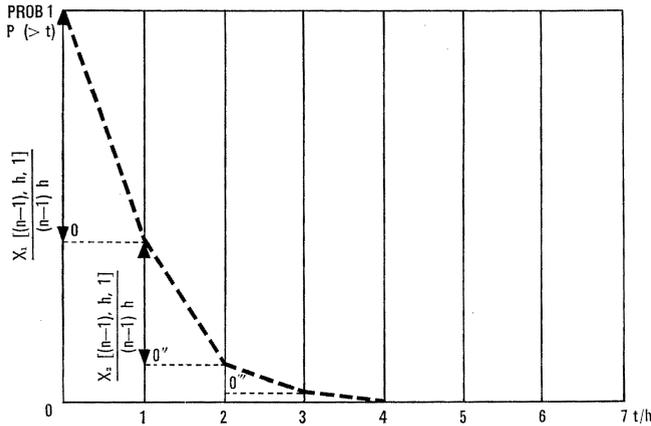


Figure 2 - Typical delay distribution function ($n = 8, b = 0.1$).

Also in this instance, the solution of the problem is facilitated by the fact that no calls can be in abeyance either at the beginning or at the end of any chain of connections, so that contributions to the delay distribution function may be calculated first per chain formation, then per chain, and finally per complete pattern of n connections, all in the manner already demonstrated in detail in the preceding text.

A delayed call is bound to arrive during some effective connection and the delay incurred depends on this moment and on the moment at which the outlet is liberated. In consequence, a delay period consists of a plurality of b and some variable interval. Both quantities depend on the imposed hunting discipline affecting the delayed calls. The delay period of a delayed call is, therefore, determined by the indication of the section in which it is placed and the variable contribution calculated from the beginning of this section. This is the notation used below.

In the various delay distribution functions the variable interval t is always expressed in units of b , which permits the application of our customary symbol for the Bernoulli law.

If only a single call arrives during a connection, the probability of its arriving after an interval greater than t equals $(1-t/b)$. If it arrives at the moment τ , it will be delayed for a time $(b-\tau)$, so that the distribution function of this single call's waiting period reads:

$$p(> b-t) = B_0\left(1, \frac{b-t}{b}\right), \text{ or}$$

$$p(> t) = B_0(1, t/b).$$

In case two calls should arrive during the first connection of a chain, it appears to be necessary to make some assumption concerning the order in which these two calls are served. The distribution function of the moments at which they arrive reads:

$$\begin{aligned} \text{1st call: } p(> t) &= B_0(2, t/b), \\ \text{2nd call: } p(> t) &= B_0(2, t/b) + B_1(2, t/b). \end{aligned}$$

If the rule "first come, first served" is followed, the contribution of the first call to the first section of the delay distribution function will be:

$$\begin{aligned} p(> b-t) &= B_0\left(2, \frac{b-t}{b}\right), \text{ or} \\ p(> t) &= B_0(2, t/b) + B_1(2, t/b). \end{aligned}$$

Similarly, the contribution of the second call to the second section of the delay distribution function amounts to:

$$\begin{aligned} p(> b-t) &= B_0\left(2, \frac{b-t}{b}\right) + B_1\left(2, \frac{b-t}{b}\right), \text{ or} \\ p(> t) &= B_0(2, t/b). \end{aligned}$$

In general, the contribution of the call numbered a out of k calls that arrive during the same connection amounts to:

$$p_k(> t) = \sum_{i=0}^{k-a} B_i(k, t/b) = \text{the symbol } k^{\circ} a.$$

The question of the section of the delay distribution function to which this contribution will go is to be determined case by case. For purposes of illustration, Figures 3 and 4 show the distribution functions relating to the arrival of 3 calls.

These two figures show the relation between the distribution functions under discussion if the waiting calls are served in the order of their arrival. If, on the other hand, it is assumed that the waiting calls are served in a random order, there is no priority; the contributions to the first and the second sections are equal and amount to:

$$p(> t) = (1-t/b) = B_0(1, t/b).$$

In conclusion, we observe that, when the waiting calls are served in a random order, the distribution function of the delay periods consists of a string of straight lines. In this instance, the function is determined by the coordinates of the points of juncture of the sections.

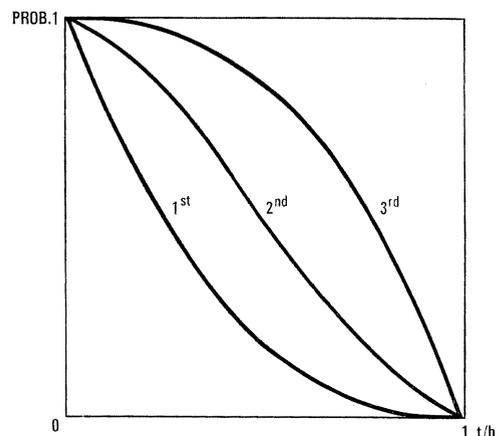


Figure 3 - Distribution functions relating to the arrival of three calls during a period of duration b .

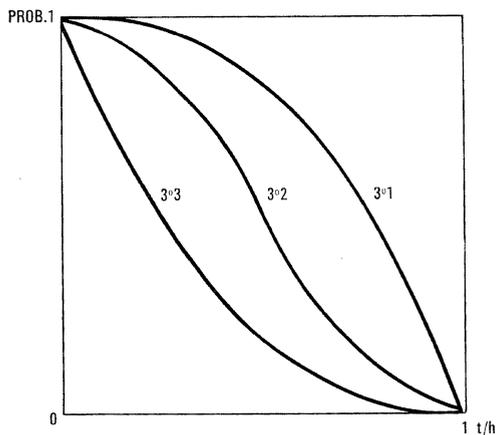


Figure 4 - Variable part of the delay distribution functions relating to three calls arriving during the time interval *b*.

Having calculated the delay distribution functions of groups of calls that arrive during the same interval *b*, we shall now continue with the investigation of the functions that pertain to complete chains of connections. This requires consideration of the various chain formations.

2-connection chain

Provides the contribution $(1-t/b)$ during the first section of the delay distribution function.

3-connection chain

— formation 2-0-0

Since the calls are served in the order of their arrival, the call that arrives first contributes $2^0 1$ to section 1 and call 2 adds $2^0 2$ to section 2.

— formation 1-1-0

Both calls contribute $1^0 1$ to section 1.

Since these two formations appear in the proportions 1/3 and 2/3, the totals for the 3-connection chain amount to:

1st section: $2^0 1/6 + 2 \times 1^0 1/3$

2nd section: $2^0 2/6$

The following chains of connections were treated in this manner:

4-connection chain

1st section: $(3^0 1 + 9 \times 2^0 1 + 24 \times 1^0 1)/48$

2nd section: $(3^0 2 + 9 \times 2^0 2 + 3 \times 1^0 1)/48$

3rd section: $(3^0 3)/48$

5-connection chain

1st section: $(4^0 1 + 16 \times 3^0 1 + 90 \times 2^0 1 + 200 \times 1^0 1)/500$

2nd section: $(4^0 2 + 16 \times 3^0 2 + 90 \times 2^0 2 + 6 \times 2^0 1 + 52 \times 1^0 1)/500$

3rd section: $(4^0 3 + 16 \times 3^0 3 + 6 \times 2^0 2 + 4 \times 1^0 1)/500$

4th section: $(4^0 4)/500$

6-connection chain

1st section: $(5^0 1 + 25 \times 4^0 1 + 240 \times 3^0 1 + 1080 \times 2^0 1 + 2160 \times 1^0 1)/6480$

2nd section: $(5^0 2 + 25 \times 4^0 2 + 240 \times 3^0 2 + 1080 \times 2^0 2 + 10 \times 3^0 1 + 160 \times 2^0 1 + 825 \times 1^0 1)/6480$

3rd section: $(5^0 3 + 25 \times 4^0 3 + 240 \times 3^0 3 + 10 \times 3^0 2 + 160 \times 2^0 2 + 10 \times 2^0 1 + 135 \times 1^0 1)/6480$

4th section: $(5^0 4 + 25 \times 4^0 4 + 10 \times 3^0 3 + 10 \times 2^0 2 + 5 \times 1^0 1)/6480$

5th section: $(5^0 5)/6480$.

The detailed calculation of the 4-connection chain may serve as an illustration of the above. It should be considered that the expressions represented by the symbols $1^0 1$, $2^0 1$, $2^0 2$, etc. relate to several calls, whereas the delay distribution function bears on a single call.

Symbol of formation	1st section $t/b = 0$ to 1	2nd section $t/b = 1$ to 2	3rd section $t/b = 2$ to 3	Proportion
3-0-0-0	$(3^0 1)/3$	$(3^0 2)/3$	$(3^0 3)/3$	1/16
2-1-0-0	$(2^0 1)/3$	$(2^0 2 + 1^0 1)/3$	—	3/16
2-0-1-0	$(2^0 1 + 1^0 1)/3$	$(2^0 2)/3$	—	3/16
1-2-0-0	$(1^0 1 + 2^0 1)/3$	$(2^0 2)/3$	—	3/16
1-1-1-0	$1^0 1$	—	—	6/16

The combination of the above expressions for the *m*-connection chains with the average number of times the chains appear in the *n*-connection traffic patterns situated on the single outlet provides the complete and exact result for the delay distribution function. These appearance numbers of the chains are provided by the Equation (3). The method of application follows the lines indicated in Section 8.

However, the expressions thus obtained are lengthy and lack clarity. For this reason, a limitation of the problem is introduced, in that we abandon the solution of the exact expressions for the sectional curves of which the delay distribution is composed and limit ourselves to the expressions for the exact location of the points of juncture. It would seem that this is quite acceptable, since it facilitates the calculations without abandoning the general exact shape of the function.

This means that $t/b = 0$ is introduced into the above contributions of the various chains. In that case they reduce to:

Chain length	Section				
	1	2	3	4	5
2	1	—	—	—	—
3	5/6	1/6	—	—	—
4	34/48	13/48	1/48	—	—
5	307/500	165/500	27/500	1/500	—
6	3506/6480	2341/6480	581/6480	51/6480	1/6480
etc.					

These values represent the contributions of the various chains to the height of the curve segments of which the delay distribution function is composed (Figure 2). This is confirmed by the fact that the sum of the fractions of every horizontal line always amounts to 1.

The combination of these contributions of the chains with the average number of times they appear in the traffic patterns given by Equation (3) provides the answer.

As an example, we have calculated the heights of the curve segments for the case where $n = 6$, with the following results:

$$\begin{aligned} 3506/6480 & (6b)^5 (1-6b)^{-1} (1-6b) 5 + \\ 307/500 & 6 (5b)^4 (1-5b)^0 (1-6b) 4 + \\ 34/48 & 15 (4b)^3 (1-4b) (1-6b) 3 + \\ 5/6 & 20 (3b)^2 (1-3b)^2 (1-6b) 2 + \\ 1 & 15 (2b) (1-2b)^3 (1-6b) 1 \\ & = 30b - 60b^2 - 120b^3 - 90b^4 - 24b^5 \end{aligned}$$

$$\begin{aligned} 2341/6480 & (6b)^5 (1-6b)^{-1} (1-6b) 5 + \\ 165/500 & 6 (5b)^4 (1-5b)^0 (1-6b) 4 + \\ 13/48 & 15 (4b)^3 (1-4b) (1-6b) 3 + \\ 1/6 & 20 (3b)^2 (1-3b)^2 (1-6b) 2 + \\ & = 60b^2 + 60b^3 - 150b^4 - 174b^5 \end{aligned}$$

$$\begin{aligned} 581/6480 & (6b)^5 (1-6b)^{-1} (1-6b) 5 + \\ 27/500 & 6 (5b)^4 (1-5b)^0 (1-6b) 4 + \\ 1/48 & 15 (4b)^3 (1-4b) (1-6b) 3 + \\ & = 60b^3 + 210b^4 + 66b^5 \end{aligned}$$

$$\begin{aligned} 51/6480 & (6b)^5 (1-6b)^{-1} (1-6b) 5 + \\ 1/500 & 6 (5b)^4 (1-5b)^0 (1-6b) 4 \\ & = 30b^4 + 126b^5 \end{aligned}$$

$$\begin{aligned} 1/6480 & (6b)^5 (1-6b)^{-1} (1-6b) 5 \\ & = 6b^5. \end{aligned}$$

Each line contains three expressions. The first is taken from the preceding table stating the contributions per chain. The second follows from Equation (3), and the third states the number of delayed calls for that particular chain of connections. The first of these five results relates to the height of the first segment of the delay distribution function situated between the limits $t/b = 0$ to 1, the second to that between $t/b = 1$ to 2, etc.

These five results are still to be divided by the total average number of delayed calls per hour, which in conformity with Equation (5) amounts to $30b$.

Thus the vertical distances between the points of juncture of the delay distribution function appear to amount to:

$$\begin{aligned} \text{1st section:} & (5b - 10b^2 - 20b^3 - 15b^4 - 4b^5)/5b \\ \text{2nd section:} & (10b^2 + 10b^3 - 25b^4 - 29b^5)/5b \\ \text{3rd section:} & (10b^3 + 35b^4 + 11b^5)/5b \\ \text{4th section:} & (5b^4 + 21b^5)/5b \\ \text{5th section:} & b^5/5b. \end{aligned}$$

These results arouse our attention if they are compared with the probabilities preceding Equation (6). We note that they are equal to:

$$X_1 (5, b, 1)/5b \text{ to } X_5 (5, b, 1)/5b.$$

This conclusion appears to be general, so that for every delay distribution function, n points are determined in an exact manner. When counting from the left to the right, i. e. from $t/b = 0$ to $(n-1)$, the coordinates of point u are:

$$t/b = u; p(>t) = \sum_{i=u+1}^{n-1} X_i (n-1, b, 1)/(n-1) b. \quad (9)$$

This equation is valid for values of u ranging from 0 to $(n-2)$. For $u = (n-1)$, $p(>t) = 0$. Figure 2 shows a typical delay distribution function for $n = 8$ and $b = 1$.

The points situated on the verticals $t/b = 0, 1$, etc., are exact and these have been joined arbitrarily by straight lines. Actually, the top curve segments are slightly concave and those of the lower part are slightly convex. As a whole, the function differs but slightly from the exponential one.

Equation (9), as already stated, is valid only when the delayed calls are handled in the order in which they arrived. If it is assumed that the calls are served in a random order, the variable parts of the contributions to the function under discussion are always equal to 1^n . In this respect the investigation of the delay distribution function is simplified, but the calculation of the coordinates of the points of juncture becomes less clear. Dealing with this problem in the same order applied above, we find for the contributions of the various chains the following values shown in Table 4.

From these data and with the help of Equation (2), the vertical distances between the points of juncture situated on the vertical lines $t/b = 1, 2$, etc., were derived for various values of n . The method is identical to the one already demonstrated above.

$$\begin{aligned} n = 3: & (3b - 3b^2 - b^3/2)/3b \\ & (3b^2 - 2b^3)/3b \\ & (5b^3/2)/3b \end{aligned}$$

$$\begin{aligned} n = 4: & (4b - 6b^2 - 2b^3 - 5b^4/3) / 4b \\ & (6b^2 - 8b^3 + 2b^4) / 4b \\ & (10b^3 - 11b^4) / 4b \\ & (32b^4/3) / 4b \end{aligned}$$

$$\begin{aligned} n = 5: & (5b - 10b^2 - 5b^3 - 25b^4/3 - 73b^5/12) / 5b \\ & (10b^2 - 20b^3 + 10b^4 - 97b^5/18) / 5b \\ & (25b^3 - 55b^4 + 211b^5/6) / 5b \\ & (160b^4/3 - 539b^5/6) / 5b \\ & (2381b^5/36) / 5b. \end{aligned}$$

No attempt has been made to derive from the above information the general expressions of the vertical distances between the points of juncture of the delay distribution function. In general, it can be stated that with this

Table 4

Chain length	Section				
	1	2	3	4	5
2	1	-	-	-	-
3	5/6	1/6	-	-	-
4	71/96	20/96	5/96	-	-
5	1015/1500	336/1500	117/1500	32/1500	-
6	147021/233280	53806/233280	21786/233280	8286/233280	2381/233280

type of service the quality is slightly better for shorter delays but somewhat worse for the longer delays.

11. Fry's Space Distribution Function

The Poisson function $P_c(y) = y^c e^{-y}/c!$ constitutes the border value of the Bernoulli function $B_c(n, b)$, when within any observation period of one hour, the number of calls is increased to an infinite value while maintaining the product of n and b at the value y . The accuracy of this statement has been proved by several known mathematical methods.

It is of interest, however, to mention here another interpretation of the transition from Bernoulli to Poisson functions, which is based on the assumption that the number of calls that arrive during the hour constitutes an average instead of exactly n calls. This can be proved in the following manner.

We increase the observation period from one hour to z hours. The number of calls that arrive during this period is proportionally increased to zn calls, whereas the proportion between the duration of one hour and the observation period amounts to $1/z$. The probability of finding exactly i calls during an arbitrary hour amounts to $B_i(zn, 1/z)$ and the probability of finding within such an hour c calls during an interval t , amounts to $B_c(i, t)$. The total probability of finding c calls within any interval t of the z hours is equal to:

$$\sum_{i=0}^{zn} B_i(zn, 1/z) \times B_c(i, t) = B_c(zn, t/z). \tag{10}$$

This identity is based on Lemma (1c) mentioned in my previous contribution to this periodical [3]. If z is increased to assume an infinite value, the above Bernoulli expression converts to a Poisson expression.

Thus we dispose of two distinct mathematical proofs for passing from the Bernoulli law to the Poisson law, the one by maintaining the hour and reducing the duration of the holding time, and the other by increasing the number of hours to an infinite value and maintaining the duration of the holding time at a measurable value.

It will, however, be realized that the two methods are identical, differing only with respect to the scale of the processes.

With either of the two interpretations, the Equation (6) converts to:

$$F_0(y, 1) = (1-y)$$

$$F_c(y, 1) = (1-y) \left\{ \sum_{i=0}^c P_i[-(c-i)y] - \sum_{i=0}^{c-1} P_i[-(c-i-1)y] \right\},$$

where $c \geq 1$. (11)

Although a different notation is used, the above result is directly comparable with that obtained by Fry. The probability is denoted by F , the first letter of the name of the author. The traffic that appears in this equation is negative.

The introduction of the same limit into Equation (7) leads to the equivalent of Fry's Equation (198):

$$D = y^2/2 (1-y) \tag{12}$$

The alternation of positive and negative terms in Equation (11) causes laborious evaluation, which can be obviated by the application of the principle already men-

tioned at the end of Section 8 in connection with the corresponding Bernoulli Equation (6). We then have, in accordance with Lemma (D):

$$\sum_{i=0}^c P_i[(i-c)y] = \frac{1}{1-y} - \sum_{i=c+1}^{\infty} P_i[(i-c)y].$$

Introducing this in Equation (11), we have:

$$F_c(y, 1) = (1-y) \left\{ \sum_{i=c}^{\infty} P_i[(i-c+1)y] - \sum_{i=c+1}^{\infty} P_i[(i-c)y] \right\}. \tag{11a}$$

In this equation, the traffic values are positive and, similarly, so are all Poisson terms.

12. Erlang's Equations

When introducing the limit $n = \infty$ into the above Equation (9), which states the value of a number of coordinates associated with a number of points situated on the delay distribution function, we obtain for point u , counting from left to right, the coordinates:

$$t/b = u$$

$$p(>t) = \frac{1}{y} - \frac{(1-y)}{y} \sum_{i=0}^u P_i[-(u-i)y] \tag{13}$$

u ranges from 0 to ∞ , and $y < 1$.

The same result is arrived at when introducing $t/b = 0$ into Erlang's equations appearing on pages 146 and 147 of the publication mentioned above [1]. Erlang's equations are, therefore, based on the discipline that the waiting calls are served in the order of their arrival.

By applying Lemma (D), Expression (13) may also take the form:

$$t/b = u$$

$$p(>t) = \frac{1-y}{y} \sum_{i=u+1}^{\infty} P_i[(i-u)y]. \tag{13a}$$

13. Lemmas

A number of lemmas have either been applied in the preceding sections or form the conclusion that can be drawn from the various equations derived.

Equation (1) expresses the probability of the occurrence of some traffic pattern that comprises a specific group of chains, composed of specific numbers of connections. In conclusion, the sum of the probabilities of all the types of traffic patterns that are possible must amount to one. The natural subdivision of the totality of these traffic patterns is the one according to the subgroups that comprise the same number of chains K . In elaborating the summation of the probabilities of a subgroup, the following simple expression is obtained:

$$B_{n-K}(n-1, nb).$$

n represents, as before, the number of calls offered per hour to the single outlet and b is the invariable holding time of the connections.

This result provides a number of lemmas, the principal one being the one valid for the subgroup comprising two chains. In accordance with Equation (1) we then have:

$$\sum_{i=1}^{n-1} \frac{1}{2} \binom{n}{i} (ib)^{i-1} (nb-ib)^{n-i-1} (1-nb)$$

$$= B_{n-2}(n-1, nb).$$

The ensuing lemma reads:

$$\sum_{i=1}^{n-1} B_i(n, i/n) / B_1(2, i/n) = (n-1). \tag{A}$$

From Equation (3) it appears that:

$$\sum_{m=1}^n m \binom{n}{m} (mb)^{m-1} (1-mb)^{n-m-1} (1-nb) = n$$

It follows that:

$$\sum_{m=1}^n \frac{1}{(1-mb)} B_m(n, mb) = \frac{nb}{1-nb}, \text{ where } nb < 1. \tag{B}$$

From Equations (3) and (4) it appears that:

$$\sum_{m=1}^n \binom{n}{m} (mb)^{m-1} (1-mb)^{n-m-1} (1-nb) = n [1-(n-1)b], \text{ or:}$$

$$\sum_{m=1}^n \frac{1}{(1-mb)mb} B_m(n, mb) = n + \frac{nb}{1-nb}.$$

Combination with Lemma (B) leads to:

$$\sum_{m=1}^n \frac{1}{mb} B_m(n, mb) = n, \text{ where } nb < 1. \tag{C}$$

On pages 130 and 161 of the publication mentioned above [1], Erlang refers to a theorem that was first published by J. L. W. V. Jensen [4]. We add it here merely for the sake of completeness. It reads:

$$\sum_{i=0}^{\infty} P_i(a + bi) = \sum_{j=0}^{\infty} b^j = 1/(1-b), \text{ where } b < 1. \tag{D}$$

There exists a Bernoulli counterpart of this lemma, which reads:

$$\sum_{j=0}^n B_j[n, (a + bi)] = \sum_{j=0}^n \frac{n!}{(n-j)!} b^j, \text{ where } nb < 1. \tag{E}$$

Finally we have:

$$\sum_{i=1}^{\infty} \frac{1}{i} P_i(bi) = b.$$

which is the Poisson counterpart of Lemma (C).

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[1] The Life and Works of A. K. Erlang, published by Copenhagen Telephone Company, pp. 133, 145, and 156.
 [2] T. C. Fry: Probability and its Engineering Uses, D. Van Nostrand (1928), pp. 374 and ff.
 [3] J. Kruithof: On the Relation between Time, Space, and Holding-Time Distribution Functions, Electrical Communication, volume 41, no. 3, 1966, pp. 252-265.
 [4] J. L. W. V. Jensen, Acta Mathematica, 1902, p. 306.

J. Kruithof was born in Rotterdam, The Netherlands, in November 6, 1894. In 1922 he received the degree of Electrotechnical Engineer from the Delft Technical University and in 1945 the degree of Doctor of Applied Sciences from Ghent University in Belgium.

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United States Patents Issued to International Telephone and Telegraph System, February—April 1967.

Between 1 February and 30 April 1967, the United States Patent Office issued 68 patents to the International System. The names of the inventors, company affiliations, subjects, and patent numbers are listed below.

P. R. Adams, C. C. Miller, Jr., and R. S. Bovitz, ITT Laboratories, *Gyroscope*, 3 309 931.

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J. Battista, ITT Kellogg, *Power Supply Heat Sink*, 3 305 704.

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Designs

Time Shared Telegraph Transmission System Including Sequence Transmission With Reduction of Start and Stop Signals

3 310 626

F. T. Cassidy, Jr.

A telegraph channel-divider system is described in which signals from a number of

subchannels may be transmitted over a single transmission line and redistributed to the corresponding subchannels at the receiving end with a smaller number of signal units than is possible with present systems. The system is self-synchronous between terminals and, accordingly, does not require initial or operational phasing, is completely transistorized resulting in low power consumption, and is such that any printer channel may be divided into subchannels of fractional speed.

Method and Apparatus for Optionally Writing-In And Reading-Out Variable Length Information Blocks in Circulating Memories

3 303 484

H. Endres, H. Bohme, and G. Jung

A method and apparatus is described for optionally writing-in and reading-out variable-length information blocks in circulating memories. By allotting two of each seven-bit word spaces on a magnetic drum track for storage of "busy-idle" signals, beginning, intermediate, and end word positions in each block of stored information are readily distinguished with a minimum amount of address logic circuitry. Each track is organized into two groups, A and B, to which are assigned alternate word positions on the track so that alternately all of the "busy-idle" central characters in one of the groups are addressed while those in the remaining group are ignored.

Method of Making Coaxial Cables

3 306 793

R. Y. Gill and G. A. Morel

This is a method of manufacturing a coaxial cable in which polythene spacers are formed as triangular projections across a continuous length of polythene strip. The strip is folded around the inner conductor to form a tube with the projections supporting the inner conductor along the center of the tube at spaced intervals.

Blocking Oscillator with Turn-off Effected by Magnetizing Current In A Self-Inductance Coil

3 303 352

A. Mingaud

Use is made of a blocking-oscillator circuit that is capable of producing accurately controlled pulses and is particularly suited for determining the exact instant wherein an item of information is to be written or read on a magnetic memory. A trigger pulse applied to the base circuit of a normally blocked transistor initiates conduction in the transistor and a feedback path quickly brings the transistor to saturation. An inductance in the feedback circuit provides accurate control of the pulse duration and rapid turnoff of the pulse.

Automatic Drilling Machine

3 302 494

P. A. Taysom and R. E. Kirk

In this automatic drilling machine a framework has a pair of clamps for holding one edge of the work piece and a spindle that supports the work piece directly opposite the point where the drill bit is positioned. This construction makes it possible to change the attitude of the drill bit relative to the work piece by moving the spindle which backs up the work piece.

Jitter Compensating Circuit For Angle Moding Apparatus

3 312 903

R. C. Webb

A circuit is provided for minimizing the amplitude and phase jitter contained in the electrical signal output of mechanical transducers and caused by mechanical imperfections in the transducers. The circuit is particularly adaptable to transducers used to provide a digital output indication of the angular position of a rotating shaft. The circuit utilizes amplitude and phase detectors to ample the jitter and either adds or subtracts corrective voltages to compensate for the jitter variations.

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This list includes papers published in other periodicals, and lectures presented at meetings. Some of these latter are also available in a written form, either printed in proceedings or edited internally. Where the publication is indicated, requests for reprints should be made directly to its editor, not to *Electrical Communication*. In the other cases, such requests should be made to the nearest editor of *Electrical Communication* or its associated language versions, as it may happen that a limited number of copies could be made available.

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Recent Achievements

Large Intercommunication System for the University Clinic in Frankfurt on Main — The installation has begun of the ITT-411 intercommunication system with a capacity of 360 stations provided with push-button calling, 20 connecting links and direct-call facility for 10 groups. It is further intended to connect 11 secretarial systems.

The system has the following characteristics:

- all units are of the plug-in type
- the system is accommodated in closed cabinets and is arranged for easy supervision
- it may be easily expanded in steps of 20 lines
- speech paths are switched through by crossbar switches
- push-button dialing facilities

The installation is in the multi-storey block of the University Clinic serving facilities such as diagnosis rooms, operating theatres, dispensary, blood bank, and so on.

Standard Elektrik Lorenz AG, Germany

Closed-Circuit Television — A total-system concept of closed-circuit television now being marketed means supplying camera, monitor and ancillary equipment, and backing up with systems know-how in transmission techniques.

The camera uses 625-line scanning at 50 fields a second. Remote control is included for focusing, lens positions, aperture, zoom, pan and tilt, and screen wiper for weather-proof camera housings.

Display monitors are 19- or 11-inch (48- or 28-centimeter) diameter tubes with a black-tinted glass cover to the tube face to improve contrast. The monitor can be supplied in a number of stands or fixtures.

The ancillary equipment includes synchronizing generators, distribution amplifiers, video amplifiers, and so on.

Standard Telephones and Cables Ltd., United Kingdom



The castor-mounted cradle floor stand enables this 19-inch closed-circuit-television monitor to be given almost any orientation.

FM 120—4000 Radio-Link Equipment — A new transistorized radio-link equipment for use in the 4-gigahertz band with a capacity of 120 frequency-division-multiplex telephone channels, was developed in 1966, and the prototype successfully tested in 1967.

The main characteristics of this equipment are:

- system figure of merit: 146.2 decibels minimum in the worst channel
- radio-frequency output power: 500 milliwatts
- intermodulation noise: less than 200 picowatts
- "hot" standby equipment on same frequency

Different mechanical designs are available for mounting in normal bay frame or transportable box. The dimensions of an equipment, comprising a transmitter-receiver with hot standby on same frequency are approximately: 720 millimeters (28 inches) in height, 540 millimeters (21 inches) in width and 200 millimeters (7.8 inches) in depth.

Factory production has started.

Bell Telephone Manufacturing Co., Belgium

Satellite Earth Terminals Expanded — High performance receiving and transmitting equipment has been developed and installed in the earth terminal stations at Andover, Maine; Brewster Flat, Washington; and Paumalu, Hawaii: of the Communications Satellite Corporation. Included in the contract was the training of operating and maintenance personnel.

The design and installation of the original Brewster Flat and Paumalu terminals was completed by us in 1960. All three earth stations handle intercontinental traffic via satellites of the Intelsat II series.

ITT Defense Communications Division,
United States of America

Message Switching Network — Stewarts & Lloyds, one of the United Kingdom's largest steel founders, has ordered the 6300-ADX® computer-based message-switching system to improve communications between country-wide locations. The new exchange will be at an existing control center in Birmingham. At first 20-input and 24-output channels will be used but the system can be increased to 32-duplex channels when needed.

The new system will replace the existing torn-tape equipment. Messages received will be sensed electronically, passed to the computer magnetic-store, and then transmitted over lines according to the computer program that makes all routing and queuing decisions.

The new system will be rented from Data Systems Division and will include telegraph multiplex equipment from the Transmission Group and teleprinters from Creed and Company.

Standard Telephones and Cables Ltd., United Kingdom

Image Dissector in Advanced Technology Satellite — An experimental space camera developed under the direction of the Goddard Space Flight Center of the National Aeronautics and Space Administration employs an image-dissector tube to convert photographic images into electrical signals for transmission to earth.

Aboard the Advanced Technology Satellite (ATS-111) stationed in a fixed position at the equator over South America, the camera photographs the whole of North and South America every 13 minutes during the daylight hours. Details include clouds only 5 miles (8 kilometers) in extent.

ITT Industrial Laboratories, United States of America

Improved High-Frequency Transmission — The TM 2/3 Lincompex (linked compressor and expander) system, originally conceived by the British Post Office provides noise-reduced high-quality communication over long-distance high-frequency radio links. Full-duplex operation reduces noise, interference, and fading. A high signal-to-noise ratio is obtained during quiet speech and

® Registered trademark

Recent Achievements

during non-speech intervals, noise is almost eliminated. Voice-operated anti-singing devices are no longer needed to prevent echoes.

On the transmit side a fast-acting compressor in the speech path, controlled by an amplitude assessor in the control path, smooths out variations of speech volume. An assessor-generated control signal outside the normal speech range varies in frequency according to the speech amplitude and is combined with the constant-amplitude speech signal for transmission in a normal 3-kilohertz channel. The control signal is transmitted using frequency modulation to reduce effects of fading.

At the receiving end the speech and control signals are separated by filters and the original volume variations in the speech signal restored according to the frequency of the control signal that was related to the transmit level.

Standard Telephones and Cables Ltd., United Kingdom

New Pentaconta® Installations — On October 21, 1967, M. Yves Guena, Minister of the French PTT and M. Jacques Chaban-Delmas, President of the Assemblée Nationale and Mayor of Bordeaux, inaugurated the new 4-wire transit center at Bordeaux and a 6000-subscriber-line extension in the Palais-Gallien exchange. The Bordeaux "CT4", the fourth of its kind in France, was designed to automatically handle the regional and national incoming and outgoing calls.

The new University exchange of Talence-Pessac, officially inaugurated on October 23, 1967 serves 600 stations connected to the PTT network by 32 trunk lines.

Additional equipment delivered to the PTT includes:

- 3000 lines for the Paris—Auteuil VI exchange
 - 4000 lines for the Paris—Chenier exchange
- extending the capacity of these two exchanges respectively to 8000 and 10 000 lines.

Lastly, 6000 lines are on order for two exchanges in Lyons.

A number of orders have also been received for overseas countries, notably:

- 5450 lines for the Ivory Coast
- 47 private exchanges for Jamaica, totaling 3750 internal lines plus 570 trunk lines and 20 operators' desks
- a 2000-line extension to the Phnom Penh exchange in Cambodia
- a 200-line private exchange for Cameroun
- other installations for electricity and gas companies in Tunisia, Ivory Coast, and Cameroun.

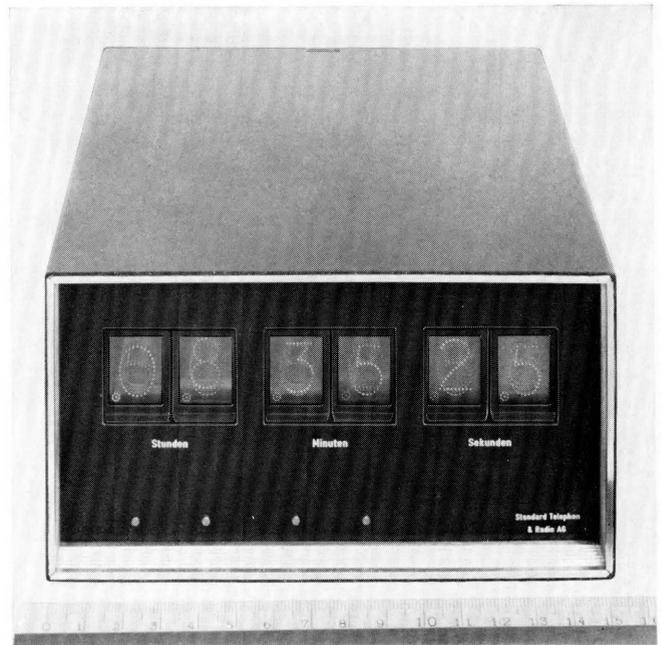
These latter installations utilize high-voltage lines as transmission circuits in accordance with CIGRE (International Congress of Large Electric Networks) recommendations and the operational conditions required by the French and foreign governments. With these equipments, the distribution centers will continuously control the state of the power distribution networks.

Compagnie Générale de Constructions Téléphoniques, France

Luminous Numerical-Display Clock — The compact electronic clock EDU-1 has the following features and technical characteristics:

- it displays time using 6 luminous numerals corresponding to hours, minutes, and seconds from 00.00.00 to 23.59.59 (alternatively, tenths and hundredths of minutes instead of seconds)
- a buffer, so that the clock can deliver non-ambiguous time information while numerals are changing over
- six binary-coded decimal outputs (1-2-4-8), the "1" level corresponding to 1 volt minimum, 2.5 milliamperes per bit maximum, and the "0" level to 0.3 volt maximum with respect to the negative pole of the supply; these outputs are compatible with *RTL*, *DTL*, and *T²L* logics and accessible on a 24-point connector
- the accuracy is equal to that of the mains frequency
- every display position can be reset to any figure using a push button

® Registered trademark



Luminous display clock EDU-1.

- the power supply is 12 volts, 35 volt-amperes, unregulated (since a regulator is incorporated into the clock)
- temperature range is 15 to 45 degrees Celsius
- dimensions are: 160 millimeters (6.3 inches) in width, 85 millimeters (3.3 inches) in height, 242 millimeters (9.5 inches) in depth
- high reliability and operation result from the exclusive use of integrated circuits.

This clock is an indispensable piece of equipment for data collection centers, where data are functions of time as, for instance, in motor-temperature recording, distribution-voltage recording, fault recording in automatic telephone exchanges, and so on.

Standard Téléphone et Radio SA., Switzerland

ITT-411 Loudspeaking Intercommunication System — A number of orders have been received for the ITT-411 loudspeaking intercommunication system. The Danish Government Hospital of Copenhagen order may be one of the largest in the world for a loudspeaking intercommunication system. The order includes delivery and installation of loudspeaking sets and a 1200-line crossbar exchange together with 24 smaller installations for intensive-treatment departments.

Another order for a 400-line installation is intended for the British European Airways' (BEA) new administration building at London Airport. The exchange will interwork with other ITT-411 exchanges over British Post Office lines.

Standard Radio & Telefon AB, Barkarby, Sweden

Punched-Card Unit for Automatic-Line-Test Equipment — The testing of telephone lines in subscriber long-distance dialing has been automated. Up to 1000 punched cards can be fed one-by-one into the reading unit where the information on the punched card is translated into electric signals. After the completion of a line test, the time, results and any other relevant data may be printed on the punched card. The unit subsequently distributes the cards into four bins according to test results.

The punched-card unit is intended for installation in telephone racks. The card feeder, reader, printing units, and the card distribution mechanism may be connected externally via 135 connection points.

A high degree of operational reliability has been achieved, as the system is to be in operation chiefly at night.

Standard Elektrik Lorenz AG, Germany

Selecto-Call System for Large Areas — A recently developed a selecto-call system enables a subscriber using his car to be called by or maintain contact with a transmitter.

The receiver installed in a car works with the car antenna and battery but is portable and will operate with its own telescopic antenna and battery which are automatically-connected when the set is taken out of the car.

The fixed central equipment, generally a 1-kilowatt frequency-modulated transmitter, continually radiates a signal so that the mobile subscriber can measure the field strength and suitably park his car for operation. The subscriber is informed of a call by an acoustic signal associated with a signaling lamp. The call is repeated after an interval of one minute to minimize lost calls. A coverage up to approximately 30 kilometers (18.5 miles) can be expected.

Technical data:

— frequency range	68 to 87.5 megahertz
— channel spacing	50 kilohertz minimum
— receiver sensitivity	0.5 microvolt
— number of code frequencies	14 maximum
— capacity with 4-frequency code	24 000 subscribers per channel
— call transmission delay	400 milliseconds
— receiver dimensions	16 by 13.8 by 6.7 centimeters (6.3 by 5.4 by 2.65 inches)
— receiver weight	1.25 kilograms (2.7 pounds)

Standard Eléctrica, SA, Spain

PABX 4 for Gas Board — The latest type of private automatic branch telephone exchange, the PABX 4, has been cut over at the headquarters of the East Midlands Gas Board (EMGB). It has 550 internal extensions, 88 lines to the public network, and 24 private lines to other EMGB establishments.

Four operators work the exchange dealing with up to 2500 incoming public calls per day. Up to 3300 outgoing calls to the public network are dialed directly by the 550 extensions. Automatic equipment will handle 1800 calls a day among the extensions.

The exchange offers selective answering because calls awaiting an answer are given a visual identity enabling the operator to select which call to deal with first.

If a call routed to an extension is not answered within 30 seconds a visual warning alerts the operator. The exchange also has automatic recall and automatic transfer facilities.

Standard Telephones and Cables Ltd., United Kingdom

Contract for New Submarine Cable — The new 720-channel (3-kilohertz channels) submarine cable link between Florida and the Virgin Islands will come into operation in late 1968. It will use 1350 nautical miles of a new design of larger diameter lightweight cable made in the Southampton factory of Standard Telephones and Cables, now modified and expanded to produce the cable. The cable of 1.5-inch (38-millimeter) diameter was developed by Bell Telephone Laboratories, the research and development organisation of American Telephone and Telegraph Corp., in the United States. The transistorized repeaters for the cable link will be made by the Western Electric Company in United States.

Standard Telephones and Cables Ltd., United Kingdom

VOR S Very-High-Frequency Omnidirectional Radio Beacon — A very-high-frequency omnidirectional radio beacon based on the latest technical achievements and employing advanced components that requires practically no maintenance is under development.

The system is entirely solid-state with no moving mechanical parts, operates on 40-volts direct current from a mains power unit with automatic changeover to battery supply in the event of

a power failure. Low-power consumption also allows the use of wind-driven generators, thermo-electric cells, or similar devices as primary power sources.

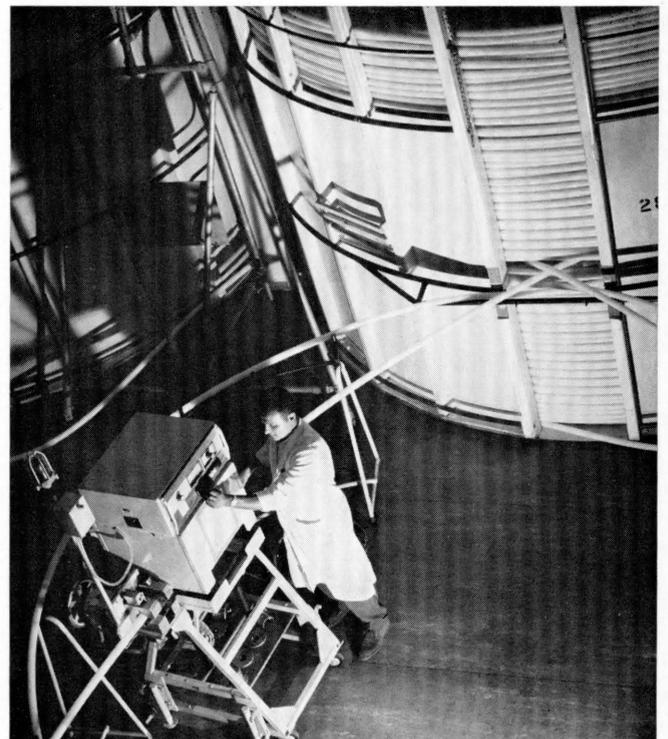
The flexible design of the new omnidirectional radio beacon permits easy modification to meet various operational requirements. It fulfils all the recommendations of the International Civil Aviation Organization (ICAO).

The whole equipment is accommodated in a housing covering approximately 2.7×2.7 meters (8.9×8.9 feet) of ground, with a height of 2.2 meters (7.2 feet). The round roof, 4.5 meters (14.8 feet) in diameter, serves as a counterpoise for the antenna.

Standard Elektrik Lorenz AG, Germany

Portable and Mobile Transceiver, Type TR-TM-4 — The military transceiver TR-TM-4 is a high-frequency radio-communication equipment, in the band 2 to 12 megahertz intended for tactical transmissions at distances greater than those usually obtained from very-high-frequency frequency-modulated sets. It can be modulated for continuous-wave telegraphy A1, for frequency-shift telegraphy F1, for single-sideband compatible with restored carrier A3H, and double sideband A3. It is produced in several versions: as a portable set, fed by a rechargeable battery, or by a crank generator, or as a mobile vehicular set. It can be connected either to a 3- to 5-meter whip antenna, or to longer antennas and dipoles. The transmitted power is 10 to 15 watts, the useful range through ground-wave propagation is about 40 kilometers (25 miles) when whip antennas are used; wire and dipole antennas increase the range beyond 1000 kilometers (621 miles) through space-wave propagation.

The TR-TM-4 which is fully solid-state includes a high-accuracy frequency synthesizer. The error related to the displayed selected frequency does not exceed a few hertz over the temperature range of -40 to +55 degrees Celsius and a variation in supply voltage of 22 to 30 volts. The pilot-oscillator frequency needs resetting once a year by reference to a known external standard frequency.



Lightweight 1.5-inch (38-centimeter) submarine cable manufacture at STC's Southampton factory. The intermediate storage frames house lengths of cable that have reached the stage where the copper outer conductor will next be added. These lengths have to be joined together. The operator is inspecting the joints using special X-ray apparatus.

Recent Achievements

The tuning operation consists of selecting the desired frequency by adjusting the 4 knobs associated with the switching of the following digits: megahertz; hundreds, tens, and units of kilohertz. This operation gives 10 000 available channels. The frequency accuracy allows the instantaneous entry into a conference network and the use of narrow-band multifrequency telegraph systems like the Coquelet system. A switchable-frequency interpolator allows communication with obsolete equipments whose frequency accuracy and stability are below standard. The set includes an antenna coupling unit for the whip antenna tuned by two knobs and an antenna-current meter.

The first TR-TM-4 sets, produced for the French Army were type approved in December 1967, after several months of satisfactory tests made in the factory as well as in the field.

Le Matériel Téléphonique, France

The Pentaconta Toll Transit Exchange of Bologna — The transit center of Bologna represents an important step in the development of subscriber-to-subscriber direct toll service. This exchange operating on a 4-wire basis is supplied with Pentaconta equipment. It was cut into service in August 1967.

Because the Bologna area, with its 18 district centers, handles a very heavy toll traffic, it was necessary to install a transit exchange there. It is expected that approximately 8000 trunk circuits, incoming and outgoing, will be connected to it over a 20-year period.

The exchange consists of two selection stages:

- the first stage only distributes traffic; it will ultimately reach 22 selection elements (as of now 3 have been installed)
- the second stage performs the selection operations and will ultimately be equipped with 14 selection elements (as of now 2 have been installed).

Six control units equipped with 400 transit registers will supervise the whole exchange (as of now 44 registers have been cut into service).

At the beginning of 1969, other Pentaconta sections will be added to replace existing rotary 7D equipment. At first, these new Pentaconta sections will operate with the older rotary equipment which will be replaced as it becomes obsolete.

In a not-too-distant future, the following Pentaconta exchanges will be realized:

- local tandem exchange for the local transit network
- toll exchange for the traffic
- toll exchanges for the district and interdistrict traffic.

Eventually, the whole center will switch more than 20 000 incoming and outgoing trunks and will then be one of the largest Pentaconta switching centers in Europe.

Fabbrica Apparecchiature Per Comunicazioni Elettriche, Italy

Control of Power Station Using Dry-Reed Relays — Encapsulated dry-reed relays are the switching element used at the Dungeness Nuclear Power Station to control sequential start-up and shut-down of the plant. The control system supplied by Integrated Electronic Systems Division includes automatic monitoring devices interconnected to prevent malfunction. The system controls 1320 megawatts of electrical power.

Standard Telephones and Cables Ltd., United Kingdom

24 000-Trunk Toll Exchange in Madrid — A new toll exchange was cut over in Madrid in December 1967. Madrid-Alcántara is the largest transit office in Spain and is constructed with Pentaconta crossbar equipment.

The switching network consists of two independent selection chains. The first one deals with the incoming (terminating and transit) traffic which comes through 2755 toll trunks. The outgoing trunks are 750 toll and 2326 urban.

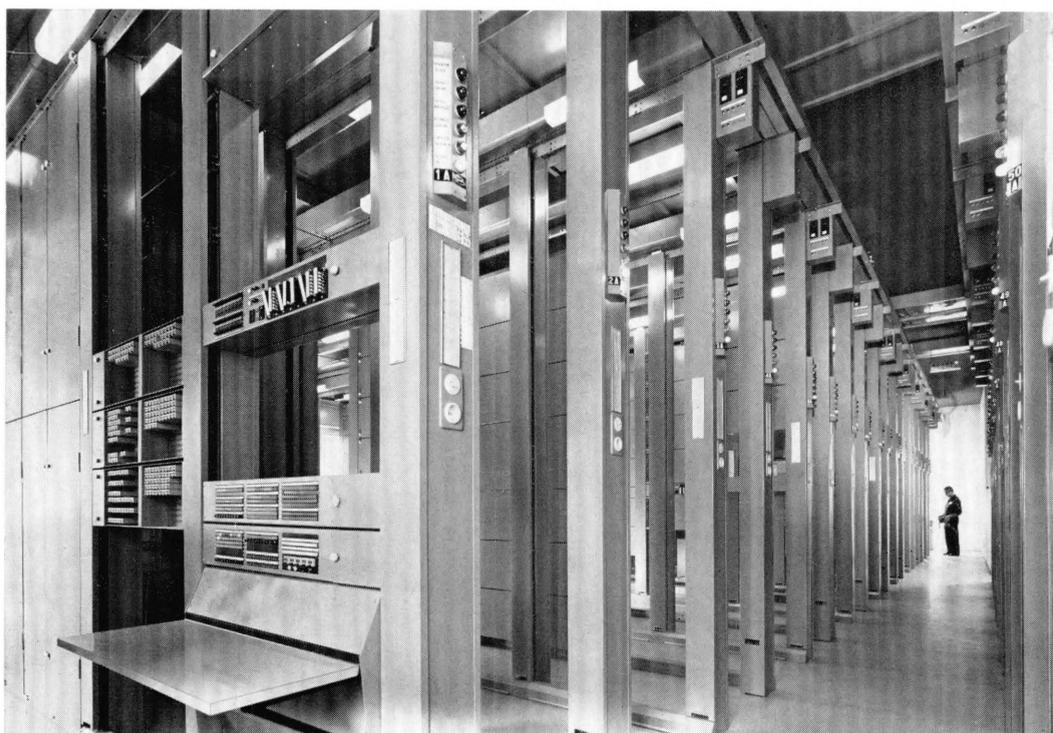
The second chain handles the outgoing traffic originated from 2356 urban junctions and directed to 2045 outgoing toll trunks. This gives a total of approximately 5000 plus 5000 trunks, the final capacity being of 12 000 plus 12 000.

The switching of all classes of calls is made on a 4-wire basis, the hybrid coils being in the 2-wire trunks.

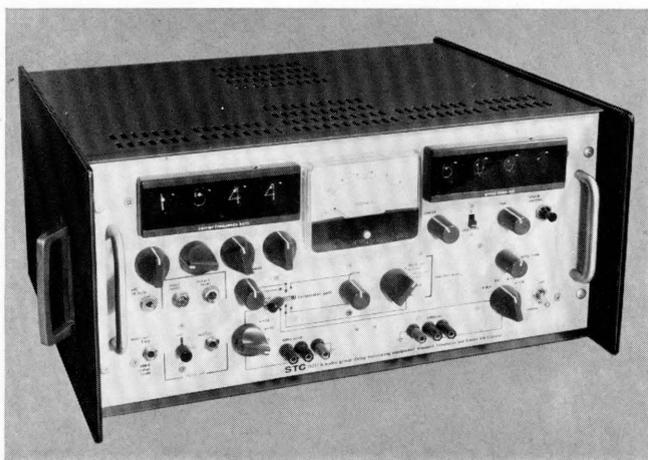
Line signaling is in-band, using 2500 hertz.

Register signaling makes use of multi-frequency code for Pentaconta offices and CCITT No. 3 code for the existing network.

Standard Eléctrica, S. A., Spain



Toll transit exchange of Bologna.



74257 group-delay measuring equipment.

Group-Delay Measuring Equipment Suitable for Data Transmission — The 74257 group-delay measuring equipment can be used on audio, broadcast or multicircuit telephone systems where the group delay is a significant parameter for data transmission.

The testing signal is obtained from an internal oscillator in the range 200 hertz to 29.99 kilohertz or from an external oscillator in the range 200 hertz to 120 kilohertz.

The delay-time measurement and oscillator frequency are both displayed digitally by indicator tubes.

Loop measurements can be made on a relative or absolute basis and end-to-end measurements by two equipments separated by many miles.

The group-delay measuring range is 0 ± 20 milliseconds in 0.01-millisecond steps. The instrument is portable and uses solid-state circuitry throughout.

Standard Telephones and Cables Ltd., United Kingdom

Antenna on Submarine for Satellite Communication — An antenna has been developed to permit a submarine to communicate with a satellite and through it with other stations in contact with that satellite. The antenna is steerable in azimuth and elevation, yet is compact enough to fit in the superstructure of the submarine.

An experimental installation on a submarine permitted successful participation in recent inter-service satellite communication tests via LES-5, an experimental communication satellite. This is the latest development in a program of designing submarine antennas for operation on the very-low through the ultra-high-frequency range.

ITT Defense Communications Division,
United States of America

Hundred Pentaconta Exchanges in Service in Spain — Five years after the first Pentaconta exchange was put into service in Spain, the 100th exchange has been cut over.

This expansion began with the simple installation of Igalada, an isolated 2000 line exchange installed in 1962. The sophisticated 100th exchange of Bilbao-Gran Via, with 10 000 lines initially and a projected capacity of 40 000 lines will serve the highly-industrialized town of Bilbao. Pentaconta switching equipment will meet the many requirements of a complex network including interconnection with the various existing rotary systems, as well as urban and toll tandem exchanges with thousands of trunks.

At the cutover of this exchange in October 1967, the total number of lines in service reached 265 300. All of them, except for Igalada, have been engineered by Standard Eléctrica, SA,

and manufactured in the Villaverde plant, which was specially built for the manufacture of Pentaconta equipment at the rate of more than 200 000 lines per year.

Standard Eléctrica, SA, Spain

Major Iranian Contract Completed — Two control centers and eleven outstations have been completed as part of the "CHAM" project for the Iranian Oil Refining Company by Integrated Electronic Systems Division.

The Selectronic® equipment is a fast time-division multiplex system using digital techniques and solid-state components (see Electrical Communication 43/1 1968). The outstations communicate over 4-wire telephone circuits with the master stations at Abadan and Bandar Mah Shahr, 57-miles (92-kilometers) distant. The two control centers are connected by a microwave link for repeating some of the measurements received from the outstations.

For outstations near the control centers simple direct-wire techniques are used to relay information but for the more distant stations various forms of multiplex signaling are used.

Standard Telephones and Cables Ltd., United Kingdom

Spain—United States Direct-Trunking Via Satellite — At the beginning of 1968, Compania Telefónica Nacional de España put into service the first semi-automatic telephone links for direct traffic between Madrid and New York. These both-way intercontinental lines via satellite use CCITT No. 5 signaling.

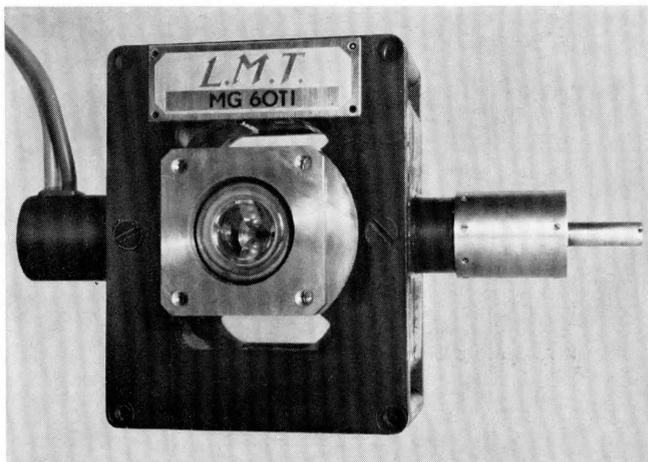
The Pentaconta switching network has been installed in the Gran Via exchange; the both-way intercontinental selector of the 7A trunk exchange operates through the incoming intercontinental selector of the international center. The incoming intercontinental calls carried over the both-way lines are automatically directed to the subscribers of the Spanish network through the trunk exchange 7A.

The Gran Via exchange joins the intercontinental exchange of Buitrago via the trunk exchange of Alcántara and Leganés by means of a coaxial cable and a radio link.

The ground station of Buitrago transmits and receives traffic through the active synchronous satellite INTELSAT II F3.

Standard Eléctrica, SA, Spain

X-Band MG 60T1 Magnetron — This tunable pulsed magnetron delivering a peak power of 10 kilowatts has as its main characteristic a high stability of operation necessary for moving-target-indication radar (stationary target suppression). High stability of pulse length, amplitude, and frequency has been achieved by the accurate machining of the anode block (hobbing) and a special design of the cathode.



MG 60T1 Magnetron.

Recent Achievements

Life tests over 1000 hours have shown negligible changes of the characteristics. Manufacture will begin at the end of 1968.

Le Matériel Téléphonique, France

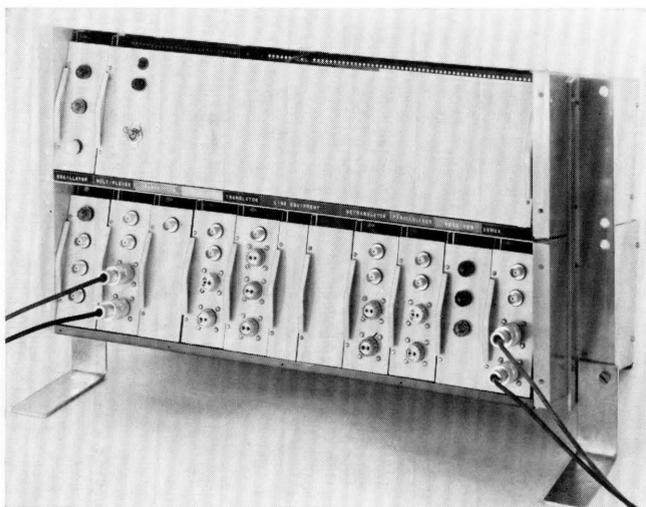
Multiplexing of Pulse-Code-Modulation Systems — A family of compatible multiplexers has been planned with capacities of 96, 384, 1536, and 6144 channels, respectively. The latest addition to this hierarchy is a prototype of a 1536-channel 100-megahertz multiplexer which is shown below.

The different lower-order multiplex groups are interleaved digit-by-digit and need not be synchronous. The incoming groups have their bit rates equalized with redundant digits by "pulse stuffing" before being multiplexed synchronously. The multiplex frame rate is 2 percent larger than the sum of the incoming group rates to allow for the insertion of the control digits allocated to the multiplex frame synchronization, the redundant digits, and the redundant-digit location codes.

The multiplexer is combined with a line translator which converts groups of four binary-digits to three ternary-digits more suitable for line transmission. These ternary codes are selected in such a way that direct-current balance and an adequate timing-content is guaranteed in the line signal. The translator is fed in parallel from the multiplexer so that all the equipment works at the incoming group rate, except for the output stage. This generates a ternary signal at 75 megabaud, conveying information at the rate of 100 megabit per second.

Pulse-code-modulation encoded speech was fed through the complete 96-, 384-, 1536-channel synchronous multiplex-demultiplex hierarchy with negligible impairment. The equipment uses integrated circuits throughout.

Standard Telecommunication Laboratories, United Kingdom



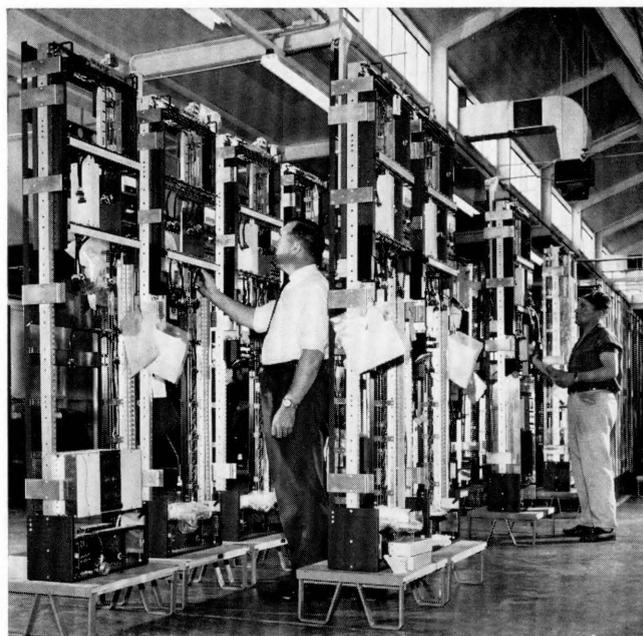
Prototype of a 1536-channel pulse-code-modulation multiplexer.

Selbond Color Picture Tube — The Selbond® design of television picture tube described in this journal (vol. 41, number 4; 1966) is now available in a color picture tube type A55-14X. By permitting the tube face to protrude slightly from the front of the cabinet, the depth of the cabinet can be reduced. Shallow cabinets are preferred by the customers. The reduction is particularly important for tubes using 90-degree deflection of the beam.

The A55-14X with a 55-centimeter (21-inch) screen diagonal operates on the shadow-mask principle with 25 kilovolts on the anode. It requires the same deflection technique as color picture tube A63-11X. The depth of the A55-14X with 90-degree deflection is 483 millimeters (19 inches).

Standard Elektrik Lorenz AG, Germany

® Registered trademark



Assembly of RL6D, 6-GHz microwave equipment at the STC Basildon factory.

Microwave Link from BPO Tower in London — The British Post Office has ordered the RL6D 6-gigahertz equipment for a microwave link to connect the BPO Museum tower in London with Norwich, 111-miles (179-kilometers) distant. The link will provide six broad-band channels, each able to carry 960 telephone circuits or color television. The link will be repeated at 30-mile (48-kilometer) intervals en route at stations in Essex and Suffolk.

The RL6D equipment provides a 10-watt output from a periodic-permanent-magnet-focusing travelling-wave tube. The antennas are Cassegrain-type for single- or double-field operation.

In this link it is intended that two channels will carry television; two will carry telephone circuits, and two will remain on standby.

Standard Telephones and Cables Ltd., United Kingdom

Pentaconta 32 Rural System in Spain — Compañía Telefónica Nacional de España has ordered 300 000 to 400 000 lines of rural switching equipment for delivery over a period of 8 years. The equipment will be manufactured in the Madrid factories of Standard Electrica and will be of the Pentaconta 32 type. Exchanges will vary from 30 to 700 lines and include terminal and transit centers with provision for toll charging.

Standard Eléctrica, SA, Spain

800-Hertz Level Discriminator — Since attenuation measurements in telephone channels are carried out periodically in automatic exchanges, an equipment has been developed for this purpose. It provides facilities for rapid measurements and recording, and consists of:

- an 800-hertz signal generator with an internal impedance of 600 ohms and balanced output; the output level is 0 decibel relative to 1 milliwatt with an accuracy of ± 0.1 decibel;
- an 800-hertz tuned receiver selecting the received range from six levels 4.0, 1.7, 1.0, -1.0, -1.7 and -4.0 decibels, with an accuracy of ± 0.1 decibel relative to one milliwatt. Seven output terminals are provided accordingly. When a signal is detected a voltage appears on the corresponding terminal to operate a lamp or a recording device.

This equipment, fully transistorized, operates with a 48-volt direct current over a temperature range of 10 to 60 degrees Celsius.

ITT Laboratories of Spain, Spain

Pentaconta A1 Switching System
Herkomat Electronic-Control Private Automatic Branch Exchange
A 24-channel PCM Junction Carrier System
Mastergroup and Supermastergroup Carrier System
Doppler VOR Ground Equipment
Air-Navigation Horizontal Situation Display
Computation of Urban Trunking Networks with Alternate Routing by Computer
Numerical Evaluation of the Erlang Function through a
Continued-Fraction Algorithm
European Color Television Standards — The SECAM System
Traffic Unbalances in Small Groups of Subscribers
Delay Problems Relating to an Invariable Number of Connections per Hour,
Having Invariable Holding Time, and Carried by a Single Outlet

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