Applications Handbook

Volume 4

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## ADSP-2100 Family Applications Handbook, Volume 4

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## Literature

## ADSP-2100 FAMILY MANUALS

ADSP-2100 User's Manual/Architecture
ADSP-2101 User's Manual/Architecture
ADSP-2111 User's Manual/Architecture
Complete descriptions of architecture and system interface.

ADSP-2100 Cross-Software Manual<br>ADSP-2101 Cross-Software Manual<br>Complete programmer's references, including optional C compiler.

ADSP-2100 Emulator Manual
ADSP-2101 Emulator Manual
ADSP-2101 EZ-ICE ${ }^{\text {TM }}$ Manual
User's manuals for in-circuit Emulators.
ADSP-2100A Evaluation Board Manual
A guide to the Evaluation Board including schematics for prototyping.
ADSP-2101 EZ-LAB ${ }^{\text {TM }}$ Manual
A guide to the EZ-LAB demonstration board and programs.

## APPLICATIONS INFORMATION

Digital Signal Processing Applications Using the ADSP-2100 Family (Formerly the ADSP-2100 Family Applications Handbook, Volumes 1, 2 and 3.) Topics include arithmetic, filters, FFTs, linear predictive coding, modem algorithms, graphics, pulse-code modulation, multirate filters, DTMF, multiprocessing, host interface and sonar.

ADSP-2100 Family Applications Handbook, Volume 4
Topics include V. 32 modem implementation, quadrature amplitude modulation (QAM), frequency shift keying (FSK) modulation, echo cancellation, and adaptive equalization.

## SPECIFICATIONS INFORMATION

ADSP-2100A/ADSP-2100 Data Sheet
ADSP-2101 Data Sheet
ADSP-2111 Data Sheet (preliminary)
ADSP-2105 Data Sheet (preliminary)

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## Introduction ■ 1

### 1.1 OVERVIEW

This is Volume 4 of our Applications Handbooks for the ADSP-2100 family of DSP microprocessors. It presents a compilation of software for a variety of data communication applications, mostly related to the CCITT V. 32 modem recommendation. These examples may be used as they are or they may serve as a starting point for the development of your particular application. Each application is prefaced by a brief discussion of the algorithm that underlies the code.

Along with the specific applications, the routines in this handbook demonstrate a variety of programming tactics. We believe that readers will want to scan every chapter, even if their application interests are confined to a single topic.

This handbook does not explain the architecture or instruction set of the ADSP-2100. Refer to the literature page at the front of this book for additional publications describing the processors in the ADSP-2100 family and their hardware and software development tools. Contact your local Analog Devices Sales Representative for these materials if you need them.

We do not attempt to explain the theory of any application area completely. We have assumed that our readers already understand the theory and practice of their own application areas. The references included in each chapter provide additional background.

### 1.2 ADSP-2100 FAMILY OF PROCESSORS

The processors in the ADSP-2100 family share the core architecture of the ADSP-2100 device. The ADSP-2100 contains three independent computational units-arithmetic/logic unit (ALU), multiplier/ accumulator (MAC) and barrel shifter-that operate on 16-bit fixed-point data. The ADSP-2100 supports a modified Harvard architecture in which

## 1 Introduction

data memory stores data and program memory stores both instructions and data. Its program sequencer and two address generators provide flexible addressing for performing DSP algorithms.

- The ADSP-2100A is a version of the ADSP-2100 fabricated in $1.0 \mu \mathrm{~m}$ CMOS. It is pin- and code-compatible with the ADSP-2100. It is available (currently) in 10 MHz and 12.5 MHz versions, whereas the ADSP-2100 is offered in 6 MHz and 8 MHz versions.
- The ADSP-2101 is a programmable single-chip microcomputer based on the ADSP-2100. Like the ADSP-2100, the ADSP-2101 contains an ALU, a multiplier/accumulator, and a barrel shifter, as well as a program sequencer and dual address generators. Additionally, there are 1 K words of data memory and 2 K words of program memory on chip, two serial ports, a timer, boot circuitry (for loading on-chip program memory at reset), and enhanced interrupt capabilities.
- The ADSP-2102 is identical to the ADSP-2101 with program memory ROM instead of RAM.
- The ADSP-2105 is the same as the ADSP-2101 with half the on-chip memory ( 512 words of data memory and 1 K words of program memory) and one serial port instead of two. It is pin- and codecompatible with the ADSP-2101.
- The ADSP-2111 contains all the features of the ADSP-2101 plus a host interface for direct connection (no glue logic) to a host processor. For example, the Motorola 68000, the Intel 8051, and the ADSP-2101 can all be connected easily to the ADSP-2111. The ADSP-2111 also provides additional flag outputs.

All references to the ADSP-2100 in this handbook apply to all members of the ADSP-2100 family of processors, unless otherwise indicated. Because the processors are code-compatible, the programs in this handbook can be executed on any member of the family (some modifications for interrupt vectors may be necessary), although a program may not take advantage of specific functions, such as the serial ports of the ADSP-2101.

### 1.3 SUMMARY OF VOLUMES 1, 2 AND 3

Volumes 1,2 and 3 of the ADSP-2100 Family Applications Handbooks, formerly published as three separate books, are now available in a single book, Digital Signal Processing Applications Using the ADSP-2100 Family: This book presents information on the following topics:

- Fixed-point arithmetic operations
- Floating-point arithmetic operations
- Function approximations
- Digital filters
- Fast Fourier transforms (FFTs), both one- and two-dimensional
- Image processing
- Graphics
- Linear predictive speech coding (LPC)
- Pulse code modulation (PCM)
- Adaptive differential pulse code modulation (ADPCM)
- High-speed modem algorithms
- Dual-tone multifrequency (DTMF) coding and detection
- Sonar beamforming
- Memory interface
- Multiprocessing
- Host interface


### 1.4 CONVENTIONS OF NOTATION

The following conventions are used throughout this handbook:

- In listings, all keywords are uppercase; user-defined names (such as labels, variables, and data buffers) are lowercase. In text, keywords are uppercase and user-defined names are lowercase and italicized. Note that this convention is for readability only; the Cross-Software modules do not distinguish between uppercase and lowercase letters.
- In comments, register values are indicated by " $=$ " if the register contains the value or " $->$ " if the register points to the value in memory.
- All numbers are decimal unless otherwise specified. In listings, constant values are specified in binary, octal, decimal, or hexadecimal by the prefixes $\mathrm{B} \#, \mathrm{O} \#, \mathrm{D} \#$, and $\mathrm{H} \#$, respectively.


## 1 Introduction

## $1.5 \quad$ PROGRAMS ON DISK

An IBM PC 5-1/4 inch diskette containing the source code in this book is available. Consult your local Analog Devices Sales Office for a copy. As with the printed routines, we cannot guarantee their suitability for your application.

## V. 32 Modems $\boldsymbol{\square} 2$

### 2.1 OVERVIEW

The International Telegraph and Telephone Consultative Committee (CCITT), which determines protocols and standards for telephone and telegraph equipment, has authored a number of recommendations describing modem operation. This chapter surveys the fundamental algorithms of the V. 32 modem recommendation, which describes the operation of a high-speed modem. Implementations of the algorithms on the ADSP-2100 family of DSP microprocessors are shown.

A modem is an electronic device that incorporates both a modulator and a demodulator into a single piece of signal conversion equipment. Interfacing directly to the communication channel, modems establish communication links between various computer systems and terminal equipment. In most cases the communications channel is the general switched telephone network (GSTN) or a two- or four-wire leased circuit. The GSTN is, for the most part, a copper wire network. The bandwidth of this channel is limited to 200 Hz to 3400 Hz .

Traditionally, a modem was implemented using analog discrete components. Today, digital circuits centered around a high performance digital signal processor can meet the demands of modem algorithms without the difficulties associated with analog circuitry. A digital modem implementation offers programmability, temperature insensitivity, ease of design and often reduced cost when compared with analog implementations.

### 2.2 V. 32 MODEM DEFINITION

The V. 32 recommendation describes a full duplex synchronous modem that operates on the general switched telephone network (GSTN) as well as point-to-point leased circuits. The V. 32 modem communicates at a rate of 9600 bits per second (with a 4800 bit per second slow down mode) utilizing quadrature amplitude modulation (QAM). (QAM is discussed in detail in Chapter 3.) Four-bit symbols (bauds) modulate a carrier frequency of 1800 Hz with a modulation rate of 2400 bauds per second. The modulation of 4-bit symbols at a rate of 2400 symbols per second yields the 9600 bit per second specification.

## 2 V. 32 Modems

There are three signal coding modes to choose from in the V. 32 recommendation.

- $9600 \mathrm{bit} /$ second 16-point QAM. Four bits per symbol are transmitted.
- 9600 bit/second 32-point trellis-coded QAM. Transmitted symbols contain four information bits and an additional trellis encoded bit for error correction.
- $4800 \mathrm{bit} /$ second 4 -point QAM.

The second method, which produces a redundant bit for error correction, is the method used in the implementation described in this chapter.

Channel separation is achieved through echo cancellation. Echo cancellers are subject to CCITT specification G.165. An ADSP-2100 family implementation of an echo canceller is described in Chapter 4.

The V. 32 modem transmits with a carrier frequency of $1800 \pm 1 \mathrm{~Hz}$ and must be able to operate with received carrier frequency offsets of $\pm 7 \mathrm{~Hz}$. The V. 32 recommendation also specifies the transmitted spectrum.

### 2.2.1 Transmitter Algorithms

A block diagram of the transmitter section of the V. 32 modem implemented in this chapter is shown in Figure 2.1. The input serial bit stream is subject to a number of algorithms prior to modulation and transmission. Each step is described briefly below and in greater detail in the following sections.

Scrambler. The input serial bit stream is first scrambled by a selfsynchronizing (requires no clock signal) scrambler. Scrambling takes the input serial bit stream and produces a pseudo-random sequence. The purpose of the scrambler is to whiten the spectrum of the transmitted data. Without the scrambler, a long series of identical symbols could cause the receiver to lose carrier lock. Scrambling makes the transmitted spectrum resemble white noise, to utilize the bandwidth of the channel more efficiently, makes carrier recovery and timing synchronization easy and makes adaptive equalization and echo cancellation possible.

Encoders. The scrambled bit stream is divided into groups of four bits. The first two bits of each 4 -bit group are first differentially encoded and then convolutionally encoded. This produces a 5 -bit symbol in which the first bit is a redundantly coded bit.

## V. 32 Modems 2



Figure 2.1 Transmitter Block Diagram

Signal Mapping. The 5-bit symbols are mapped into the signal space (defined in the V. 32 recommendation) for modulation. The signal space mapping produces two coordinates, one for the real part of the QAM modulator and one for the imaginary part.

Pulse Shape Filters. The pulse shape filter is based on the impulse response of a raised cosine function. Used prior to modulation, these filters attenuate frequencies above the Nyquist frequency that are generated in the signal mapping process. The filters are designed to have zero crossings at the appropriate frequencies to cancel intersymbol interference.

Modulation. The modulation for all coding schemes in the V. 32 modem recommendation is quadrature amplitude modulation (QAM). A QAM implementation on the ADSP-2100 family is described in Chapter 3. The carrier frequency is 1800 Hz and the modulation rate is 2400 symbols/ second.

After modulation, the samples are converted to an analog signal. The analog output is filtered through a smoothing filter.

## 2 V. 32 Modems

### 2.2.2 Receiver Algorithms

A block diagram of the receiver section of the V. 32 modem described in this chapter is shown in Figure 2.2. Each step is described briefly below and in greater detail in the following sections.


Figure 2.2 Receiver Block Diagram

Input Filter. The received analog signal is oversampled by a factor of 4 at 9600 samples per second. The sampled input is filtered with a raised cosine pulse shape filter. The output is then decimated by a factor of 2 .

Demodulation. Multiplication by $\mathrm{e}^{-\mathrm{j}\left(2 \pi \mathrm{f} \mathrm{C}^{\mathrm{nT}} / 2\right)}$ demodulates the signal. QAM demodulation techniques are described in Chapter 3.

Adaptive Equalizer. An adaptive equalizer compensates for distortions introduced in the communications channel. A 64-tap fractionally spaced equalizer provides the performance necessary for V. 32 applications. The

## V. 32 Modems 2

equalizer also feeds a timing loop which adjusts the 4 X sampling input and the 2 X sampling output of the input filter. An ADSP-2100 family implementation of an adaptive equalizer is described in Chapter 5.

Viterbi Decoder. The decoder takes as input a demodulated, pulse shaped, equalized signal. The Viterbi algorithm is employed as a decoder in order to determine the appropriate signal constellation point received. This algorithm is a soft-decision maximum likelihood sequence decoder. By keeping a past history of 20 or so baud, the decoder can determine the signal point received in noisy conditions. The phase detector and delay adjust the feedback from the Viterbi decoder to the equalizer, which is constantly adapting in response to the received data.

Differential Decoder and Descrambler. Once the amplitude and phase of the signal point received is known, the corresponding symbol must be backmapped to decode the encoded bits. The decoded 4-bit symbol is then descrambled utilizing the same generating polynomials as the scrambler.

### 2.3 SCRAMBLER

The V. 32 modem recommendation calls for the use of a scrambler in the transmit section of the modem and descrambler in the receive section of the modem. The scrambler and descrambler are based on simple polynomials. Each transmission direction uses a different scrambler, i.e., a different generating polynomial, as specified in the V. 32 specification. The calling or call mode modem uses the following generating polynomial (GPC):
$\mathrm{GPC}=1+\mathrm{x}^{-18}+\mathrm{x}^{-23}$
where $x$ is the input sample and the exponent on $x$ indicates a time delay, e. g., $\mathrm{x}^{-23}$ is the twenty-third previous sample. The answering or answer mode modem uses a similar scrambler with the following generating polynomial (GPA):
$\mathrm{GPA}=1+\mathrm{x}^{-5}+\mathrm{x}^{-23}$
The additions are modulus 2 additions, that is, the bitwise exclusive-OR of the data values. The transmitting modem scrambles the input data sequence by dividing the message sequence by the generating polynomial. The receiving modem multiplies the scrambled sequence by the same polynomial to descramble and recover the original message sequence.

## 2 V. 32 Modems

These polynomials can be thought of as digital filters. The scrambler has an all pole transfer function and the descrambler has an all zero transfer function.

The scrambler output is pseudo-random. For a repetitive input signal, the scrambler output is also repetitive with a maximum period of $2^{\mathrm{k}}-1$ samples, where $k$ is the order of the generating polynomial ( 23 in the case of the V. 32 scrambler). In order to maximize the period of the pseudorandom output patterns, the specified GPC and GPA are irreducible and primitive.

A block diagram of the call mode scrambler is shown in Figure 2.3; $\mathrm{x}_{\mathrm{in}}$ is the serial bit input stream and $D_{s}$ is the scrambled data bit stream. Each delay block corresponds to a serial port cycle and each addition block is an exclusive OR operation.


Figure 2.3 Call Mode Scrambler

The answer mode scrambler block diagram (Figure 2.4) is similar. The fifth delay line sample, $x^{-5}$, is used in the answer mode scrambler rather than the eighteenth delay line value as in the call mode scrambler.

### 2.4 DESCRAMBLING

The descrambler is implemented using a delay line, similar to the scrambler. The descrambler is the last functional block that the data passes through in the receiver. The data that is input to the descrambler is in effect multiplied by the appropriate generating polynomial. This multiplication performs the inverse operation of the scrambler.

## V. 32 Modems 2



Figure 2.4 Answer Mode Scrambler

There are two versions of the descrambler, one for call mode and one for answer mode. Block diagrams for the call mode and answer mode descramblers are shown in Figures 2.5 and 2.6.


Figure 2.5 Call Mode Descrambler


Figure 2.6 Answer Mode Descrambler

## 2 V. 32 Modems

### 2.4.1 ADSP-2100 Family Implementation

Fundamentally, the implementation of the generating polynomials for scrambling and descrambling is the management of a delay line. The scrambler generates its output from the current input bit and two delayed outputs. The call mode uses the eighteenth and twenty-third previous outputs, while the answer mode uses the fifth and twenty-third previous outputs.

The ADSP-2100 family processors have two key features to facilitate efficient delay line management. First, each of two independent data address generators (DAGs) has four independent data pointers. An index register pointer can be programmed to handle each of the delay values and can be separately updated. Second, the DAGs support circular buffers into which delay lines are easily mapped.

In either scrambler, the twenty-third value is the oldest value, and once used is no longer needed. Thus the newest value can be written over it, so the circular buffer always contains only the 23 most recent values. Figure 2.7 illustrates the circular buffer implementation and shows the appropriate pointers.


Figure 2.7 Circular Buffer Implementation for Scrambler

# V. 32 Modems 2 

The value $x^{0}$ is the current input value. This value is put into an ALU register. The delayed value, $\mathrm{D}_{\mathrm{s}} \bullet \mathrm{x}^{-18}$, is read from the circular buffer using the address supplied by a pointer (represented in the above diagram with an arrow). Once the location is read, the pointer is decremented to the next location in the buffer, shown with the light arrow. The oldest value is then written to an ALU register; the pointer's address is not yet modified. The necessary XOR operations are performed and the result is output, as well as written to the last buffer location. This pointer is now decremented to the next value, now the oldest.

This process is repeated with each new input bit. When a pointer comes to the first location in the circular buffer and is decremented, it wraps around to the last location in the circular buffer. Eighteen and twentythree unit delays are maintained in the circular buffer, with no need to move data values, just pointer addresses.

The answer mode scrambler works similarly, except with a delay of five units instead of eighteen units. The descrambler, for both call and answer modes, also uses the same basic structure, but with a different flow of data to accomplish the inverse operation.

### 2.4.2 Scrambler/Descrambler Programs

The code in Listings 2.1 and 2.2 implements the V. 32 scrambler (call mode) on the ADSP-2100 family processors. There are two modules, a main module and a scrambler module. The main module sets up interrupts, initializes the appropriate registers for interrupt control, initializes index registers for maintenance of the circular buffer, clears the circular buffer to zero and waits in an infinite loop for an interrupt. The only interrupt active in this program is IRQ3. This is the highest priority interrupt, and in this case it corresponds to a sampling interrupt. When a sample is ready to be scrambled, this interrupt is asserted.

The second program module is the actual scrambling routine. Included as part of this module is the bits subroutine, which takes 16-bit data values and strips off bits one at a time. The output of this subroutine is a string of simulated serial data values in the most significant bit position of 16-bit words. That is, a 16 -bit word is input and 16 words (each of whose value is either $\mathrm{H} \# 8000$ or $\mathrm{H} \# 0000$ ) are output. These simulated serial bits are then passed to the scrambler. The scrambler output is in the AR register at the end of each pass and is written to the data memory location dac.

The descrambler program, in Listing 2.3, has the same fundamental structure as the scrambler program, performing the inverse operation of the scrambler.

## 2 V. 32 Modems

```
.MODULE/RAM/ABS=0 cms_main_routine;
{ This module initializes registers, clears a buffer}
{ of length 23 for the call mode scrambler, sets IMASK}
{ and waits in a loop for sampling interrupt}
{ CALLS: initial, clear_buffer}
{ INTERRUPTS: only interrupt 3 active}
.CONST no_bits_per_word=16;
.VAR/DM/RAM/CIRC buffer[23], input_buffer [no_bits_per_word];
.GLOBAL input_buffer;
.PORT cntl_port;
.EXTERNAL start_scramble;
{interrupt jump table}
    RTI; {only INT3 is used}
    RTI;
    RTI;
    JUMP start_scramble; {INT3 8 kHz from codec}
{main routine}
    CALL initial;
    CALL clear_buffer;
    IMASK=H#8; {enable interrupt 3}
mainloop: JUMP mainloop; {loop until interrupted}
{—_INIT SUBROUTINE-_}
{One time initialization subroutine, sets up registers}
initial: IMASK=B#0000; {disable interrupts}
ICNTL=H#F; {edge sensitive interrupts}
SI=0;
DM(cntl_port)=SI; {load codec control register}
L0=%buffer; {length registers}
L1=%buffer;
{circular buffer length 23}
L2=%buffer;
L3=0; {no other index circ buffer}
L4=0;
L5=0;
L6=0;
L7=0;
    {index registers}
    IO=^buffer; {ds(n-5)}
I1=^buffer + 17; {ds(n-18)}
I2=^buffer + 22; {ds(n-23)}
```


## V. 32 Modems 2

```
    I 3 =0000;
I4=^input_buffer + 15;
    MO=0; {modify registers}
M1 = - 1;
M2=1;
M4=-1;
M5=1;
SE=4; {SE for nibble pack}
RTS;
{—CLEAR BUFFER SUBROUTINE-_ }
{initialize scramble buffer to zero}
clear_buffer: CNTR=%buffer;
    DO clear UNTIL CE;
clear: DM(I0,M1)=0;
    RTS;
. ENDMOD;
```

Listing 2.1 Call Mode Scrambler Main Routine

## 2 V. 32 Modems

```
.MODULE
call_mode_scrambler;
{ This module performs V.32 call mode scrambling }
{ The generating polynomial is: xin + y(n-18) + y(n-23) }
{ CALLS: bits}
```

```
.EXTERNAL input_buffer;
```

.EXTERNAL input_buffer;
.CONST no_bits_per_word = 16;
.CONST no_bits_per_word = 16;
.PORT codec;
.PORT codec;
.PORT dac;
.PORT dac;
.ENTRY start_scramble;
.ENTRY start_scramble;
start_scramble: AY0=DM(codec); {read from port}
start_scramble: AY0=DM(codec); {read from port}
CALL bits; {show as serial stream}
CALL bits; {show as serial stream}
CNTR=no_bits_per_word; {scramble 16 times}
CNTR=no_bits_per_word; {scramble 16 times}
{once for every bit of input}
{once for every bit of input}
DO scrambl UNTIL CE;
DO scrambl UNTIL CE;
AY0=DM(I4,M5);
AY0=DM(I4,M5);
AX0=DM(I1,M1); {d(n-18)}
AX0=DM(I1,M1); {d(n-18)}
AY1=DM(I2,M0); {d(n-23)}
AY1=DM(I2,M0); {d(n-23)}
AR=AX0 XOR AY1; {d(n-18) + d(n-23)}
AR=AX0 XOR AY1; {d(n-18) + d(n-23)}
AR=AR XOR AYO; {d(n) + d(n-18) + d(n-23)}
AR=AR XOR AYO; {d(n) + d(n-18) + d(n-23)}
DM(I2,M1)=AR; {store scramble in buffer}
DM(I2,M1)=AR; {store scramble in buffer}
{write new value over oldest}
{write new value over oldest}
DM(dac)=AR; {out to dac}
DM(dac)=AR; {out to dac}
MODIFY(I4,M4); {reset pointer to last buffer}
MODIFY(I4,M4); {reset pointer to last buffer}
{value for next input word}
{value for next input word}
scrambl: NOP;

```
scrambl: NOP;
```

        RTI;
    \(\{—\) BITS SUBROUTINE——\}
    \{ takes output from u_expand (16-bit word) and separates out \}
\{ the bits; stores as MSB in a 16 -word buffer 'input_buffer'\}
\{ The most significant bit of the input word is at the top of \}
\{ the buffer\}.
bits: AXO=AY0; \{expanded output into ALU\}
SE=15;
CNTR=no_bits_per_word;
AY0 $=\mathrm{H} \# 8000$;
DO bit_loop UNTIL CE;
AR=AX0;
SR=LSHIFT AR (LO); \{shift so next bit is\}
\{MSB in reg SR0\}

# V. 32 Modems 2 

```
    AR=ARO AND AY0; {mask out all except MS}
    DM(I4,M4)=AR;
    AY1=SE; {decrement SE for next}
    AR=AY1-1;
bit_loop:
    SE=AR;
I4=^input_buffer;
SE=4;
RTS;
```

.ENDMOD;
Listing 2.2 Call Mode Scrambler Scrambling Routine

## 2 V. 32 Modems

```
.MODULE/RAM/ABS=0 main_routine;
{ Descrambling Routine }
{ Call Mode Functions implemented:}
{ d(n)=di(n)+d(n-18) + d(n-23)}
{ System file: fullpm.sys}
{ CALLS: initial, clear_buffer, output}
.VAR/DM/RAM/CIRC buffer[23];
.PORT codec;
.PORT dac;
.PORT cntl_port;
    RTI; RTI; RTI; {int0-2 not used}
    JUMP start_descramble; {INT3 8 kHz from codec}
    CALL initial;
    CALL clear_buffer;
    IMASK=h#8; {enable interrupts}
mainloop: JUMP mainloop; {loop until interrupted}
{__ descramble subroutine ___}
{addressing circular buffer with 2 pointers for modem scrambler}
start_descramble: AYO=DM(codec); {read from port}
    AX0=DM(I1,M1); {d(n-18)}
    AY1=DM(I2,M0); {d(n-23)}
    AR=AX0 XOR AY1; {d(n-18) + d(n-23)}
    AR=AR XOR AYO; {d(n) + d(n-18) + d(n-23)}
    DM(I2,M1)=AY0; {store scramble in buffer}
    {input stored... not output}.
    CALL output;
    AR=0; {clear AR for next time}
    RTI;
    {— initialize subroutine -_}
{initialize registers}
\begin{tabular}{|c|c|c|}
\hline \multirow[t]{6}{*}{initial:} & IMASK=B\#0000; & \{disable interrupts\} \\
\hline & ICNTL= \(\mathrm{H} \# \mathrm{~F}\); & \{edge level interrupts\} \\
\hline & SI=0; & \\
\hline & DM (cntl_port) =SI; & \{load codec control reg\} \\
\hline & L0 \(=\frac{\%}{\circ}\) buffer; & \{circular buffer length 23\} \\
\hline & L1=\%buffer; & \\
\hline
\end{tabular}
```


## V. 32 Modems 2

```
    L2=%buffer;
    L3=0;
    L4 =0;
    L5=0;
    L6=0;
    L7=0;
    I0=^buffer;
    II=^buffer + 17;
    I2=^buffer + 22;
    M0=0;
    M1=-1;
    SR0=0;
    SR1=0;
    SE=16;
    RTS;
{- clear buffer subroutine -
{initialize buffer to zero}
clear_buffer: CNTR=%buffer;
    DO clear UNTIL CE;
clear: DM(I0,M1)=0;
    RTS;
{-_output routine packs serial into 16 bit words -_}
output: SR=SR OR LSHIFT AR(LO);
    AYO=SE;
    AR=AY0 -1;
    SE=AR;
    IF EQ CALL out;
    RTS;
out:
DM (dac) \(=\) SR1;
SR0 \(=0\);
SR1=0;
SE=16;
RTS;
```

.ENDMOD;
Listing 2.3 Call Mode Descrambler Routine

## 2 V. 32 Modems

### 2.5 RAISED COSINE FILTER

For the V. 32 modem recommendation, 5 -bit symbols are modulated by a carrier of 1800 Hz . This modulation is performed digitally. Coupled with the modulator and the demodulator are pulse shaping low pass filters. These digital filters eliminate intersymbol interference (ISI) on the bandlimited GSTN.

A brief development of the theory of pulse shaping filters follows. For a more complete theoretical discussion of pulse shaping filters, see
"References" at the end of this chapter: Bingham, Lee and Messerschmitt, Proakis.

Low pass transmitted signals can be shown to have the form

$$
\sum_{n=0}^{\infty} \quad I_{n} g(t-n T)
$$

where $I_{n}$ is the discrete code word and $g(t)$ is a pulse. For the bandlimited channel, we desire a transmitted pulse $g(t)$ that produces no ISI. If the channel is ideally bandlimited, then an ideally bandlimited pulse can be used. In the frequency domain, this ideally bandlimited pulse can be described as:
$G(f)=T$ for $f<1 / 2 T$ 0 for $f \geq 1 / 2 T$

This spectrum has an ideal rectangular shape.
In the time domain, this ideal spectrum shape is the sinc function:
$g(t)=\sin (\pi t / T) /(\pi t / T)$
The nulls (zero values of the pulse function) occur at multiples of $T$, the baud rate. Because of the placement of the nulls, there is no additive interference due to previous symbols; there is no ISI.

The ideal pulse shaping filter is not practical to implement. The ideally bandlimited frequency response has a corresponding infinite impulse response. Although the impulse response has a zero value at all multiples of T, any mistiming in the modem produces an infinite series of ISI terms.

## V. 32 Modems 2

A pulse shaping filter that is practical and widely used in digital communications is the raised cosine pulse shaping filter. The raised cosine pulse shaping filter is realizable, unlike the ideal pulse shaping filter. The raised cosine function has tails that decay proportional to $1 / t^{3}$, whereas the ideal pulse tails off proportional to $1 / \mathrm{t}$. Mistiming errors in sampling in the modem therefore have a much less dramatic effect on the amount of ISI in the raised cosine pulse filter.

A generic formula for the impulse response of the raised cosine filter, $p(t)$, is shown below. $T$ is the symbol rate in $\mathrm{Hz}, t$ is the sampling rate in Hz , and $\alpha$ is the rolloff factor.

$$
p(t)=\frac{\sin (\pi t / T) \cdot \cos (\alpha \pi t / T)}{(\pi t / T) \cdot\left(1-(2 \alpha \pi t / T)^{2)}\right.}
$$

The rolloff factor, $\alpha$, represents the amount of excess bandwidth required. A raised cosine with a rolloff factor of 0 needs the least excess bandwidth. As $\alpha$ varies from 0 to 1 , the amount of excess bandwidth required increases from 0 to $100 \%$. For purposes of this implementation, a common rolloff factor of 0.25 is used. For the V. 32 modem, the symbol rate, T, is specified at 2400 symbols per second. The sampling rate, $t$, is usually 9600 Hz . The frequency response of the raised cosine pulse shaping filter with these parameter values is shown in Figure 2.8, on the following page.

The pulse shaping filter usually spans four baud intervals. For a sampling rate of 9600 Hz and a symbol rate of 2400 Hz , a 17-tap FIR filter can be used.

### 2.5.1 ADSP-2100 Family Implementation

The raised cosine pulse shaping filter can be implemented in the modem as a simple FIR filter. Implementation of FIR filters on the ADSP-2100 family is straightforward. The dual DAGs with circular buffering and the on-chip Harvard architecture allows for efficient realization of FIR filter structures. A complete description of FIR filters as well as other fixedcoefficient filters can be found in Digital Signal Processing Applications Using the ADSP-2100 Family, Chapter 5 (see "Literature" at the beginning of this book).

Filter coefficients are arrived at using the formula above, generated with a $C$ program. The coefficients are scaled to provide a filter with 0 dB gain.

## 2 V. 32 Modems

Impulse response


Figure 2.8 Raised Cosine Pulse Shaping Filter, $\alpha=0.25$

The coefficients represent a rolloff factor of 0.25 , and the generated impulse response spans four baud intervals.

For the V. 32 modem, the filter input is a digitally modulated value ( 1800 Hz carrier). Samples are processed at the baud rate ( 2400 baud ) and are interpolated, zero-filled, to provide filter input at a rate of 9600 Hz . Samples are processed in quadrature. Figure 2.9 shows the relationship of the filter to the digital modulator and the data rates.

Listing 2.4 contains the ADSP-2100 family code for implementation of the raised cosine filter. The coefficients can be found in the data file coef.dat.

## V. 32 Modems 2



Figure 2.9 Modem Transmitter

```
.MODULE/boot=0 fir_sub;
{-
    Pulse Shape filter routine for V. }3
    ICASSP DEMO
    Rev History 2/8/90 take APP VOL I FIR routine
    adapt for V. }3
}
```

.ENTRY
. CONST
. EXTERNAL
.EXTERNAL
. VAR/DM
. VAR/DM
. VAR/DM
. VAR/DM
. VAR/DM
. VAR/DM
PSF_length = 89;
Real_PSF_delay_line, Imag_PSF_delay_line, Pulse_Shape_coeff;
real_PSF_i0, imag_PSF_i0;
psf_save_IO;
psf__save_L0;
psf_save_I4;
psf__save_L4;
test_psf1;
test_psf2;

```
```

```
pulse_shape;
```

```
```

pulse_shape;

```

\section*{2 V. 32 Modems}
```

pulse_shape: }\quad\begin{array}{ll}{\mathrm{ DM(psf_save_I0) = I0; DM(psf_save_L0) = L0; {save I0,L0,I4,L4}}}<br>{\mathrm{ DM(psf_save_I4) = I4; DM(psf_save_L4) = L4; }}<br>{}<br>{I0=DM(real_PSF_i0); }
{-D Do real part of the filter. ax0 contains the x value
from the signal map module. }
DM(I0,M2) = AXO; {dump new vals into delay line}
CNTR = PSF_Length - 1;
MR=0, MXO=DM(IO,M2), MYO=PM(I4,M5);
sop:
MR=MR+MX0*MY0(SS), MX0=DM(I0,M2), MY0=PM(I4,M5);
IF NOT CE JUMP sop;
MR=MR+MXO *MYO (RND);
IF MV SAT MR;
AX0 = MR1; {filtered X in ax0}
DM(real_PSF_i0) = I0;
{- Do the imaginary part of the Pulse Shape filter. ax1 contains
the imaginary part of the point from the signal map module. }
IO = DM(imag_PSF_i0);
DM(IO,M2) = \overline{AX1; ; {dump new vals into delay line}}
CNTR = PSF_Length - 1;
MR=0, MX0=DM(IO,M2), MY0=PM(I4,M5);
imag_sop: MR=MR+MX0*MY0(SS), MX0=DM(I0,M2), MY0=PM(I4,M5);
IF NOT CE JUMP imag_sop;
MR=MR+MXO *MYO (RND);
IF MV SAT MR;
AX1 = MR1; {filtered Y in ax1}
DM(imag_PSF_i0) = IO;
IO = DM(psf_save_I0); L0 = DM(psf_save_L0);
I4 = DM(psf_save_I4); L4 = DM(psf_save_L4);
RTS;
.ENDMOD;

```

\section*{Listing 2.4 Raised Cosine Filter}

\title{
V. 32 Modems 2
}

\subsection*{2.6 TRELLIS ENCODING}

The GSTN was intended for voiceband transmission and is bandlimited 200 Hz to 3400 Hz . Data rates in excess of the upper band limit can be realized only by the transmission of multiple bits per symbol interval. Data rates of 9.6 Kbits per second can be achieved on unconditioned circuits and data rates of up to 16.8 Kbits per second can be realized on conditioned leased lines using the technique known as trellis coded modulation (TCM).

The V. 32 modem recommendation specifies trellis encoding as an option. Four-bit symbols are encoded into 5-bit symbols that are made up of four information bits and a redundant bit. These 5-bit symbols are used with a 32 carrier state QAM modulator. A 2400 baud rate is used and 9600 information bits per second are transmitted. A trellis encoded scheme offers much better performance than a non-encoded scheme. It results in a much higher immunity to noise for a given error rate and can reduce the block error rate by three orders of magnitude for a given signal-to-noise ratio.

There are two fundamental types of codes used in channel encoding. Linear block codes include Hamming codes, BCH (Bose-ChadhuriHocquenghem) codes, Reed-Solomon codes, Galay codes and many others. The convolutional code, which is specified for V. 32 modems can be implemented using a shift register and can be described using a diagram called a trellis diagram.

Suppose we can achieve a certain \(P_{e}\) (probability of error) in an uncoded system operating on a bandlimited channel. We can attempt to improve system performance by coding. If we add a single redundant bit to a binary symbol with \(k\) bits, we increase the number of waveforms that the modulator must produce from \(2^{k}\) to \(2^{k+1}\). An increase in alphabet size on the same bandwidth requires a 3 dB increase in the signal to noise ratio to achieve the same \(P_{e}\). That is, coding alone decreases the performance of the system.

Trellis coded modulation employs signal set partitioning in addition to redundant coding in order to increase the system performance. In the case of the V. 32 modem, there are 32 modulator states. Of the four input bits to the encoder, only two are encoded. Two bits pass through uncoded and two bits are encoded into three output bits. The three bits provide a mechanism for dividing the 32 modulator states into 8 subsets of 4 modulator carrier states. The coded bits identify the subset of the 32

\section*{2 V. 32 Modems}
modulator states and the uncoded bits select a point within the subset. Figure 2.10 shows the input and output bits of the trellis encoder. Bits Q1 through Q4 are the input bits. Bits Q3 and Q4 pass through the encoder unchanged. Bits Q1 and Q2 are encoded to give Y1, Y2 and the redundant error correcting bit Y0. Bits Y0, Y1, Y2 identify the subset while the bits Q3 and Q4 identify the point within the subset.


Figure 2.10 Encoder Block Diagram

The signal set for the V. 32 modem (and other TCM schemes) has been designed so that there is a large distance between the members of each subset. The 32 -state signal constellation for the V .32 modem is shown in Figure 2.11. Bits are ordered on this diagram left to right, most significant to least significant: Y0 Y1 Y2 Q3 Q4. The signal space mapping for the redundant coding is from Figure 3/V. 32 of the V. 32 recommendation.

The signal set is located on a quadratic grid known as a \(Z_{2}\) lattice and the signal set type is known as 32 CROSS. In order to transmit \(m\) bits per signalling interval, \(2^{\mathrm{m}+1}\) signals are needed. The coding gain (performance of the coded signals versus uncoded signals) is approximately 4 dB for any \(m\). The closest distance between any two points on the signal set is \(\Delta_{0}\). The closest distance between any two points in a subset (i.e.. points that have the same \(\mathrm{Y} 0, \mathrm{Y} 1\) and Y 2 bits) is \(\sqrt{8} \Delta_{0}\) for the 32 CROSS signal set.

All bit patterns that begin with the same three bits are spread out on the signal constellation. This signal set partitioning along with the redundant coding are the fundamentals of TCM.

\section*{V. 32 Modems 2}


Figure 2.11 V. 32 Signal Constellation

\subsection*{2.6.1 ADSP-2100 Family Implementation}

Trellis encoding for the V. 32 modem consists of two encoding operations: a differential encoder, implemented as a lookup table and a convolutional encoder, performed using a shift register and Boolean logic. Together, these two encoders generate a 5-bit symbol from a 4-bit input word.

The serial input bits to the encoder are Q1, Q2, Q3 and Q4 (Q1 first, Q4 last). Three of the output bits are Y0, Y1 and Y2, and the other two output

\section*{2 V. 32 Modems}
bits are Q3 and Q4, unchanged from the input. Y1 and Y2 are generated in the differential encoder. Y0, the redundant bit for error correction, is generated in the convolutional encoder.

The differential encoder takes as input the first two bits, Q1 and Q2, and produces two output bits, Y 1 and Y 2. Previous output bits, \(\mathrm{Y} 1(\mathrm{n}-1)\) and \(\mathrm{Y} 2(\mathrm{n}-1)\) are also used in the differential encoder. The encoder is easily implemented on the ADSP-2100 family as a lookup table. The input bits and the previous output bits are combined to a 4 -bit value that serves as a pointer into the lookup table. For example, assume that the current input bits are \(\mathrm{Q} 1=1, \mathrm{Q} 2=0, \mathrm{Y} 1(\mathrm{n}-1)=0\) and \(\mathrm{Y} 2(\mathrm{n}-1)=1\), for a 4 -bit value of 1001 . This corresponds to the 1001 (ninth) entry in the lookup table, from which the current Y1 and Y2 outputs are read. Table 2.1 shows the lookup table for differential encoding.
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{2}{|c|}{Inputs} & \multicolumn{2}{|l|}{Previous Outputs} & \multicolumn{2}{|c|}{Outputs} \\
\hline Q1 & Q2 & Y1(n-1) & Y2(n-1) & Y1 & Y2 \\
\hline 0 & 0 & 0 & 0 & 0 & 0 \\
\hline 0 & 0 & 0 & 1 & 0 & 1 \\
\hline 0 & 0 & 1 & 0 & 1 & 0 \\
\hline 0 & 0 & 1 & 1 & 1 & 1 \\
\hline 0 & 1 & 0 & 0 & 0 & 1 \\
\hline 0 & 1 & 0 & 1 & 0 & 0 \\
\hline 0 & 1 & 1 & 0 & 1 & 1 \\
\hline 0 & 1 & 1 & 1 & 1 & 0 \\
\hline 1 & 0 & 0 & 0 & 1 & 0 \\
\hline 1 & 0 & 0 & 1 & 1 & 1 \\
\hline 1 & 0 & 1 & 0 & 0 & 1 \\
\hline 1 & 0 & 1 & 1 & 0 & 0 \\
\hline 1 & 1 & 0 & 0 & 1 & 1 \\
\hline 1 & 1 & 0 & 1 & 1 & 0 \\
\hline 1 & 1 & 1 & 0 & 0 & 0 \\
\hline 1 & 1 & 1 & 1 & 0 & 1 \\
\hline
\end{tabular}

Table 2.1 Differential Encoder Lookup Table

\section*{V. 32 Modems 2}

The convolutional encoder (Figure 2.12) uses a shift register structure to examine the four incoming bits (the output of the differential encoder) and build a 5-bit symbol. The five output bits of the convolutional encoder consist of the four input bits plus an additional redundantly coded fifth bit. This additional bit increases the complexity of the signal set, but limits the number of possible transitions between bit patterns. For any given 5bit convolutionally encoded word, only half of the signal states can follow. In other words, the process of convolutional encoding prohibits transitions from any particular signal state to only half of the possibilities. This property is exploited in the Viterbi decoder in the receiver.


Figure 2.12 Convolutional Encoder Block Diagram

Listing 2.5 contains a ADSP-2100 family subroutine that provides both the differential encoder and the convolutional encoder. The input is assumed to be a single bit residing in the most significant bit position of a 16-bit word. Listing 2.6 shows the convolutional encoder routine that is called by the program in Listing 2.5, and Listing 2.7 contains the routine that performs signal mapping on the encoded data.

\section*{2 V. 32 Modems}
```

.MODULE/RAM trellis;
.VAR/DM/RAM t_table[16];
.VAR/DM/RAM last_ys;
.VAR/DM/RAM bit_count;
.VAR/DM/RAM diff_out;
.VAR/DM/RAM
.VAR/DM/RAM
.VAR/DM/RAM
.VAR/DM/RAM
.VAR/DM/RAM
.INIT
.ENTRY
.PORT
.PORT
.GLOBAL
{-bit count is intially 4-}
trellis_encode: SE=DM(bit_count);
SI=DM(adc); {take in new 8000 or 0000}
Q1Q2_pack: SR=SR OR LSHIFT SI (LO); {count up 4 bits,}
AYO=SE; {shift into SR register}
AR=AYO -1;
SE=AR;
DM(bit_count)=SE; {store decremented count}
IF EQ JUMP packed;
RTI;
packed:
AX0=SR1; {stored as 4 bits}
AX1=4; {Q1 Q2 Q3 Q4}
DM(bit_count)=AX1;
SR0=0;
SR1=0;
CALL d_encode;
RTI;

```

\section*{V. 32 Modems 2}
```

{\longrightarrow_ENCODE-_
{input: AX0 -> 0 0 0 X where X -> bits 0 0 0 0 Q1Q2Q3Q4}
d_encode: I3=^t_table;
AYO=h\#000C; {mask to keep Q1 Q2}
AR=AXO AND AYO;
AY1=DM(last_ys); {last output Y1 Y2}
AR=AR XOR AY1; {AR is Q1 Q2 Y1 Y2}
M3=AR; {address in lookup}
MODIFY(I3,M3); {for new Y1 Y2}
SI=DM(I3,M0);
DM(last_ys)=SI; {AY0 ->encoded Y1 Y2 }
AY1=3;
AF=AX0 AND AY1; {keep Q3 Q4}
SR=LSHIFT SI BY 2(LO);
AR=SR0+AF; {AR ->Y1 Y2 Q3 Q4}
DM(diff_out)=AR; {store output of diff encode}
DM(dac)=AR;
CALL c_encode; {call convolutional encode}
RTS;

```
.ENDMOD;

\section*{Listing 2.5 Trellis Encoder Program}

\section*{2 V. 32 Modems}


\section*{V. 32 Modems 2}
```

AR=AXO XOR AY0; {op \#1}
AY1=DM(delay_val_3);
AR=AR XOR AY1; - {op \#2}
DM(intermed_1)=AR;
AX0=DM(delay_val_1);
AR=AX0 XOR AY0; {delay val 1 XOR Y2 op \#5}
DM(intermed_2)=AR;
AY0=DM(delay_val_2);
DM(delay_val_3)=\overline{A}Y0; {update delay val 3}
AR=AR AND AYO;
AY1=DM(intermed_1);
AR=AR XOR AY1;
DM(delay_val_1)=AR; {update delay_val_1}
AX1=DM(Y1);
AR=AX1 AND AY0; {and_2}
AY0=DM(intermed_2);
AR=AR XOR AYO;
DM(delay_val_2)=AR; {update delay val 2}
DM(YO) =AR;
CALL pack_up_5_bits;
RTS;
{- OUTPUT FORMATTER - } }
pack_up_5_bits: SR0=0; {pack up bits as Y0Y1Y2Q3Q4}
SR1=0; {clear SR}
SR1=DM(diff_out);
SI=DM(YO);
SR=SR OR LSHIFT SI BY 4 (HI);
DM(conv_out)=SR1;
DM(dac)=SR1;
SR0=0;
SR1=0;
CALL sig_map;
RTS;

```

\section*{2 V. 32 Modems}
```

.MODULE signal_map;
{ This module takes the output of the convolutional encoder,
that is, a five bit code residing in the LSBs of the data
memory location "conv_out", and looks up the x and y coordinates
as defined by the CCITT spec for the V. }32\mathrm{ modem.
The coordinates are given in the CCITT spec as whole integers.
They are represented in a 16-bit fixed format as follows:

| integer |  | hexadecimal |
| :--- | :--- | :--- |
| 0 | 0000 |  |
| 1 | 2000 |  |
| 2 | 4000 |  |
| 3 | 6000 |  |
| 4 | $7 F F F$ |  |
| -1 | E000 |  |
| -2 | C000 |  |
| -3 | A000 |  |
| -4 | 8000 |  |

    Registers used:
    }
.VAR/DM x_table[32];
.VAR/DM Y_table[32];
.INIT x_table: H\#8000, H\#0000, H\#0000, H\#7FFF, H\#7FFF,
H\#0000, H\#0000, H\#8000, H\#C000, H\#COOO, H\#4000,
H\#4000, H\#4000, H\#4000, H\#C000, H\#COOO, H\#A000,
H\#2000, H\#A000, H\#2000, H\#6000, H\#E000, H\#6000,
H\#EOOO, H\#2000, H\#A000, H\#2000, H\#2000, H\#E000,
H\#6000, H\#E000, H\#E000;
.INIT y_table: H\#2000, H\#A000, H\#2000, H\#2000, H\#E000,
H\#6000, H\#E000, H\#E000, H\#6000, H\#E000, H\#6000,
H\#E000, H\#A000, H\#2000, H\#A000, H\#2000, H\#COOO,
H\#C000, H\#4000, H\#4000, H\#4000, H\#4000, H\#C000,
H\#C000, H\#7FFF, H\#0000, H\#0000, H\#8000, H\#8000,
H\#0000, H\#0000, H\#7FFF;

```
```

.EXTERNAL Conv_out, dac;
.ENTRY sig_map;
sig_map: II=^x_table;
I2=^y_table;
MO=0;
M1=DM(conv_out);
MODIFY(I1,M1);
MODIFY(I2,M1);
AXO=DM(I1,MO); {x value in ax0}
AX1=DM(I2,M0); {y value in ax1}
DM(dac)=ax0;
DM(dac)=ax1;
RTS;
.ENDMOD;

```

\section*{Listing 2.7 Signal Mapping Routine}

\subsection*{2.7 VITERBI DECODING}

The V. 32 recommendation specifies a trellis or convolutional encoding of data before transmission. The most common technique used for decoding received data is Viterbi decoding. The Viterbi algorithm is a general purpose technique for making an error-corrected decision. Viterbi decoding provides a certain degree of error correction by determining from the received bit pattern the value that was the most likely to have been transmitted. The Viterbi algorithm can be used for many applications where error correcting is required. Its application in the V. 32 modem is similar to that used in other digital data communication schemes, such as digital telephones.

In order for the Viterbi algorithm to decode received data properly, the model for encoding the transmitted data must be known. In trellis encoding, it is assumed that the three delay elements of the encoder contain zeros initially. At each time period, a new 2-bit input is presented. The contents of the delay elements are changed accordingly and a 3-bit output is produced. If the three delay elements are treated as a 3-bit word, where delay element 1 is the most significant bit and delay element 3 is

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the least significant bit, then the state of the delay elements collectively can be represented by that 3-bit value.

It is possible to derive a state diagram or table from this specification. The three delay elements in the encoder are labelled from left to right as element 1, 2 and 3, respectively, in Figure 2.12 (on page 2-25). At any moment, each delay element has stored in it a 1 or a 0 . The possible combinations of bits in the three delay elements or the possible states is eight. The state table shows the eight possible states of these three storage elements. It also shows that for any 2-bit input to the encoder, the three delay elements go to some new state and the encoder also produces an output. The state table showing the state transitions with the encoder inputs and outputs is shown in Table 2.2.
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Beginning} & \multicolumn{2}{|r|}{End} & \multicolumn{2}{|l|}{Beginning} & \multirow[b]{2}{*}{Output} & \multirow[t]{2}{*}{\begin{tabular}{l}
End \\
State
\end{tabular}} \\
\hline State & Input & Output & State & State & Input & & \\
\hline 000 & 00 & 000 & 000 & 100 & 00 & 000 & 010 \\
\hline 000 & 01 & 101 & 011 & 100 & 01 & 101 & 001 \\
\hline 000 & 10 & 010 & 010 & 100 & 10 & 010 & 000 \\
\hline 000 & 11 & 111 & 001 & 100 & 11 & 111 & 011 \\
\hline 001 & 00 & 000 & 100 & 101 & 00 & 000 & 110 \\
\hline 001 & 01 & 101 & 101 & 101 & 01 & 101 & 111 \\
\hline 001 & 10 & 110 & 111 & 101 & 10 & 110 & 101 \\
\hline 001 & 11 & 011 & 110 & 101 & 11 & 011 & 100 \\
\hline 010 & 00 & 100 & 001 & 110 & 00 & 100 & 011 \\
\hline 010 & 01 & 001 & 010 & 110 & 01 & 001 & 000 \\
\hline 010 & 10 & 110 & 011 & 110 & 10 & 110 & 001 \\
\hline 010 & 11 & 011 & 000 & 110 & 11 & 011 & 010 \\
\hline 011 & 00 & 100 & 111 & 111 & 00 & 100 & 101 \\
\hline 011 & 01 & 001 & 110 & 111 & 01 & 001 & 100 \\
\hline 011 & 10 & 010 & 100 & 111 & 10 & 010 & 110 \\
\hline 011 & 11 & 111 & 101 & 111 & 11 & 111 & 111 \\
\hline
\end{tabular}

Table 2.2 State Table for Convolutional Encoder

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Table 2.2 can also be used to derive a trellis diagram. The trellis diagram and the state diagram convey equivalent information. The trellis diagram for the convolutional encoder of the V. 32 modem is shown in Figure 2.13.

Each node of the trellis represents a state and each node is labelled with the three-bit value of that particular state out of the eight possible states. A line is drawn from a state in one time window to a state of the next time window and represents the transition from one state to another for any given 2-bit input. Figure 2.13 shows some of the trellis paths labelled with the 3-bit output that was produced as the delay elements went from one state to another.

Time window 1


Figure 2.13 Trellis Diagram for Convolutional Encoding

It is assumed that at time \(t=0\), the contents of each delay element is 0 . Therefore the starting point for the trellis is at state 000. There are four possible combinations of 2-bit inputs and therefore, four lines that come out of state 000 and connect to the corresponding states at time window 2 as specified by the state table. For example, an input of 01 results in a

\title{
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}
change in the state of the delay elements from 000 to 011 with an output of 101. This information is conveyed in the trellis diagram by a line from state 000 to 011 labelled 101. The trellis diagram in Figure 2.13 has some of the branches labelled with the output value that is produced for a specific state transition; the rest can be determined from the state table.

\subsection*{2.7.1 Data Constellation}

A 2-bit input to the convolutional encoder produces a 3-bit output containing a redundant bit. Because of redundancy, this 3-bit data value can be corrected for errors that occur during transmission.

In the transmission of information in a V. 32 modem, the three bits from the output of the convolutional encoder are combined with two bits coming directly from the data bit stream. In essence, four bits from the data stream are being encoded to five bits (one redundant bit is added to the four original bits).

To modulate a carrier with this information, a constellation is created that maps any 5-bit data value to an \(X\) and \(Y\) coordinate or a real and imaginary term associated. The real and imaginary terms are used to modulate sine and cosine carriers for quadrature amplitude modulation (see Chapter 3). Figure 2.14 shows the V. 32 constellation with the 3-bit output of the convolutional encoder underlined.

The demodulated carrier yields the original X and Y coordinates which determine the original 5-bit data value. Since the transmission medium for the carrier is noisy, the demodulated data may not be correct. The Viterbi algorithm corrects errors introduced in transmission.

\subsection*{2.7.2 Viterbi Algorithm}

The Viterbi algorithm decides whether demodulated data is the data that was sent and if not, corrects it. It works by analyzing the pattern of data values received over a period of time to deduce the data value that is most likely to have occurred at the beginning of the period.

The received carrier is demodulated to produce \(X\) and \(Y\) coordinates of a point on the signal constellation. The distances from that point on the constellation to the nearest eight points that all have different leading three bits are calculated. These Euclidean distances are then used to label the branches of the trellis diagram. After a number of samples have been received and mapped to the trellis diagram in this fashion, the diagram can be read to determine the shortest path back to the original state, which determines the data value that has the highest probability of having been transmitted at that time.

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Figure 2.14 Signal Constellation Showing Convolutional Encoder Output

For example, assume that the received signal at time window 1 is mapped into the constellation at coordinate \(2,2(x, y)\). This does not correspond to a five-bit code on the constellation. The Euclidean distances from this point to the nearest eight points are calculated. Because of the way the signal map is configured, each of these points has a different value for its first three bits (underlined in Figure 2.14).

In the trellis diagram, the line connecting state 000 to state 011 in time window 1 is labelled 101. The point in the signal constellation that is nearest to 2,2 and has the value 101 as its first three bits is 10100, at coordinate 3, 2. The Euclidean distance between coordinate 2, 2 and 3,2 is:
\[
\left[(2-3)^{2}+(2-2)^{2}\right]^{1 / 2}=1
\]

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Therefore, the branch of the trellis diagram going from state 000 to state 011 is labelled 1 . This process is repeated to label the other branches on the trellis diagram. As a new sample is received in each time window, the trellis branches are labelled with the corresponding Euclidean distances.

After a given number of time windows have elapsed, the shortest path back to the start of the first time window is calculated. The branch of the shortest path in the first time window represents the original data value that was transmitted.

Since the data point is determined only after a given number of time windows has elapsed, a delay of (number of time window multiplied by the symbol rate) is incurred. The more time windows that elapse before a decision is made, the more accurate the decision. Thus there is a tradeoff between accuracy and execution time.

\subsection*{2.7.3 ADSP-2100 Family Implementation}

The first task of the program is to determine which eight points in the data constellation are the nearest to the X and Y coordinates produced by the demodulator. This is done using a lookup table. Each group in the lookup table contains the X and Y coordinates of the four points in the constellation that have the same 3 -bit leading sequences. There are 32 points in the constellation, and therefore eight groups. Because the ADSP2100 is a 16-bit machine, the \(X\) and \(Y\) values are normalized for 16-bit data. A negative full scale value of \(\mathrm{H} \# 8000\) and a positive full scale value of H\#7FFF are used for both the \(X\) and \(Y\) values.

For example, \(\underline{00000}, \underline{00001, ~} \underline{00010}\) and \(\underline{00011}\) are in group 0. The Euclidean distance between the received point and the points in the group 0 are calculated. The shortest distance is then written into another table called min_dist in which the first location holds the shortest distance of the first group, the second location holds the shortest distance of the second group, etc. Table 2.3 shows the X and Y coordinates in each of the eight groups.

\section*{V. 32 Modems 2}
\begin{tabular}{|c|c|c|c|}
\hline \multirow[t]{5}{*}{\[
\begin{aligned}
& \text { Group } \\
& 000
\end{aligned}
\]} & \(X \quad Y\) & Group & \(X \quad Y\) \\
\hline & 41 & 100 & 12 \\
\hline & 01 & & -3 2 \\
\hline & -4 1 & & \(1-2\) \\
\hline & \(0 \begin{array}{ll}0 & -3\end{array}\) & & -3-2 \\
\hline \multirow[t]{4}{*}{001} & \(4-1\) & 101 & 32 \\
\hline & \(0-1\) & & -1 2 \\
\hline & -4-1 & & \(3-2\) \\
\hline & 03 & & 10 \\
\hline \multirow[t]{4}{*}{010} & 23 & 110 & 10 \\
\hline & -2 3 & & 14 \\
\hline & \(2-1\) & & -3 0 \\
\hline & -2-1 & & \(1-4\) \\
\hline \multirow[t]{4}{*}{011} & 21 & 111 & 30 \\
\hline & -2 1 & & -1 0 \\
\hline & \(2-3\) & & -14 \\
\hline & -2 -3 & & -1 -4 \\
\hline
\end{tabular}

Table 2.3 Lookup Table of \(X\) and \(Y\) Coordinates

\subsection*{2.7.4 Shortest Path Through Trellis Diagram}

After the distance from the received point for the current time window to the closest point in each group is known, the total distance back to the beginning of the trellis diagram can be calculated. Each time, only the incremental distance for the time window, not the total distance, is calculated.

An 8-location table acc_dist stores the accumulated distance through the trellis diagram. Because the trellis diagram starts at state 000, the first location of the table is initialized with a 0 and all other locations with the positive full scale value. This ensures that, for the first time window, all paths converge back to state 000 , since this state starts with the shortest accumulated distance.

At each time window, the surviving path to each state is determined and the accumulated distance table is updated with the accumulated distance of each of the eight surviving paths. The surviving path is determined by taking the length of all of the possible paths going into a state and adding that distance to the accumulated distance of the state at the other end of the path.

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For example, Figure 2.15 shows the four paths that lead into state 001 . The length of each path is added to the accumulated distance of the state from where the path emanates. The length of path 111 is added to the accumulated distance of state 000 , the length of path 100 to is added the accumulated distance of state 010, the length of path 101 to is added the accumulated distance of state 100, and the length of path 110 is added to the accumulated distance of state 110. The lengths of these paths are read from the min_dist table.

The minimum of these four distances becomes the new accumulated distance to state 001 and is written into the appropriate location of the accumulated distance table (acc_dist). As each surviving path leg is determined, a table is filled with the distance of the path and the state from which it came, to allow the program to trace back along the surviving path to the beginning of the trellis diagram.


Figure 2.15 Accumulated Distance Table Update Example

\section*{V. 32 Modems 2}

After all eight accumulated distances are updated, the shortest of the eight accumulated distances is determined. This path is traced back the given number of time windows. The distance of the branch in the first time window determines the data value most likely to have been transmitted. The point in the data constellation that is this distance from the received point represents the error-corrected symbol.

\subsection*{2.7.5 Viterbi Program}

The example program uses \(\mathrm{N}=20\) time windows. In general, a value of N which is greater than or equal to three times the constraint length gives good results. In this case, the constraint length is 3, the number of bits needed to describe the possible states at each time window. The larger the value of N , the better the performance of the Viterbi algorithm, but the longer the execution time and the larger the table sizes.

\subsection*{2.7.5.1 Initialization}

The first part of the program declares buffers and initializes variables. A buffer to store input data, eight tables holding the coordinates of the eight data groups, eight tables holding the 5-bit codes for the eight data groups, the accumulated distance buffer, eight state-tracing tables, eight buffers to hold surviving path distances and some pointer tables are all declared in the initialization section.

\subsection*{2.7.5.2 Data Input and Euclidean Distance}

Data values are placed in registers \(A X 0\) and \(A X 1\) as \(X\) and \(Y\) coordinates, respectively, for input to the Viterbi program. The code starting at find_dist calculates the distances by calling the subroutine dist (which calculates the Euclidean distance squared) followed by the subroutine sqrt. This subroutine is repeated for each data group. The table min_dist is filled with the shortest distance for each group.

\subsection*{2.7.5.3 Shortest Path}

The code starting at short_path determines the shortest surviving path to each state for the current time window. It also fills the eight state tables with the distance of the surviving branch and the state from which the branch came. The subroutine min_calc compares the four possible surviving paths and determine the shortest.

\subsection*{2.7.5.4 Last Surviving Path}

After the accumulated distances to all eight states are calculated, the shortest is determined. The code starting at search determines the shortest path and traces this path back to the start of the trellis diagram.

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\subsection*{2.7.5.5 Determination of Error Corrected Data}

When the surviving branch of the first time window is determined, the closest point of the data constellation in that data group is found. This 5bit code is put into the SR1 register.
```

.MODULE/RAM viterbi;
{Viterbi decoder program for convolutional encoded data for a V. }32\mathrm{ modem. This program
decodes information using N=20 levels or time windows of Viterbi decoding.
Demodulated data is stored as input to this routine in registers AX0 and AX1 as
follows;

```
```

AXO = X coordinate

```
AXO = X coordinate
AX1 = Y coordinate
This data is used as input.
The 5-bit data word output by this routine is placed in register SR1.}
.CONST N=20;
.CONST base=h#0D49, sqrt2=h#5A82; {required for square root}
.VAR/PM/RAM
    sqrt_coeff[5];
.INIT sqrt_coeff: h#5D1D00, h#A9ED00, h#46D600,
                                h#DDAA00, h#072D00;
{table for storing last N inputs, as X and Y coordinate
table will contain alternating X, Y for each time window}
.VAR/DM/RAM/CIRC inputs[N+N];
{variables to hold new X and Y inputs}
.VAR/DM/RAM x_input;
.VAR/DM/RAM y_input;
```


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\{tables for $X$ and $Y$ coordinates of data constellation points. Coordinates of both axes are $-4,-3,-2,-1,0,1,2,3,4$. They are represented in binary as:

```
    -4 H#8000
    -3 H#AOOO
    -2 H#C0OO
    -1 H#E000
    OHOOOO
    H#2000
    H#4000
    H#6000
    H H#7FFF
}
\begin{tabular}{ll}
.VAR/PM/RAM & group0 [8]; \\
.VAR/PM/RAM & group1 [8]; \\
.VAR/PM/RAM & group2[8]; \\
.VAR/PM/RAM & group3[8]; \\
.VAR/PM/RAM & group4[8]; \\
.VAR/PM/RAM & group5 [8]; \\
.VAR/PM/RAM & group6[8]; \\
.VAR/PM/RAM & group7[8];
\end{tabular}
. INIT group \(0: H \# 7 F F F 00, H \# 200000, H \# 000000, H \# 200000\), H\#800000, H\#200000, H\#OOOOOO, H\#A00000;
. INIT groupl: H\#7FFF00, H\#E00000, H\#000000, H\#E00000, H\#800000, H\#E00000, H\#000000, H\#600000;
. INIT group2: H\#400000, H\#600000, H\#C00000, H\#600000, H\# 400000 , H\#E00000, H\#C00000, H\#E00000;
. INIT group3: H\#400000, H\#200000, H\#C00000, H\#200000, H\# 400000 , H\#A00000, H\#C00000, H\#AOOOOO;
. INIT group \(4: H \# 200000, H \# 400000\), H\#A00000, H\#400000, H\#200000, H\#C00000, H\#A00000, H\#COOOOO;
. INIT group5: H\#600000, H\#400000, H\#E00000, H\#400000, H\#600000, H\#COOOOO, H\#E00000, H\#C00000;
. INIT group6: H\#200000, H\#000000, H\#200000, H\#7FFF00, H\#AOOOOO, H\#OOOOOO, H\#200000, H\#800000;
.INIT group7: H\#600000, H\#000000, H\#EOOOOO, H\#000000, H\#E00000, H\#7FFFOO, H\#E00000, H\#800000;
\{lookup table to get proper group\}
.VAR/DM/RAM group table[8];
.INIT group_table: ^group0, ^group1, ^group2, ^group3, ^group4, ^group5, ^group6, ^group7;
```


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```
{eight tables which show the 5-bit codes that correspond to the X and Y coordinates in
the 8 group tables}
.VAR/DM/RAM codes0[4];
.VAR/DM/RAM codes1[4];
.VAR/DM/RAM codes2[4];
.VAR/DM/RAM codes3[4];
.VAR/DM/RAM codes4[4];
.VAR/DM/RAM codes5[4];
.VAR/DM/RAM codes6[4];
.VAR/DM/RAM codes7[4];
```


\{table for accumulated distances at each state\}
.VAR/DM/RAM/CIRC acc_dist[8];
.VAR/DM/RAM temp_dist[8];
\{eight tables where each table contains the possible states from where a path could come for each of the eight states\}
.VAR/DM/RAM
.VAR/DM/RAM
.VAR/DM/RAM
.VAR/DM/RAM
.VAR/DM/RAM
.VAR/DM/RAM
.VAR/DM/RAM
.VAR/DM/RAM

$$
\begin{aligned}
& \text { to_state0 [4]; } \\
& \text { to_state1[4]; } \\
& \text { to_state2[4]; } \\
& \text { to_state3[4]; } \\
& \text { to_state4[4]; } \\
& \text { to_state5[4]; } \\
& \text { to_state6[4]; } \\
& \text { to_state7[4]; }
\end{aligned}
$$

## V. 32 Modems 2

```
{table is stored with state numbers in backwards order}
.INIT to state0: 2,4,6,0;
.INIT to state1: 0,6,4,2;
.INIT to_state2: 6,0,2,4;
.INIT to_state3: 4,2,0,6;
.INIT to_state4: 5,3,7,1;
.INIT to_state5: 3,5,1,7;
.INIT to_state6: 1,7,3,5;
.INIT to_state7: 7,1,5,3;
\{eight tables, each with \(N\) entries, where each entry contains the label of the leg of the surviving path for a given time window\}
. VAR/DM/RAM/CIRC state0[N];
. VAR/DM/RAM/CIRC state1[N];
.VAR/DM/RAM/CIRC state2[N];
.VAR/DM/RAM/CIRC state3[N];
.VAR/DM/RAM/CIRC state4[N];
.VAR/DM/RAM/CIRC state5[N];
. VAR/DM/RAM/CIRC state6[N];
. VAR/DM/RAM/CIRC state7[N];
\{eight variables to hold the most recent pointer into the eight state tables above\}
\begin{tabular}{|c|c|}
\hline . VAR/DM/RAM & pointer0; \\
\hline . VAR/DM/RAM & pointer1; \\
\hline . VAR/DM/RAM & pointer2; \\
\hline . VAR/DM/RAM & pointer3; \\
\hline . VAR/DM/RAM & pointer4; \\
\hline . VAR/DM/RAM & pointer5; \\
\hline . VAR/DM/RAM & pointer6; \\
\hline . VAR/DM/RAM & pointer7; \\
\hline
\end{tabular}
. INIT pointer0:^state0;
.INIT pointer1: ^statel;
. INIT pointer2:^state2;
. INIT pointer3:^state3;
. INIT pointer4: ^state4;
.INIT pointer5: ^state5;
.INIT pointer6:^state6;
.INIT pointer7:^state7;
\{table used to look up pointers declared above\}
.VAR/DM/RAM point_table[8];
```


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```
{initialize table with the addresses of the pointers}
.INIT point_table: ^pointer0, ^pointer1, ^pointer2,
    ^pointer3, ^pointer4, ^pointer5,
    ^pointer6, ^pointer7;
{table to hold the eight possible distances, minimum of each group}
.VAR/DM/RAM min_dist[8];
{interrupt vectors}
    RTI;
    RTI;
    RTI;
    JUMP decode;
    IMASK=0; {disable all interrupts}
    ICNTL=8; {interrupts edge sensitive, non-nested}
    ENA AR_SAT;
    IO=^inputs; {init. IO to start of input buffer}
    LO=%inputs; {init. LO to size of input buffer}
    M0=1;
    M1=0;
    M3=-1;
    L3=N;
    L5=0;
{initialize input buffer to all 0s}
    CNTR=%inputs; {load counter with size of buffer}
    SI=0; {put a 0 into register si}
    DO clear_buf UNTIL CE;
clear_buf: DM(IO,MO)=SI; {transfer 0 into buffer location}
{initialize accumulated distance table}
    I1=^acc_dist;
    L1=%acc_dist;
    DM(I1,MO)=0;
    CNTR=%acc_dist - 1;
    DO clear_acc UNTIL CE;
clear_acc: DM(I1,M0)=h#7FFF;
```


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```
{initialize eight tables with 0}
    I2=^state0;
    L2=%state0;
    CNTR=N;
    DO init_table0 UNTIL CE;
init_table0: DM(I2,M0)=SI;
    I2=^state1;
    L2=%state1;
    CNTR=N;
    DO init_table1 UNTIL CE;
init_table1: DM(I2,M0)=SI;
    I2=^state2;
    L2=%state2;
    CNTR=N;
    DO init_table2 UNTIL CE;
        DM(I2,M0)=SI;
    I2=^state3;
    L2=%state3;
    CNTR=N;
    DO init_table3 UNTIL CE;
init_table3: DM(I2,M0)=SI;
    I2=^state4;
    L2=%state4;
    CNTR=N;
    DO init table4 UNTIL CE;
init_table4: DM(I\overline{2},M0)=SI;
    I2=^state5;
    L2=%state5;
    CNTR=N;
    DO init_table5 UNTIL CE;
        DM(I2,M0)=SI;
    I2=^state6;
    L2=%state6;
    CNTR=N;
    DO init_table6 UNTIL CE;
        DM(I2,M0)=SI;
    I2=^state7;
    L2=%state7;
    CNTR=N;
    DO init_table7 UNTIL CE;
```


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```
init_table7: DM(I2,M0)=SI;
```

```
L2=0;
IMASK=8; {enable interrupt 3}
waitlp: JUMP waitlp;
```



```
decode: \(\quad \mathrm{AXO}=\mathrm{DM}\) (codec);
AX1=DM (codec);
\(\operatorname{DM}(I 0, M O)=A X 0 ; \quad\) \{store \(X\) input in input buffer\}
\(D M(I 0, M 0)=A X 1 ; \quad\) \{store \(Y\) input in input buffer\}
DM (x_input) \(=A X 0\);
DM (y_input) \(=\) AX1;
```

\{Calculate Euclidean distances from received point to 32 points of data constellation. The shortest distance in each data group is saved and will represent the distance for the trellis branch for the current time window\}
find_dist: $\quad$ M4 $=1$;
$\mathrm{L} 4=0$;
I4 =^group0;
CALL dist;
$A R=P A S S$ AF; $\quad\{$ put distance squared into $A R\}$
MR0 $=0$;
MR1=AR;
CALL sqrt;
DM(min_dist)=SR1; \{store shortest dist in table\}

I4 =^group1;
CALL dist;
AR=PASS AF; $\{$ put distance squared into AR\}
MR0 $=0$;
$\mathrm{MR} 1=\mathrm{AR}$;
CALL sqrt;
$\mathrm{DM}($ min_dist +1$)=$ SR1; $\{$ store shortest dist in table\}
I4 =^group2;
CALL dist;
$A R=P A S S$ AF; $\quad\{p u t$ distance squared into AR\}
MR0 $=0$;
MR1=AR;
CALL sqrt;
DM(min_dist+2)=SR1; \{store shortest dist in table\}

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```
I4=^group3;
CALL dist;
AR=PASS AF; {put distance squared into AR}
MR0=0;
MR1=AR;
CALL sqrt;
DM(min_dist+3)=SR1; {store shortest dist in table}
I4=^group4;
CALL dist;
AR=PASS AF; {put distance squared into AR}
MR0=0;
MR1=AR;
CALL sqrt;
DM(min_dist+4)=SR1; {store shortest dist in table}
I4=^group5;
CALL dist;
AR=PASS AF; {put distance squared into AR}
MR0=0;
MR1=AR;
CALL sqrt;
DM(min_dist+5)=SR1; {store shortest dist in table}
I4=^group6;
CALL dist;
AR=PASS AF; {put distance squared into AR}
MR0=0;
MR1=AR;
CALL sqrt;
DM(min_dist+6)=SRI; {store shortest dist in table}
I4=^group7;
CALL dist;
AR=PASS AF; {put distance squared into AR}
MR0=0;
MR1=AR;
CALL sqrt;
DM(min_dist+7)=SR1; {store shortest dist in table}
SR1=H#7fff;
DM(min dist+8)=SR1;
```

\{Add each path distance to accumulated distance to yield 4 accumulated distances for each state. The shortest accumulated distance becomes the new accumulated distance to that state.\}

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$\{$ Find shortest path into state 0 . Choose from 0, 1, 2, 3 of min_dist table; these correspond to paths back to states $0,6,4,2$ respectively. The accumulated distances to these states are added with the paths of the current time window to determine the shortest accumulated path to this point.\}
short_path: I2=^min_dist;
I3 $=$ ^to_state0 +3 ;
CNTR=4;
CALL min_calc;
DM(temp_dist)=AR; \{store temporarily\}
$\mathrm{AXO}=4$;
$A Y 0=S I ;$
$A R=A X 0-A Y 0 ; \quad\{c a l c$. label from index of survivor\}
SR1=AR; \{store label into SR1, pack later\}
\{find the state from which the shortest path came\}
I2=^to_state0 - 1;
\{point to 1 before start of table\}
M2 $=$ SI; $\quad$ get index into table $\}$
MODIFY(I2,M2); \{point into table\}
SI=DM(I2,M1); \{get state at end of surviving path\}
\{now that state at end of path is known, store for later along with the 3-bit output label of the suriving path; pack both into 1 word; state in high byte, label low byte\}

```
SR=SR OR LSHIFT SI BY 8 (HI);
I3=DM(pointer0); {get pointer for state path}
DM(I3,M0)=SR1; {store state for current time window}
DM(pointer0)=I3; {store new pointer}
```

\{find shortest path into state 1, choose from 4, 5, 6, 7 of min_dist table these correspond to paths back to states $2,4,6,0$ respectively\}

I2 $=$ ^ min _dist +4 ;
I3 $=$ ^to_state $1+3$;
CNTR=4;
CALL min_calc;
DM (temp_dist + 1)=AR; \{store temporarily\}

AX0 $=8$;
$A Y 0=S I$;
AR=AXO-AY0; \{calc. label from index of survivor\}
SR1=AR; \{store label into SR1, pack later\}
\{find the state from which the shortest path came.\}

## V. 32 Modems 2

```
I2=^to_state1 - 1; {point to start of table}
M2=SI; {get index into table}
MODIFY(I2,M2); {point into table}
SI=DM(I2,M1); {get state at end of surviving path}
```

\{now that state at end of path is known, store for later use along with the 3-bit output label of the suriving path pack both into 1 word state is in high byte, label lo byte.\}

```
SR=SR or LSHIFT SI BY 8 (HI);
I3=DM(pointer1); {get pointer for state path}
DM(I3,M0)=SR1; {store state for current time window}
DM(pointer1)=I3; {store new pointer}
```

\{find shortest path into state 2, choose from 0, 1, 2, 3 of min_dist table these correspond to paths back to states 4, 2, 0, 6 respectively\}
I2 $=^{\wedge}$ min_dist;
I3=^to_state $2+3$;
CNTR=4;
CALL min_calc;
DM (temp_dist + 2) =AR; \{store temporarily\}
$\mathrm{AXO}=4$;
$A Y 0=S I ;$
$A R=A X 0-A Y 0 ; \quad\{c a l c$. label from index of survivor\}
SR1=AR; \{store label into SR1, pack later\}
\{find the state from which the shortest path came.\}
I2 =^to state2 - 1; \{point to start of table\}
M2=SI; ${ }^{-}$\{get index into table\}
$\operatorname{MODIFY}(I 2, M 2) ; \quad\{$ point into table\}
SI=DM(I2,I1); \{get state at end of surviving path\}
\{now that state at end of path is known, store for later use along with the 3-bit output label of the suriving path pack both into 1 word state is in high byte, label lo byte.\}

SR=SR or LSHIFT SI BY 8 (HI);
I3=DM(pointer2); \{get pointer for state path\}
DM (I3, M0) $=$ SR1; $\quad\{$ store state for current time window\}
DM(pointer2)=i3; \{store new pointer\}

## 2 V. 32 Modems

\{find shortest path into state 3 , choose from 4, 5, 6, 7 of min_dist table these correspond to paths back to states $6,0,2,4$ respectively\}

```
I2=^min dist + 4;
I3=^to state3 + 3;
CNTR=4;
CALL min_calc;
DM(temp_dist + 3)=AR; {store temporarily}
AX0=8;
AYO=SI;
AR=AXO-AY0; {calc. label from index of survivor}
SRI=AR; {store label into SR1, pack later}
```

\{find the state from which the shortest path came.\}
I2=^to_state3 - 1; \{point to start of table\}
M2=SI; \{get index into table\}
MODIFY(I2,M2); \{point into table\}
SI=DM(I2,M1); \{get state at end of surviving path\}
\{now that state at end of path is known, store for later use along with the 3-bit output label of the suriving path pack both into 1 word state is in high byte, label lo byte.\}

SR=SR OR LSHIFT SI BY 8 (HI);
I3=DM(pointer3); \{get pointer for state path\}
DM(I3,M0)=SR1; \{store state for current time window\}
DM (pointer3) =I3; \{store new pointer\}
\{find shortest path into state 4, choose from 0, 1, 2, 3 of min_dist table these correspond to paths back to states 1, 7, 3, 5 respectively\}

```
I2=^min_dist;
I3=^to_state4 + 3;
CNTR=4;
CALL min_calc;
DM(temp_dist + 4)=AR; {store temporarily}
AX0=4;
AYO=SI;
AR=AXO-AYO; {calc. label from index of survivor}
SR1=AR; {store label into SR1, pack later}
```


## V. 32 Modems 2

```
{find the state from which the shortest path came.}
    I2=^to_state4 - 1; {point to start of table}
    M2=SI; {get index into table}
    MODIFY(I2,M2); {point into table}
    SI=DM(I2,M1); {get state at end of surviving path}
```

\{now that state at end of path is known, store for later use along with the 3-bit output label of the suriving path pack both into 1 word state is in high byte, label lo byte.\}

SR=SR OR LSHIFT SI BY 8 (HI);
I3=DM(pointer4); \{get pointer for state path\}
DM(I3,M0)=SR1; $\{$ store state for current time window\}
DM(pointer4)=I3; \{store new pointer\}
\{find shortest path into state 5, choose from 4, 5, 6, 7 of min dist table these correspond to paths back to states $7,1,5,3$ respectively\}

I2 $=\wedge$ min dist +4 ;
I3 =^to_state5 + 3;
CNTR=4;
CALL min_calc;
DM (temp_dist + 5) =AR; \{store temporarily\}
$\mathrm{AXO}=8$;
$A Y 0=S I ;$
AR=AXO-AY0; \{calc. label from index of survivor\}
SR1=AR; \{store label into SR1, will pack later\}
\{find the state from which the shortest path came.\}
I2=^to_state5 - 1; \{point to start of table\}
M2=SI; \{get index into table\}
MODIFY(I2,M2); \{point into table\}
SI=DM(I2,M1); \{get state at end of surviving path\}
\{now that state at end of path is known, store for later use along with the 3-bit output label of the suriving path pack both into 1 word state is in high byte, label lo byte.\}

SR=SR OR LSHIFT SI BY 8 (HI);
I3=DM(pointer5); \{get pointer for state path\}
DM (I3, MO) $=$ SR1; $\quad\{$ store state for current time window\}
DM (pointer5) =I3; \{store new pointer\}

## 2 V. 32 Modems

\{find shortest path into state 6 , choose from $0,1,2,3$ of min_dist table these correspond to paths back to states 5, 3, 7, 1 respectively\} I2 $=$ ^min_dist;
I3 $=$ ^to_state6 +3 ;
CNTR=4;
CALL min_calc;
DM(temp_dist + 6)=AR; \{store temporarily\}
$\mathrm{AXO}=4$;
AY0=SI;
AR=AXO-AYO; \{calc. label from index of survivor\}
SR1=AR; \{store label into SR1, pack later\}
\{find the state from which the shortest path came.\}
I2 $=^{\wedge}$ to_state6 - 1; \{point to start of table\}
I2=SI; $\{$ get index into table\}
MODIFY(I2,M2); \{point into table\}
SI=DM(I2,I1); \{get state at end of surviving path\}
\{now that state at end of path is known, store for later use along with the 3-bit output label of the suriving path pack both into 1 word state is in high byte, label lo byte\}

SR=SR or LSHIFT SI BY 8 (HI);
I3=DM (pointer6); \{get pointer for state path\}
DM(I3,M0) $=$ SR1; $\{$ store state for current time window\}
DM (pointer6)=I3; \{store new pointer\}
\{find shortest path into state 7, choose from 4, 5, 6, 7 of min_dist table these
correspond to paths back to states 3, 5, 1, 7 respectively\} I2 $={ }^{\wedge}$ min_dist +4 ; I3=^to_state7 +3 ; CNTR=4;
CALL min_calc;
DM (temp_dist + 7)=AR; \{store temporarily\}
AXO $=8$;
$A Y 0=S I ;$ $A R=A X 0-A Y 0 ; \quad\{c a l c . ~ l a b e l ~ f r o m ~ i n d e x ~ o f ~ s u r v i v o r\} ~$ SR1=AR; \{store label into SR1, pack later\}

## V. 32 Modems 2

```
{find the state from which the shortest path came.}
    I2=^to_state7 - 1; {point to start of table}
    M2=SI; {get index into table}
    MODIFY(I2,M2); {point into table}
    SI=DM(I2,M1); {get state at end of surviving path}
```

\{now that state at end of path is known, store for later use along with the 3-bit output label of the suriving path pack both into 1 word state is in high byte, label lo byte.\}

```
SR=SR OR LSHIFT SI BY 8 (HI);
I3=DM(pointer7); {get pointer for state path}
DM(I3,M0)=SR1; {store state for current time window}
DM(pointer7)=I3; {store new pointer}
```

\{Put data from temp_dist back into acc_dist as new accumulated distance up to this point.\}
replace: $\quad$ CNTR=8;
I2=^acc_dist;
I1=^temp_dist;
I1=0;
DO move_buf UNTIL CE;
SI=DM(I1,MO); \{read data from temp_dist\}
move_buf: $\quad D M(I 2, M 0)=S I ; \quad\{p u t$ back as new acc_dist \}
\{Search through the acc dist table for the shortest distance. This will indicate the end point of the surviving path.\}
search
I2=^acc_dist;
CNTR=8;

SI=CNTR;
AY0=h\#7FFF; \{initialize with largest number\}
AF=PASS AY0;
AX0 $=\mathrm{DM}(\mathrm{I} 2, \mathrm{MO})$;
DO short_dst UNTIL CE;
AR=AF-AX0;
IF LE JUMP short_dst; SI=CNTR; \{save index of smallest\} IF GE AF=PASS AXO; \{if smaller, update\}
short_dst: $\quad \mathrm{AXO}=\mathrm{DM}(\mathrm{I} 2, \mathrm{MO})$;
$\mathrm{AXO}=8$;
$A Y 0=S I ;$
AR=AXO-AYO; \{calc. which state is at end of surviving path\}

## 2 V. 32 Modems

\{Now that the end of surviving path is known (in $A R$ ), trace back $N$ time windows to find starting path or path of survivor in first time window.\}

\{At this point the surviving leg label is in AF and the state number in AR find the 5bit code in the group specified by value in $A F$ that is closest to the data recieved $N$ time windows ago.\}

| final_stage: | ```AR=PASS AF; MX1=AR; I2=^group_table; M2=AR; MODIFY(I2,M2); AX0=DM(I2,M1); I4=AX0;``` |
| :---: | :---: |
|  | $\begin{aligned} & \mathrm{AX} 0=\mathrm{DM}(\mathrm{I} 0, \mathrm{M} 0) ; \\ & \mathrm{M} 2=-1 ; \\ & \mathrm{AX} 1=\mathrm{DM}(\mathrm{I} 0, \mathrm{M} 2) ; \end{aligned}$ |

\{put leg label into AR\}
\{store leg label in MX1, for later\}
\{point to start of group table\}
\{get displacement into table\}
\{update pointer\}
\{get address of proper table\}
\{load i4 with start of group table\}
\{get $X$ coord. of input $N$ windows ago\}
\{get $Y$ coord. of input $N$ windows ago\}

## V. 32 Modems 2

```
    AY0=32767;
    AF=PASS AY0, AYO=PM(I4,M4);
    CNTR=4;
DO ptloop2 UNTIL CE;
    AR=AX0-AY0, AY1=PM(I4,M4);
        IF AV JUMP ptloop2;
    MY0=AR, AR=AX1-AY1;
    IF AV JUMP ptloop2;
    MY1=AR;
    MR=AR*MY1(SS), MX0=MYO;
    MR=MR+MX0*MY0 (RND);
    AR=MR1-AF;
    IF GE JUMP ptloop2;
    AF=PASS MR1;
    SI=CNTR;
ptloop2:
    AY0=PM(I4,M4);
```

\{init with max distance
$\{$ get $X$ value from table
\{init with max distance $\}$
$\{$ get $X$ value from table
\{4 points in group\}
\{do $X-X^{\prime}$ and get $Y$ \}
\{if overflow, go on\}
\{copy $\left.X-X^{\prime}, ~ d o ~ Y-Y '\right\}$
\{if overflow, go on\}
\{copy Y-Y'\}
\{square $\left.Y-Y^{\prime}, ~ c o p y ~ X-X '\right\}$
\{add square of $X-X^{\prime}$ \}
\{compare with previous\}
\{if larger, no update\}
\{if smaller, update\}
\{save index of closest point \}
$\mathrm{AXO}=4$;
$A Y 0=S I$;
AR=AXO-AYO; \{calculate index from min pointer\}
I2=^codes_table; \{point to start of codes_table\}
M2=MX1; $\{l e g ~ l a b e l ~ i s ~ o f f s e t ~ i n t o ~ t a b l e\} ~$
MODIFY(I2,M2);
SI=DM(I2,M1); \{get address of which codes buf\}
I2 $=$ SI;
M2=AR; $\{$ get index into codes table\}
MODIFY(I2,M2);
SR1=DM (I2, M1) ;
\{get 5-bit code from table\}
\{SR1 now contains the answer\}
answer: $\quad D M(d a c)=S R 1$;

RTI;

\{Calculate the Euclidean distance squared between the point specified by the $x$ and $y$ coordinates found data memory locations $x$ input and y_input and the points specified by the $x$ and $y$ coordinates found in the table pointed to by index register i4. The index denoting the table entry which is closest to the input point is left in register SI and the shortest distance squared is left in register AF.\}

```
dist: AY0=32767; {init min distance to max num}
    AX0=DM(x_input);
    AX1=DM(y_input);
    AF=PASS AY0, AY0=PM(I4,M4); {get X value from table}
    CNTR=4; {4 points in group}
```


## 2 V. 32 Modems

DO ptloop UNTIL CE;
$\mathrm{AR}=\mathrm{AXO}-\mathrm{AY0}, \mathrm{AY} 1=\mathrm{PM}(\mathrm{I} 4, \mathrm{M} 4)$; $\left\{\mathrm{do} \mathrm{X}-\mathrm{X}^{\prime}\right.$ and get Y$\}$
IF AV JUMP ptloop; \{if overflow, go on\}
$M Y 0=A R, A R=A X 1-A Y 1 ; \quad\left\{c o p y ~ X-X^{\prime}\right.$, do $\left.Y-Y^{\prime}\right\}$
IF AV JUMP ptloop; \{if overflow, go on\}
MY1=AR;
MR=AR*MY1 (SS), MX0=MY0;
\{copy Y-Y' \}
IF MV SAT MR;
$M R=M R+M X 0 * M Y 0$ (RND);
\{square $\left.Y-Y^{\prime}, ~ c o p y ~ X-X^{\prime}\right\}$

IF VM SAT MR;
AR=MR1-AF;
\{add square of $\left.X-X^{\prime}\right\}$

IF GE JUMP ptloop;
\{compare with previous\}
AF=PASS MR1;
\{if larger, no update\}
SI=CNTR; $\{$ save index of closest point \}
\{if smaller, update\}
ptloop:
$\mathrm{AY} 0=\mathrm{PM}(\mathrm{I} 4, \mathrm{M} 4)$;
\{get next $X$ value \}
RTS;

## $\{\longrightarrow$ \}

$\{$ Take a 32 -bit number whose most significant portion is in register MRI and least significant portion in register MRO and calculate the 16 -bit square root. If the input is interpreted as a 16.16 unsigned number, the output in register SR1 is in 8.8 signed format.\}
sqrt: $\quad$ 77=^sqrt_coeff; \{pointer to coeff. buffer\}
M4 = 1;
$\mathrm{L} 7=0$;
SE=EXP MR1 (HI); \{check for redundant bits\}
SE=EXP MR0 (LO);
AXO $=$ SE, $\mathrm{SR}=\mathrm{NORM}$ MR1 (HI); \{remove redundant bits\}
SR=SR OR NORM MRO (LO);
MYO = SR1, AR=PASS SR1;
IF EQ RTS;
MR=0;
MR1=base; $\{l o a d$ constant value \}
$M F=A R * M Y 0$ (RND), $M X 0=P M(I 7, M 4) ; \quad\{M F=x$ squared $\}$
$M R=M R+M X 0 * M Y O(S S), M X 0=P M(I 7, M 4) ;\{M R=$ base $+C X\}$
CNTR=4;
DO approx UNTIL CE;
$\mathrm{MR}=\mathrm{MR}+\mathrm{MX} 0$ * $\mathrm{MF}(\mathrm{SS}), \mathrm{MX} 0=\mathrm{PM}(\mathrm{I} 7, \mathrm{M} 4)$;
approx:

$$
\mathrm{MF}=\mathrm{AR} * \mathrm{MF} \text { (RND) ; }
$$

$A Y 0=15$;
$M Y 0=M R 1, \quad A R=A X 0+A Y 0 ; \quad\{S E+15=0 ?\}$
IF NE JUMP scale; \{no, compute square-root $\}$
SR=ASHIFT MR1 BY -7 (HI);
RTS;

## V. 32 Modems 2

```
scale:
    MR=0;
    MR1=sqrt2; {load 1 over square rt of 2}
    MY1=MR1, AR=ABS AR;
    AYO=AR;
    AR=AY0-1;
    IF EQ JUMP pwr_ok;
    CNTR=AR
    DO compute UNTIL CE;
compute:
pwr_ok:
IF NEG JUMP frac;
AY1=h#0080; {load a 1 in 9.23 format}
AY0=0;
DIVS AY1, MR1; {compute reciprocal MR}
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
DIVQ MR1;
MX0=AY0;
MR=0;
MR0=h#2000;
MR=MR+MX0*MYO (US);
SR=ASHIFT MR1 BY 1 (HI);
SR=SR OR LSHIFT MRO BY 1 (LO);
RTS;
frac: MR=MR1*MYO (RND);
SR=ASHIFT MR1 BY -7 (HI);
RTS;
```


## 2 V. 32 Modems


\{Take the distances found in the table pointed to by register $I 2$, add them to the accumulated distance to the state specified in the state table pointed to by register I3, and determine the shortest of these total distances. The shortest distance is placed in register $A R$ and the index of the shortest distance is placed in register SI.\}

```
min_calc: L3=0;
```

    SI=CNTR;
    AY0=h\#7FFF; \{initialize with largest number\}
    \(A F=P A S S\) AY0;
    MR1=DM (I2, MO) ;
    SR=ASHIFT MR1 BY -1 (HI); \{half scale\}
    AX0=SR1;
    DO short_dist UNTIL CE;
        AY1 \(=\mathrm{DM}(I 3, \mathrm{M} 3) ; \quad\) \{read state number \(\}\)
        I5=^acc_dist;
        M5=AY1;
        \(\operatorname{MODIFY}(I 5, M 5) ; \quad\left\{p o i n t ~ t o ~ p r o p e r ~ a c c \_d i s t ~ v a l\right\} ~\)
        MR1=DM (I5, M4) ;
        AR=MR1-AY0;
        IF EQ JUMP read_nxt;
        SR=ASHIFT MR1 BY -1 (HI);
        AY1=SR1;
    \(A R=A X 0+A Y 1 ; \quad\) \{add new path to acc_dist \}
    AX0 0 AR;
    AR=AF-AX0;
    IF LE JUMP read_nxt;
    SI=CNTR; \(\{\) save index of smallest\}
    IF GE AF=PASS AX0; \{if smaller, update\}
    read_nxt: $\quad \operatorname{MR1}=\mathrm{DM}(\mathrm{I} 2, \mathrm{MO})$;
SR=ASHIFT MR1 BY -1 (HI); \{half scale\}
short_dist:
AX0=SR1;
AX0=DM (I2, M0) ;
AR=PASS AF;
L3=N;
RTS;
.ENDMOD;
Listing 2.8 Viterbi Decoder

## V. 32 Modems 2

### 2.8 REFERENCES

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## Quadrature Amplitude $\mathbf{\square}$ Modulation

### 3.1 INTRODUCTION

The CCITT V. 32 modem recommendation calls for the use of quadrature amplitude modulation (QAM) in the transmit section and quadrature amplitude demodulation in the receive section of the modem. The encoded digital sequence to be transmitted is amplitude modulated in the digital domain and then converted to analog form (via a $\mathrm{D} / \mathrm{A}$ converter) for transmission over the telephone wires. At the receiving end of the V. 32 system, the received analog signal is digitized (via an A/D converter) and demodulated in the digital domain in order to recover the information that was sent.

This chapter describes the implementation of quadrature amplitude modulation and demodulation on the ADSP-2100 family of processors. See Chapter 2 for information on other aspects of the V. 32 modem recommendation.

### 3.2 QAM METHODOLOGY

Double-sideband quadrature amplitude modulation (QAM) is a very efficient modulation technique in terms of bandwidth usage. In QAM, two quadrature ( $90^{\circ}$ phase-shifted) carriers, $\cos \omega_{c} k$ and $\sin \omega_{c} k$, are amplitude-modulated by two separate information-bearing signals, as shown in Figure 3.1, which can be found on the following page.

The synthesized digital sequence can be expressed as:
$x(k)=m_{1}(k) \cos \omega_{c} k+m_{2}(k) \sin \omega_{c} k$
where $m_{1}(k)$ and $m_{2}(k)$ are the two separate information-bearing signals. The QAM signal sequence $x(k)$ has the spectrum:
$X(2 \pi F)=1 / 2\left[M_{1}\left(\omega-\omega_{c}\right)+M_{1}\left(\omega+\omega_{c}\right)\right]-j 1 / 2\left[M_{2}\left(\omega-\omega_{c}\right)-M_{2}\left(\omega+\omega_{c}\right)\right]$

## 3 QAM



Figure 3.1 QAM Modulator Block Diagram

The spectrum components of the information-bearing signals overlap. However, the quadrature phase relationship in the carrier components cos $\omega_{c} \mathrm{k}$ and $\sin \omega_{c} \mathrm{k}$ allows the receiving end of the V .32 system to separate the two signals.

The demodulation is performed as shown in Figure 3.2. A digital phaselocked loop is used to obtain the carrier component $\cos \omega_{c} k$ and to generate $\sin \omega_{\mathrm{c}} \mathrm{k}$.

Subsequently, the received sequence is multiplied by the two quadrature carriers. This multiplication results in two signal sequences:
$x(k) \cos \omega_{c} k=1 / 2 m_{1}(k)+1 / 2 m_{1}(k) \cos 2 \omega_{c} k+1 / 2 m_{2}(k) \sin 2 \omega_{c} k$ $x(k) \sin \omega_{c} k=1 / 2 m_{2}(k)+1 / 2 m_{2}(k) \cos 2 \omega_{c} k+1 / 2 m_{1}(k) \sin 2 \omega_{c} k$

## QAM 3

The information-bearing signal components $m_{1}(k)$ and $m_{2}(k)$ can be recovered by passing each of the sequences through a filter that rejects the double-frequency terms centered at $2 \omega$.

In this particular V. 32 implementation, the carrier frequency ( $\mathrm{F}_{\mathrm{c}}$ ) is 1800 Hz , the symbol rate is 2400 Hz and the sample rate of the modulator is 9600 Hz . Thus, the desired cosine carrier is:
$\cos \omega_{\mathrm{c}} \mathrm{k}=\cos 2 \pi \mathrm{~F}_{\mathrm{c}} \mathrm{kT} \mathrm{s}_{\mathrm{s}}=\cos 2 \pi(1800)(1 / 9600) \mathrm{k}=\cos 3 \pi / 8 \mathrm{k}$
and similarly the sine carrier is:
$\sin \omega_{c} k=\sin 3 \pi / 8 k$
Again, in this particular V. 32 implementation, the sequences $m_{1}(k)$ and $m_{2}(k)$ correspond to $i(k)$ and $q(k)$ respectively. These input streams are the filtered versions of quadrature and in-phase portions of the encoded symbols to be transmitted.


Figure 3.2 QAM Demodulator Block Diagram

## 3 QAM

### 3.3 ADSP-2100 FAMILY IMPLEMENTATION

There are two ADSP-21XX assembly modules that handle the modulation and demodulation tasks separately. These modules are arranged as interrupt service routines that can be called from a main program which is presumably managing the V. 32 modem.

Modulation is performed by the modulator routine shown on Listing 3.1. The first section of the code contains the necessary variable, constant and buffer declarations. The cosine table contains 16 discrete values of a cosine wave between 0 and $2 \pi$, in increments of $\pi / 8$. This table is used to generate the $\cos 3 \pi / 8 \mathrm{k}$ and $\sin 3 \pi / 8 \mathrm{k}$ values for the modulation process. The variable mod_ptr stores a pointer into the cosine table between interrupts. The mod_ptr points to the cosine value to be modulated with the next arriving data sample.

```
.MODULE/RAM modulator;
.VAR/PM/CIRC cosine[16]; {Declare cosine table}
.VAR
.PORT
.INIT cosine:<cosval.dat>; {Initialize the cosine table}
.INIT cos_ptr:^cosine;
q_in, i_in; {Input ports for i(k) and q(k)}
.EXTERNAL
.GLOBAL
.ENTRY
modulate:
```

```
I4=DM(cos_ptr); {Read current pointer to cosine table}
```

I4=DM(cos_ptr); {Read current pointer to cosine table}
M4=-4;
M4=-4;
M5=7;
M5=7;
L4=16;
L4=16;
MX0=PM(I4,M4); {Read current cos value}
MX0=PM(I4,M4); {Read current cos value}
MYO=DM(i_in); {Read I(k)}
MYO=DM(i_in); {Read I(k)}
MR=MX0*MY0(SS),MX0=PM(I4,M5); {cos(k)*I(k) and get -sin value}
MR=MX0*MY0(SS),MX0=PM(I4,M5); {cos(k)*I(k) and get -sin value}
MY0=DM(q_in); {Read Q (k)}
MY0=DM(q_in); {Read Q (k)}
MR=MR+MX0*MY0 (RND); {cos(k)*I(k)-sin(k)*Q(k)}
MR=MR+MX0*MY0 (RND); {cos(k)*I(k)-sin(k)*Q(k)}
SR=ASHIFT MR2 BY -1 (HI); {Scale modulated output by 1/2}
SR=ASHIFT MR2 BY -1 (HI); {Scale modulated output by 1/2}
SR=SR OR LSHIFT MR1 BY -1 (LO);
SR=SR OR LSHIFT MR1 BY -1 (LO);
DM(mod_out)=SR0; {Send scaled output}
DM(mod_out)=SR0; {Send scaled output}
DM(cos_ptr)=I4; {Save the cosine table pointer}
DM(cos_ptr)=I4; {Save the cosine table pointer}
RTI;

```
RTI;
```

.ENDMOD;

Listing 3.1 Modulator Code

## QAM

The main body of the modulator code starts at the label modulate. The current cosine pointer is read and used to fetch the proper cosine value from the table. This fetch is done using M4=-4, which modifies the I4 register to point to the proper sine value on the following program memory (PM) fetch. Next, the $i(k)$ input is read and multiplied with the cosine value. Subsequently, the proper sine value is fetched, multiplied with the $\mathrm{q}(\mathrm{k})$ input and added to the previous multiplication result. The sine value is fetched using M5=7 which modifies the I4 register to point to the proper cosine value on the following PM fetch. At this point, the MR register contains the output of the QAM modulator. Next, the contents of MR are scaled down by $1 / 2$ using the shifter. This is necessary to keep the output of the modulator within a 16-bit field without causing overflows or underflows. Finally the current I4 value is saved as mod_ptr and the output is sent to the $\mathrm{D} / \mathrm{A}$ converter.

The demodulation is handled by the demodulator routine shown in Listing 3.2 , on the next page. The first section of the code contains the necessary variable, constant and buffer declarations. This module also uses the cosine table that is declared and initialized in the modulator program. The variable demod_ptr points to the next cosine value for the demodulator, just as mod_ptr does for the modulator.

The main body of the demodulator code starts at the label demodulate. First, the current cosine pointer is read into I4. Next, the variable phase_shift is read in order to determine whether the phase-locked loop requires a phase shift in the cosine values to be used in demodulation. If a shift is required, the subroutine cos_gen is called to compute new values for the cosine table. Once this is completed, the appropriate cosine value is read from program memory using M4=-4. This value is multiplied with the input from the A/D converter and sent out to the memory location $x \cos$ which represents $x(k) \cos \omega_{c} k$. Subsequently, the proper sine value is fetched from program memory using M5=7 and multiplied with the A/D input. This result is sent to the memory location $x \sin$ which represents $x(k)$ $\sin \omega_{\mathrm{c}} \mathrm{k}$. Finally, the current I4 value is saved as demod_ptr.

### 3.4 REFERENCES

Proakis, John G. 1989. Digital Communications. Second Edition. New York, N.Y.: McGraw-Hill.

Proakis, John, G. and D.G. Manolakis. 1988. Introduction to Digital Signal Processing. New York, N.Y.: McMillan Publishing Co.

## 3 QAM


.ENDMOD;
Listing 3.2 Demodulator Code

## Echo Cancellation 4

### 4.1 INTRODUCTION

Most voiceband telephone connections involve several connections through the telephone network. The 2-wire subscriber line available at most sites is generally converted to a 4-wire signal at the telephone central office. The signal must be converted back to a 2-wire signal at the far-end subscriber line. The 2-to-4-wire interface is implemented with a circuit called a hybrid. The hybrid intentionally inserts impedance mismatches to prevent oscillations on the 4 -wire trunk line. The mismatch forces a portion of the transmitted signal to be reflected or echoed back to the transmitter. This echo can corrupt data the transmitter receives from the far-end modem.

The telephone system and sources of echo are shown in Figure 4.1. There are two types of echo in a typical voiceband telephone connection. The first echo is the reflection from the near-end hybrid, and the second echo is from the far-end hybrid.


Four Wire Trunk
Figure 4.1 Telephone Channel Block Diagram

## 4 Echo Cancellation

In long distance telephone transmissions, the transmitted signal is heterodyned to and from a carrier frequency. Since local oscillators in the network are not exactly matched, the carrier frequency of the far-end echo is offset from the frequency of the transmitted carrier signal. In modem applications this shift can affect the degree to which the echo signal can be cancelled. It is therefore desirable for the echo canceller to compensate for this frequency offset.

### 4.2 ECHO CANCELLATION ALGORITHM

A data signal produced by a modem with a two-dimensional signal constellation has the form
$\mathrm{s}(\mathrm{t})=\mathrm{RE}\left[\sum \mathrm{b}_{\mathrm{m}} \mathrm{g}(\mathrm{t}-\mathrm{mT}) \mathrm{e}^{\mathrm{j} 2 \pi \mathrm{ft}}\right]$
where $b_{m}$ is the complex data symbol and $g(t)$ is the baseband pulse shape. The frequency $f$ is the carrier frequency. The echo signal is the transmitted signal convolved with the channel transfer function, $\mathrm{H}(\mathrm{f})$. This transfer function usually involves a linear delay and some dispersive filtering. The echo signal has the form
$\mathrm{s}_{\mathrm{e}}(\mathrm{t})=\operatorname{RE}\left[\sum \mathrm{b}_{\mathrm{m}} \mathrm{h}(\mathrm{t}-\mathrm{mT}) \mathrm{e}^{\mathrm{j} 2 \pi\left(f+\mathrm{f}^{\prime}\right) \mathrm{t}}\right]$
where $f^{\prime}$ is the frequency offset (Weinstein, 1977).
If the near-end modem is transmitting a signal $\mathrm{s}(\mathrm{n})$ and the far-end modem is transmitting a signal $y(n)$, the near-end received signal is:
$\mathrm{r}(\mathrm{n})=\mathrm{y}(\mathrm{n})+\mathrm{s}_{\mathrm{ne}}(\mathrm{n})+\mathrm{s}_{\mathrm{fe}}(\mathrm{n})+\mathrm{w}(\mathrm{n})$
where $s_{n e}$ and $s_{f e}$ are the near-end and far-end echo respectively, and $w(n)$ is random noise introduced by the system.

Echo cancellation is accomplished by subtracting an estimate of the echo return signal from the actual received signal. The received signal after echo cancellation is
$\mathrm{r}^{\prime}(\mathrm{n})=\mathrm{y}(\mathrm{n})+\left(\mathrm{s}_{\mathrm{ne}}(\mathrm{n})-{ }^{\wedge} \mathrm{s}_{\mathrm{ne}}(\mathrm{n})\right)+\left(\mathrm{s}_{\mathrm{fe}}(\mathrm{n})-\mathrm{N}_{\mathrm{fe}}(\mathrm{n})\right)+\mathrm{w}(\mathrm{n})$
where ${ }^{\wedge} s_{f e}(n)$ is the estimate of the far-end echo and ${ }^{\wedge} s_{n e}(n)$ is the estimate of the near-end echo. Ideally, the estimates are equal to the echo signals and the echo terms drop out (Quatieri and O'Leary, 1989).

## Echo Cancellation 4

The estimated echo is generated by feeding the transmitted signal into an adaptive filter whose transfer function tries to model the telephone channel's (see Figure 4.2). The filter coefficients are determined using the stochastic gradient (Least Mean Squared, or LMS) algorithm (Kamilo and Messerschmitt, 1987) during a training sequence prior to full duplex communications. The LMS algorithm attempts to minimize the mean squared error $\left|\mathrm{E}(\mathrm{n})^{2}\right|$. A more detailed description of the LMS algorithm can be found in Chapter 5.

In the training sequence, because the far-end modem is not transmitting, the received signal consists of echo:
$\mathrm{r}(\mathrm{n})=\mathrm{s}_{\mathrm{ne}}(\mathrm{n})+\mathrm{s}_{\mathrm{fe}}(\mathrm{n})$
The output of the filter is an estimate of the received signal,
$\mathrm{r}^{\wedge}(\mathrm{n})={ }^{\wedge} \mathrm{S}_{\mathrm{ne}}(\mathrm{n})+{ }^{\wedge} \mathrm{S}_{\mathrm{fe}}(\mathrm{n})$
and the difference is the error term that the LMS algorithm operates on.
$\mathrm{E}(\mathrm{n})=\mathrm{r}(\mathrm{n})-\mathrm{r}^{\wedge}(\mathrm{n})$


Figure 4.2 Echo Canceller

## 4 Echo Cancellation

The adaptive filter is commonly implemented with a transverse FIR filter. The structure of this filter is shown in Figure 4.3. The LMS update equation for $\operatorname{tap} C$ at sample time $n$ is
$C(n)_{k+1}=C(n)_{k}+\beta A(n) E(n)$
where $A(n)$ is the sample transmitted at sample time $n, \mathrm{E}(\mathrm{n})$ is the residual error and $\beta$ is an adaptation constant related to the rate of convergence.


Figure 4.3 LMS Adaptive Filter

In a modem application, the filter taps are only updated during the training periods. The tap update algorithm is either disabled or the adaptation constant $\beta$ is greatly reduced during full duplex operation. In the second case, reducing $\beta$ allows the echo canceller to track a slowly changing telephone channel without retraining the modem.

### 4.2.1 ADSP-2100 Family Implementation of LMS Algorithm

Figure 4.4 shows a flowchart for implementing the LMS stochastic gradient algorithm on the ADSP-2100 family of processors. The LMS algorithm is implemented in an interrupt service routine so that the arrival of a new sample forces one iteration of the algorithm. In this example, the FIR filter and the tap update are implemented as subroutine calls from the interrupt service routine.

## Echo Cancellation 4

In applications such as V. 32 modems, the tap update algorithm gets disabled during full duplex operation.


Figure 4.4 Flowchart for LMS Stochastic Gradient Algorithm

## 4 Echo Cancellation

Listing 4.1 contains the LMS filter code. The ADSP-2100 family can execute a multiply/accumulate operation and fetch two operands in a single cycle. The FIR filter loop and the tap update loop are executed without any additional cycles for loop overhead. These features allow the FIR filter to execute in one cycle per tap and the coefficient update to execute in two cycles per tap. Table 4.1, on page 4-17, summarizes the execution speeds.

Some applications require the echo canceller to operate on complex data. A complex data implementation of the LMS algorithm is described in Chapter 5.

```
.MODULE/RAM/ABS=0
    adaptive;
{ Near and Far End Echo Canceller
    INPUT: Received Data from Channel
    Transmitted Data
    OUTPUT: To Rest of Modem
}
.PORT received_data; {Received sample from channel}
.PORT transmitted_data; {Transmitted sample from modem}
.PORT tran
.CONST A=154;
.CONST beta=H#CC;
.VAR/DM/RAM/CIRC
.VAR/DM/RAM/CIRC
    enable;
    afilt_data[A];
    {Output to rest of modem}
{Adaptive filter length}
    {Adaptation constant}
    afilt_coeff[A];
{Update enabled bit}
.VAR/PM/RAM/CIRC
    {Filter delay line}
    {Filter coefficients}
{ Each new sample asserts interrupt 3}
start: RTI;
    RTI;
    RTI;
    JUMP sample;
```


## Echo Cancellation 4

```
{ Initialize Routine: This is executed during system startup}
.ENTRY setup;
setup: ICNTL=B#01111; {Initialize Interrupts}
    MO=0; {Initialize DAGS }
    M1=1;
    M3=-1;
    M4=1;
    M5=1;
    M6=-1;
    M7=2;
    IO=^afilt__data;
    I4=^afilt_coeff;
    L0=%afilt_data;
    L4=%afilt_coeff;
    AX0=H#0000;
    AY1=H#0000; {Initialize filter to 0}
    CNTR=%afilt_data;
    DO foo3 UNTIL CE;
foo3: PM(I4,M4)=AY1,DM(I0,M1)=AX0;
    IMASK=B#1000; {Enable IRQ2}
    JUMP fevr; {Wait for Interrupt}
{ Interrupt Routine: This code processes one data sample}
sample: AYO=DM(received_data); {Received data: r(n)}
    SR0=DM(transmitted_data); {Transmitted data: A(n)}
    CALL fir; {Calculate r^(n)}
    AR=AY0-MR1; {AR=error=r-r^}
    DM(out)=AR; {Output cancelled data}
    AX0=DM(enable); {Update taps if enabled}
    AF=PASS AXO;
    IF EQ CALL update;
done: RTI;
{ FIR Filter
    INPUTS:
        IO=Start of data buffer in DM
        I4=Start of coeff buffer in PM
        SRO=Newest input value
        M1,M4=1
    OUTPUTS:
        MR=Output value
    ALTERS:
        MR, MYO, MXO
}
```


## 4 Echo Cancellation

```
.ENTRY fir;
fir: DM(I0,M1)=SR0;
    MR=0, MX0=DM(I0,M1), MY0=PM(I4,M4);
    CNTR=A-1;
    DO floop UNTIL CE;
floop: MR=MR+MX0*MYO(SS), MX0=DM(I0,M1), MY0=PM(I4,M4);
    MR=MR+MX0*MYO (RND);
    RTS;
{ Adaptive Filter Coefficient Update
    INPUTS:
        IO=Start of data buffer in DM
        I4=Start of coeff buffer in PM
        M1,M4=1
        M6=-1
        M7=+2
        AR=error of last iteration
    Executes the coeff update algorithm as follows:
    Ck+1=Ck+Beta*Error*A (n)
}
.ENTRY update;
update: MY1=beta; {Load Beta}
                                {MF=Beta*Error, Load Ck, A(n) }
    MF=AR*MY1 (RND), AY0=PM(I4,M4), MX0=DM(I0,M1);
    MR=MXO*MF (RND);
    CNTR=A; {Tap update loop}
    DO uloop UNTIL CE;
        AR=MR1+AY0, AY0=PM(I4,M6), MX0=DM(I0,M1);
uloop: PM(I4,M7)=AR, MR=MX0*MF (RND);
    MODIFY(IO,M3);
    MODIFY(I4,M6);
    RTS;
.ENDMOD;
```

Listing 4.1 LMS Stochastic Gradient Implementation

### 4.3 FREQUENCY OFFSET COMPENSATION

Frequency offset in the far-end echo can limit convergence of the adaptive filter. In order to compensate for shifts in the carrier frequency, it is necessary to shift the received signal back to the original carrier frequency. Figure 4.5 shows a block diagram for performing this operation. The

## Echo Cancellation 4

frequency shifter is a first-order digital phase locked loop (DPLL). The magnitude of the frequency shift is defined as
$\varnothing^{\wedge}(\mathrm{n}+1)=\varnothing^{\wedge}(\mathrm{n})+\beta \mathrm{A}(\mathrm{n})\left(\varnothing(\mathrm{n})-\varnothing^{\wedge}(\mathrm{n})\right) \mathrm{r}(\mathrm{n})$
where $\beta$ is the adaptation constant, $\varnothing(n)$ is the frequency offset of sample $n, \varnothing^{\wedge}(\mathrm{n})$ is the estimate of the frequency offset, $\mathrm{A}(\mathrm{n})$ is the transmitted sample, and $r(n)$ is the received sample from the echo channel (Wang and Werner, 1988).


Figure 4.5 Block Diagram of Echo Canceller with Frequency Shift

## 4 Echo Cancellation

When compensating for frequency offset, the received sample must be rotated before the error term is calculated. The new error equation is

$$
E(n)=r(n) e^{j o t}-r(n)^{\wedge}
$$

In a real system, the frequency shift is implemented in the time domain with a Hilbert transform algorithm. Figure 4.6 shows the general structure of this algorithm.


Figure 4.6 Block Diagram of Hilbert Transform

The Hilbert algorithm is best understood in the frequency domain. Consider the real, bandlimited signal shown in Figure 4.7a. The Hilbert transfer function is

$$
\begin{aligned}
H(\omega) & =-\mathrm{j} \omega>0 \\
& =+\mathrm{j} \omega<0
\end{aligned}
$$

The output of the Hilbert transform is multiplied by +j so that the frequency magnitude is real. The sum of the Hilbert transform and the original sample is complex in the time domain and contains only positive frequencies in the frequency domain. The magnitude in the frequency domain is equal to twice the magnitude of the original sample (Figure 4.7 d ).

The frequency shift is accomplished by convolving (in the frequency domain) the signal in Figure 4.7d with the desired frequency. This convolution is equivalent to multiplying the time domain signal by $e^{-j \omega_{0}}$, where $\omega_{0}$ is the desired frequency shift. The sample is converted back to a real signal by taking the real part of the complex waveform.

## Echo Cancellation 4



Figure 4.7 Spectrum of Hilbert Frequency Shift

### 4.3.1 ADSP-2100 Family Implementation of Hilbert Transform

Code implementing a Hilbert transform is shown in Listing 4.2. The received signal must be rotated before $\mathrm{E}_{\mathrm{n}^{\prime}}$ the error signal for the adaptive filter, can be calculated. The Hilbert transform is thus performed in a subroutine called from the LMS interrupt service routine.

## 4 Echo Cancellation

The Hilbert transform is implemented with a 31-tap transverse FIR filter. Since every other coefficient is zero, the circular buffers in the ADSP-2100 are programmed to access every other data sample. This is possible using multiple modify registers with a single index register in the data address generators. The 31-tap Hilbert transform executes in 20 cycles.

To compensate for the group delay in the Hilbert transform, a 15-cycle linear delay is required for the real-valued input signal. Again, the circular buffering capabilities of the ADSP-2100 family allow for a simple implementation. Once the delay line is initialized, the index registers automatically increment to the next value, even when the end of the buffer is reached. The 15-tap delay line executes in just 3 cycles per sample.

The addition operation described shown in Figure 4.6, (page 4-10), is actually summing of a real and a complex number. Since a real and imaginary number cannot be added, this operation is not implemented in the code. Instead, the real and imaginary parts are used in the complex multiplication.

The complex multiply by $\mathrm{e}^{-j \omega_{0} \mathrm{t}}$ would normally require four multiplications and two additions. In practice, the desired output is contained entirely in the real part of the product. Therefore, only two multiplications and one addition are required. The values for $\sin \left(\omega_{0} t\right)$ and $\cos \left(\omega_{0} t\right)$ must be calculated for each successive sample.

The single cycle multiply/accumulate operation on the ADSP-2100 family allows both multiplications and the addition to be executed in two cycles. Execution time is also reduced when operands are fetched from data memory in parallel with the multiplications. In transmit mode, the entire Hilbert frequency shift requires about 100 cycles to execute.

## Echo Cancellation 4

```
.MODULE/RAM/ABS=0
hilbert_rotator;
{ Hilbert Rotator
    INPUT: Received Sample
    OUTPUT: To Adaptive Filter
}
\begin{tabular}{|c|c|c|}
\hline . CONST & \(\mathrm{H}=31\); & \{Length of Hilbert xform filter\} \\
\hline . PORT & received_data; & \{Received sample from channel\} \\
\hline . PORT & out; & \{Output to rest of modem\} \\
\hline . VAR/DM/RAM/CIRC & hdelay[H]; & \{Delay line for phase matching\} \\
\hline . VAR/DM/RAM/CIRC & hil_dat [H]; & \{filter data values\} \\
\hline . VAR/PM/RAM/CIRC & hilbert_coeff[16]; & \{Hilbert filter coefficients\} \\
\hline . VAR / DM / RAM & time; & \\
\hline . VAR / DM/RAM & delta_time; & \{Delta for frequency shift\} \\
\hline . VAR/DM/RAM & high; & \\
\hline . VAR/DM/RAM & low; & \\
\hline . VAR / DM/RAM & ovr; & \\
\hline . INIT & \multicolumn{2}{|l|}{hilbert_coeff: <hilb.dat>;} \\
\hline & & \{Hilbert filter coefficients \\
\hline
\end{tabular}
{ Initialize Routine: This is executed during system startup}
.ENTRY setup;
setup: AXO=H#O0;
    DM(time) =AX0;
    AX0=H#02;
    DM(delta_time)=AX0;
    CNTR=^HIL_DAT; {Init Delay line, Hilbert data}
    DO iloop UNTIL CE;
        DM(IO,M1) =H#0000;
iloop: DM(I1,M1)=H#0000;
    IMASK=B#1000; {Enable IRQ2}
fevr: JUMP fevr; {Wait for Interrupt}
{ Interrupt Routine: This code processes one data sample}
sample: AYO=DM(received_data); {Received data: r(n)}
    CALL delay; {Insert r(n) into delay line}
    CALI hilb; {Execute Hilbert transform}
    CALL rotate2;
    AR=MR1;
    DM(out)=AR;
    RTI;
```


## 4 Echo Cancellation

```
{ 31 Tap Linear Delay Line
    INPUTS: AY0=Newest Input Value
        IO=Oldest value in delay
        M0=0
        M1=1
    OUTPUTS: AX1=Delay line output
}
.ENTRY delay;
delay: AX1=DM(I0,M0);
    DM(IO,M1)=AY0;
    RTS;
{ 31 Tap Fir Hilbert Filter
    INPUTS: AY0=Newest Input Data
        II=Oldest data value
        I4=First Coeff value
        M0=0
        M1=1
        M4 =1
    OUTPUTS: AY0=Hilbert output
}
.ENTRY hilb;
hilb: MR=0, MX0=DM(I1,M2), MY0=PM(I4,M4);
    CNTR=16;
    DO hil_loop UNTIL CE;
hil_loop: MR=MR+MX0*MY0(SS), MX0=DM(I1,M2), MY0=PM(I4,M4);
    MR=MR+MX0 *MY0 (RND);
    DM(I1,M1)=AY0;
    AY0=MR1;
    RTS;
{ Hilbert Rotator
    Perform the calculation:
        Y(t)=RE[(Xr(t)+jXi(t)* (exp (-jWt))]
    INPUTS: AY0=Xi(t)
        AX1=Xr(t)
        AY1=W in degrees-q15 format
        W*t=DM(time)=time in q15
    OUTPUTS: MR=Y(t)
}
```


## Echo Cancellation 4

```
.ENTRY rotate2;
rotate2: AX0=DM(time);
    AY1=DM(delta_time);
    AR=AXO+AY1, MY0=AY0;
    IF AC AR=PASS 0;
    DM(time)=AR;
    CALL sin;
    MR=AR*MYO(SS), MYO=AX1;
    DM(ovr)=MR2;
    DM(high)=MR1;
    DM(low)=MR0;
    AY0=H#4000;
    {Xr(t)*sin(wt+90)}
    AR=AX0+AY0;
    AX0=AR;
    CALL sin;
    MR0=DM(low);
    MR1=DM(high);
    MR2=DM(ovr);
    MR=MR+AR*MYO(RND);
    RTS;
{ Sine Calculation
    Sine Approximation: Y=Sin(x)
    INPUTS: AXO=x in scaled 1.15 format
                        M3=1
                            L3=0
    OUTPUTS: AR=y in 2.14 format
    Computation Time: 25 cycles
}
```


## 4 Echo Cancellation

```
.VAR/DM sin_coeff[5];
.INIT sin_coeff: H#3240, H#0053, H#AACC, H#08B7, H#1CCE;
.ENTRY sin;
sin: I3=^sin_coeff; {Pointer to coeff. buffer}
    AYO=H#4000;
    AR=AXO, AF=AXO AND AYO; {Check 2nd or 4th quad}
    IF NE AR=-AXO; {If yes, negate input}
    AYO=H#7FFE;
    AR=AR AND AYO; {Remove sign bit}
    MY1=AR;
    MF=AR*MY1 (RND), MX1=DM(I3,M3); {MF=x2}
    MR=MX1*MY1 (SS), MX1=DM(I3,M3); {MR=C1x }
    CNTR=3;
    DO approx UNTIL CE;
        MR=MR+MXI *MF (SS) ;
approx: MF=AR*MF (RND), MX1=DM(I3,M3);
    MR=MR+MXI*MF (SS) ;
    SR=ASHIFT MR1 BY 2(HI);
    SR=SR OR LSHIFT MRO BY 2(LO); {Convert to 2.14 format}
    AR=PASS SR1;
    IF LT AR=PASS AY0; {Saturate if needed}
    AF=PASS AXO;
    IF LT AR=-AR; {Negate output if needed}
    RTS;
.ENDMOD;
```

Listing 4.2 Hilbert Transform Implementation

## $4.4 \quad$ V. 32 MODEM IMPLEMENTATION

V. 32 modems operate in full duplex mode; both the near-end and far-end modem are transmitting data at the same time. The echo canceller is responsible for channel separation as well as cancelling the near-end and far-end echos.

The echo canceller can be implemented in the passband or the baseband. The advantage of passband cancellation is reduced computation. A baseband echo canceller must execute all algorithms on complex data. In addition, compensating for frequency shift in the baseband is difficult. The disadvantage of passband echo canceller is a longer convergence time for the adaptive filter and the digital phase locked loop. Figure 4.8 shows a block diagram of a V. 32 modem with a passband echo canceller.

## Echo Cancellation 4



Figure 4.8 V. 32 Modem Block Diagram

The CCITT specification for V. 32 modems recommends a carrier frequency of $1800 \pm 7 \mathrm{~Hz}$. The echo canceller must be able to cancel 16 ms of echo. At 9600 samples/second, a 154 -tap FIR filter is required to cancel the echo. It is recommended that the echo canceller be implemented with a minimum number of taps.

Assuming that the canceller and frequency shifter have converged during the training period, about 200 cycles are required to cancel a V. 32 signal. Benchmarks are summarized in Table 4.1.

Operation
Real FIR Filter
Complex FIR Filter
Real LMS Update (Stochastic)
Complex LMS Update (Stochastic)
154-Tap LMS Filter With Update

Cycles
@12.5 MHz
$\mathrm{N}+6$
80 ns per tap
$4(\mathrm{~N}-1)+21$
$2 \mathrm{~N}+9$
$6 \mathrm{~N}+10$
935

240 ns per tap 160 ns per tap 480 ns per tap $74.8 \mu \mathrm{~s}$
$\mathrm{N}=$ Number of Taps
Table 4.1 ADSP-2100 Family Benchmarks for Echo Cancellation

## 4 Echo Cancellation

### 4.5 REFERENCES

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Quatieri, T. and G. O'Leary. 1989. "Far-Echo Cancellation in the Presence of Frequency Offset", IEEE Transactions on Communications. Volume 37, No. 6, pp. 635-634.

Wang, J. D. and J. J. Werner. 1988. "Performance Analysis of an Echo Cancellation Arrangement that Compensates for Frequency Offset in the Far Echo", IEEE Transactions on Communications. Volume COM-36, No. 3, pp. 364-372.

Weinstein, S. 1977. "A Passband Data-Driven Echo Canceller for FullDuplex Transmission on Two-Wire Circuits", IEEE Transactions on Communications. Volume COM-25, pp. 654-665.

## Adaptive Equalizer ■ 5

## $5.1 \quad$ INTRODUCTION

This chapter presents an ADSP-2100 family implementation of an adaptive channel equalizer for a high speed modem. The CCITT's V. 32
recommendation for a 9600 bps modem specifies the use of this type of equalizer in the receiver section.

The architecture used in this equalizer is a fractionally-spaced tapped delay line with a least-mean-squared (LMS) algorithm for adapting the tap weights.

The topics discussed in this chapter are:

- Historical perspective of adaptive filters
- Applications of adaptive filters
- Channel equalization in a modem
- Equalizer structures
- Least Mean Square (LMS) Algorithm
- Program Structure
- Practical considerations


### 5.2 HISTORY OF ADAPTIVE FILTERS

Until the mid-1960s, telephone-channel equalizers were either fixed equalizers that caused performance degradation or manually adjustable equalizers that were cumbersome to adjust.

In 1965, Lucky (see "References" at the end of this chapter) introduced the zero-forcing algorithm for automatic adjustment of the equalizer tap weights. This algorithm minimizes a certain distortion, which has the effect of forcing the intersymbol interference (ISI) to zero. This breakthrough by Lucky inspired other researchers to investigate different aspects of the adaptive equalization problem, leading to new improved solutions.

## 5 Adaptive Equalizer

Proakis and Miller (1969) reformulated the adaptive equalizer problem using a new criterion known as the mean squared error (MSE). This formulation requires a relatively modest amount of computation and remains the most popular approach for data rates up to 9600 bits/s.

Three years later, Ungerboeck (1972) improved on this work by presenting a detailed mathematical analysis of the convergence properties of an adaptive transversal equalizer using the least-mean-squared (LMS) algorithm. This algorithm is described later in this chapter.

A more powerful algorithm for adjusting the tap weights based on Kalman filtering theory was developed soon afterward (Godard, 1974). This algorithm is computationally demanding, but it was later modified by Falcomer and Ljing (1978) to simplify its computational complexity.

All of these adaptive equalizer implementations are synchronous, that is, the spacing between taps is equal to the reciprocal of the symbol interval. Other possible structures include the fractionally spaced equalizer (FSE) and the decision feedback equalizer (DFE).

The FSE has the ability to better compensate for channel distortion by spacing the tap weights more closely than in the conventional synchronous equalizer. Brady (1970) did some early work on this class of equalizers and was followed by Ungerboeck (1976). The DFE, on the other hand, uses a more elaborate structure and can yield good performance in the presence of severe ISI as experienced in fading radio channels.

### 5.3 APPLICATIONS OF ADAPTIVE FILTERS

Adaptive filters offer a significant improvement in performance over fixed-tap-weight digital filters because of their ability to detect signals in environments of unknown characteristics. They are successfully used in several areas including:

## System Identification and Modeling

An adaptive transversal filter can be forced to converge to the same impulse response as an unknown linear system and then can be used to model the unknown system. To determine the taps for this filter, an excitation input drives both the unknown system and the adaptive filter. The outputs of these two systems are compared, and the error signal generated is used to adjust the tap weights of the adaptive filter to reduce the error size. After a sufficiently large number of iterations, the error is

## Adaptive Equalizer 5

reduced to some small value (in a statistical sense) and the tap weights converge to model the real system.

If the unknown system is dynamic and time-variant, the adaptive filter can track these variations provided they are sufficiently slow compared to the convergence time of the filter.

## Echo Cancellation

In telephone systems that include both 2-wire and 4-wire loops, hybrid circuits couple these lines. These hybrid circuits create impedance mismatches which in turn create signal reflections, heard at both ends of the line as echo. This echo is tolerable to some degree over long distance voice connections, but can be catastrophic in high-speed data transmission over cross-Atlantic links.

Echo cancellers, in the form of adaptive filters, model the impulse response of the echo path. Cancellation is achieved by making an estimate of the echo and subtracting it from the return signal. See Chapter 4 for a detailed discussion of echo cancellation.

## Linear Predictive Coding

In the past 20 years, digital coding of speech waveforms has become a popular technique for reducing speech degradation due to transmission. Of the speech coding techniques, linear predictive coding (LPC) stands out for its ability to produce low data rates. Basic speech parameters (e.g. pitch, vocal tract, formants) are estimated, transmitted and then used at the receiver to resynthesize the speech through a speech production model. Adaptive filters can be used to estimate speech parameters in model-based speech coding systems.

The speech quality of LPC is synthetic when compared to other coding techniques such as PCM or ADPCM; however, its significantly lower data rates make it attractive. The GSM standard for the Pan-European cellular digital mobile radio network specifies an LPC-based coding scheme.

## Adaptive Beamforming

A spatial form of adaptive signal processing finds applications in radar and sonar. By combining signals from an array of sensors, it is possible to change the directivity pattern of the array. Independent sensors (e.g. antennas or hydrophones) placed at various locations in space or water detect incoming waveforms. The collection of sensor outputs at a particular instant is analogous to the set of consecutive tap inputs in a

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transversal filter. The sensitivity and directivity of the sensor array can be adaptively adjusted. Beamforming is discussed in Chapter 15 of Digital Signal Processing Using the ADSP-2100 Family.

## Adaptive Channel Equalization for Data Transmission

 Adaptive filters used in digital communication systems as channel equalizers minimize transmission distortion and maximize the use of channel bandwidth. A typical bandlimited telephone channel or radio link suffers from intersymbol interference (ISI) and additive noise. To improve system performance in additive-noise channels, transmission power can be increased. However, increased power has no effect on ISI since it amplifies both the intended symbol sample as well as interfering ones.The traditional technique for alleviating ISI is an equalizing filter at the receiver. The receiver equalizer filter combines the channel characteristics and the transmitter filter to minimize ISI distortions. Channel characteristics, however, vary over time. An adaptive equalizer is needed to ensure a constant transmission quality.

Since the channel conditions are unknown, a training sequence is transmitted to bring up the equalizer from its initial (usually zero) state. This sequence is known at the receiver and therefore the deviation error of received samples from the expected sequence is used to adjust the equalizer tap weights. Once the training period is completed, the weights can still be continually updated in a decision directed mode. In this mode, a minimum distance detector at the receiver decides which symbol was transmitted. In normal operation these decisions have a high probability of being correct, and thus are good enough to allow the equalizer to maintain proper adjustment.

### 5.4 CHANNEL EQUALIZATION IN A MODEM

The International Telegraph and Telephone Consultative Committee (CCITT) sets standards and protocols for telephone and telegraph equipment. Its V. 32 modem recommendation specifies a fractionally spaced transversal filter as the channel equalizer in the receiver. This equalizer, along with trellis coding and quadrature amplitude modulation (QAM), maximizes data rates over the bandlimited telephone channel.

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A telephone channel can suffer from a variety of limitations as a communications medium:

- As a bandlimited channel, it creates an environment for ISI.
- Channel additive noise requires increased transmitted power to improve signal-to-noise ratio.
- Radio links create fading channels and echo in cross-Atlantic connections
- When several connections are frequency multiplexed, baseband speech signals are modulated into the passband using different carrier frequencies for transmission. Demodulating these passband signals can create frequency offsets as well as amplitude and phase distortion.
- Phase jitter (poor timing recovery).
- Envelope delay or harmonic distortion is another limitation.

These channel limitations combined with the dense symbol constellation of the V. 32 modem necessitate adaptive equalization for acceptable error rates at 9600 bits/s.

### 5.4.1 Equalization

The basic function of the equalizer is to create an ideal transmission medium from a real channel. An example channel's short impulse response $\{\mathrm{h} 1, \mathrm{~h} 2, \mathrm{~h} 3, \mathrm{~h} 4\}$ is shown in Figure 5.1. The ideal medium is characterized as a pure delay, shown in Figure 5.2 (on the next page).


Figure 5.1 Example Short Impulse Response

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Figure 5.2 Pure Delay Impulse Response

Take for example the equalizer shown in Figure 5.3 which has three taps \{c1, c2, c3\}. Convolving this response with the channel's impulse response from Figure 5.1 yields
$\left|\begin{array}{l}\mathrm{y}_{1} \\ \mathrm{y}_{2} \\ \mathrm{y}_{3} \\ \mathrm{y}_{4} \\ \mathrm{y}_{5} \\ \mathrm{y}_{6}\end{array}\right|=\left|\begin{array}{llll}\mathrm{c}_{1} & 0 & 0 & 0 \\ \mathrm{c}_{2} & c_{1} & 0 & 0 \\ c_{3} & c_{2} & c_{1} & 0 \\ 0 & c_{3} & c_{2} & c_{1} \\ 0 & 0 & c_{3} & c_{2} \\ 0 & 0 & 0 & c_{3}\end{array}\right| \times\left|\begin{array}{l}\mathrm{h}_{1} \\ \mathrm{~h}_{2} \\ \mathrm{~h}_{3} \\ \mathrm{~h}_{4}\end{array}\right|$


Figure 5.3 Equalizer Impulse Response

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The outputs $\left\{\mathrm{y}_{1^{\prime}}, \mathrm{y}_{2^{\prime}} \mathrm{y}_{3^{\prime}}, \mathrm{y}_{4^{\prime}}, \mathrm{y}_{5^{\prime}}, \mathrm{y}_{6}\right\}$ represent samples of the impulse response of the combined channel/equalizer system.

If the equalizer is to create ideal conditions for transmission, all the $y^{\prime}$ s should be zeros except for one main sample. Rewriting the equation for ideal equalization yields:
$\left|\begin{array}{l}0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0\end{array}\right|=\left|\begin{array}{cccc}c_{1} & 0 & 0 & 0 \\ c_{2} & c_{1} & 0 & 0 \\ c_{3} & c_{2} & c_{1} & 0 \\ 0 & c_{3} & c_{2} & c_{1} \\ 0 & 0 & c_{3} & c_{2} \\ 0 & 0 & 0 & c_{3}\end{array}\right| \times\left|\begin{array}{l}h_{1} \\ h_{2} \\ h_{3} \\ h_{4}\end{array}\right|$
or
$0=\mathrm{c}_{1} \mathrm{~h}_{1}$
$0=c_{1} h_{2}+c_{2} h_{1}$
$1=c_{1} h_{3}+c_{2} h_{2}+c_{3} h_{1}$
$0=c_{1} h_{4}+c_{2} h_{3}+c_{3} h_{2}$
$0=c_{2} h_{4}+c_{3} h_{3}$
$0=c_{3} h_{4}$
The system of equations above has only three controllable variables (unknowns) but six simultaneous equations. The system is overdetermined and can only be solved approximately. To approximate this solution, a reformulation of a recursive technique known as method of steepest descent can be used. This iterative algorithm is defined by the equation:
(1) $\mathrm{C}_{\mathrm{k}+1}=\mathrm{C}_{\mathrm{k}}-\Delta \partial \mathrm{E} / \partial \mathrm{C}_{\mathrm{k}}$
where $E$ is a defined performance index to be optimized. It is a function of some controllable parameters (tap weights $C_{k}$ ). E is minimized by adjusting the tap weights in small steps $(\Delta)$. The gradient vector $\partial E / \partial C_{k}$ indicates the direction of the adjustment required to minimize E . This method converges to an optimum solution when $\partial \mathrm{E} / \partial \mathrm{C}_{\mathrm{k}}$ is zero.

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### 5.4.2 Performance Index

It is important to choose a meaningful performance index that is a linear function of the tap weights and that defines a smooth error surface (bowl) in the space spanned by the tap weight vector. This ensures the convergence of the algorithm to the lowest point (minimum) of the error surface.

In some cases, a desirable performance index is a nonlinear function of the adjustable parameters and the solution is unrealizable. As an example, consider the probability of error in a digital communication system. Even though this is a meaningful measure of system performance, it is a highly nonlinear function of the equalizer tap weights. Using the method of steepest descent, it cannot be determined whether the adaptive equalizer has converged to the optimum solution or to one of the relative minima of the surface. For this reason some desirable performance indices must be rejected.

A practical and popular index for performance is the mean squared error (MSE). The error is measured as the difference between the received signal and the ideal signal value. The MSE index is a measure of the energy in this error signal averaged over a signaling interval. It results in a quadratic performance surface as a function of the filter coefficients and thus has a single minimum (optimal solution). An implementation of an MSE-based iterative adaptation algorithm is developed for the ADSP-2100 processor family in this chapter; it is discussed in a later section.

### 5.5 EQUALIZER ARCHITECTURES

The preferred form of a linear equalizer is a tapped delay line. The delay line consists of delay elements in a feedforward path and possibly a feedback path.

If the delay line has feedforward delays only, its transfer function can be expressed as a single polynomial in $\mathrm{Z}^{-1}$ and therefore the equalizer has a finite impulse response (FIR). This type of equalizer is often called a nonrecursive or transversal equalizer (Figure 5.4).

If the delay line also has feedback delay elements, its transfer function is a rational function of $\mathrm{Z}^{-1}$ and the equalizer has an infinite impulse response (IIR) due to its nonzero poles (Figure 5.5).

The V. 32 modem equalizer has no feedback delay elements and is therefore an FIR equalizer.

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Figure 5.4 Transversal (FIR) Delay Line

### 5.5.1 Real Or Complex

In a one-dimensional communication system (e.g. pulse amplitude modulation or PAM), the signal is real and the equalizer has real coefficients. The V. 32 modem, which uses quadrature amplitude modulation (QAM), transmits complex data by modulating two


Figure 5.5 IIR Delay Line

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orthogonal carrier signals. Because of cross-distortion between the inphase and quadrature channels in this two-dimensional communication system, an equalizer with complex tap coefficients is required.

Algorithms for the complex equalizer are essentially the same as for the real equalizer with the added burden of complex arithmetic. A complex equalizer typically requires four times as many multiplications and introduces the complex conjugation operator in recursive algorithms such as LMS adaptation.

### 5.5.2 Sampling Rates

It is often advantageous to space the delay elements in an equalizer more closely than the symbol rate, as shown in Figure 5.6. This has the effect of oversampling the input to the filter and thus increasing the effective bandwidth of the equalizer. The input is pushed onto the delay line twice for every one output computed. Fractionally spaced equalizers have superior performance because of wider bandwidth, and they simplify the problem of phase synchronization between transmitter and receiver. They do, however, suffer from stability problems in low noise conditions and are more computationally demanding (Ungerboeck, 1976).

X(KT)


Figure 5.6 Fractionally Spaced Delay Line (FSE)
A fractionally spaced filter can be designed the same way as a T-spaced delay line filter. The basic delay line structure is the same for both. For a $\mathrm{T} / 2$ FSE filter, the samples are shifted in at $2 \mathrm{f}_{\mathrm{s}}$ (twice the sampling frequency) but the output is only computed at $f_{s^{\prime}}$, i.e. every other input time.

The ADSP-2100 routine to implement the delay line with complex tap weights is in Listing 5.1.

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This Complex Fractionally Spaced Filter (FSE) Subroutine is used in the V32 equalizer. The basic structure for the delay line is the same as that of a T-Spaced Filter (TSE). In the FSE case, however, samples are shifted in at 2 Fs ( $\mathrm{Fs}=$ Sampling Frequency) and the output is computed at Fs, i.e. at alternate times. This subroutine will therefore be called after 2 new input samples have been pushed onto the delay line.

```
Calling Parameters
    IO -> Oldest data value in real delay line (Xr's)
    LO = filter length (N)
    I1 -> Oldest data value in imag. delay line (Xi's)
    L1 = filter length (N)
    I4 -> Beginning of real coefficient table (Cr's)
    L4 = filter length (N)
    I5 -> Beginning of imaginary coefficient table (Ci's)
    L5 = filter length (N)
    M0,M6 = 1
    AXO = filter length minus one (N-1)
    CNTR = filter length minus one (N-1)
Return Values
    IO -> Oldest data value in real delay line
    I1 -> Oldest data value in imaginary delay line
    I4 -> Beginning of real coefficient table
    I5 -> Beginning of imaginary coefficient table
    SR1 = real output (rounded, cond. saturated)
    MR1 = imaginary output (rounded, cond. saturated)
Altered Registers
    MXO,MYO,MR,SR1
Computation Time
    2*(N-1) + 2*(N-1) + 13 + 8 cycles
```

All coefficients and data values are assumed to be in 1.15 format. \}

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```
fir: MR=0, MX0=DM(I1,M0), MY0=PM(I5,M6);
    DO realloop UNTIL CE;
        MR=MR-MXO*MYO(SS), MX0=DM(IO,M0), MYO=PM(I4,M6); {Xi * Ci}
realloop: MR=MR+MX0*MYO(SS), MX0=DM(I1,M0), MY0=PM(I5,M6); {Xr * Cr}
    MR=MR-MX0*MYO(SS), MX0=DM(I0,M0), MYO=PM(I4,M6); {last Xi * Ci}
    MR=MR+MX0*MYO (RND) ; {last Xr * Cr}
    IF MV SAT MR;
    SR1=MR1; {Store Yr}
    MR=0, MXO=DM(IO,MO), MYO=PM(I5,M6);
    CNTR=AX0;
    DO imagloop UNTIL CE;
        MR=MR+MX0*MYO(SS), MX0=DM(I1,M0), MY0=PM(I4,M6); {Xr * Ci}
imagloop: MR=MR+MX0*MY0(SS), MX0=DM(IO,M0), MYO=PM(I5,M6); {Xi * Cr}
    MR=MR+MX0*MYO(SS), MX0=DM(I1,M0), MYO=PM(I4,M6); {last Xr * Ci}
    MR=MR+MX0*MYO (RND); {last Xi * Cr}
    IF MV SAT MR;
    {MR1=Yi}
```

    RTS;
    
## Listing 5.1 Delay Line Routine, Complex Tap Weights

### 5.6 LEAST MEAN SQUARED (LMS) ALGORITHM

Since the mean squared error (MSE) performance index is a convex function of the tap weights (has a bowl-shaped surface), the optimum tap weights can be obtained by the steepest descent algorithm. In this algorithm, tap weights are assumed to have an arbitrary initial setup and are moved in the direction of optimum value when MSE is minimized. The direction is determined by the gradient of the objective function of performance,

$$
\begin{equation*}
E=|e(k t)|^{2} \tag{2}
\end{equation*}
$$

where e(kt) is the error between the estimated symbol and the received sample and the bar above the expression denotes time averaging. Optimum tap weights are determined when the derivative of the MSE surface with respect to all the tap weights is zero.

$$
\begin{equation*}
\partial \mathrm{E} / \partial \mathrm{C}_{\mathrm{k}}=0 \quad 1 \leq \mathrm{k} \leq \mathrm{N}, \text { for an N-tap filter } \tag{3}
\end{equation*}
$$

The error function E is a complex quadratic function because of the 2dimensional modulation scheme (QAM). The derivative expression is:

$$
\begin{equation*}
\partial \mathrm{E} / \partial \mathrm{C}_{\mathrm{n}}(\mathrm{k})=-2 \overline{\mathrm{e}(\mathrm{kt}) \mathrm{y}\left(\mathrm{kT}_{\mathrm{sym}}-\mathrm{nT}_{\mathrm{taps}}\right)} \tag{4}
\end{equation*}
$$

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where $\quad T_{\text {taps }}$ is the spacing between the taps $\mathrm{T}_{\text {sym }}$ is the spacing between symbols

Combining with equation (1) yields:

$$
\begin{equation*}
C_{n}(k+1)=C_{n}(k)+\beta \overline{e(k t) y^{*}\left(k T_{\text {sym }}-n T_{\text {taps }}\right)} \tag{5}
\end{equation*}
$$

The implementation of the steepest descent algorithm requires the evaluation of the cross-correlation of error signal $\mathrm{e}(\mathrm{kt})$ and received signal $y(t)$. Cross-correlation requires time-averaging, which is not a viable option considering the real time requirements of the equalizer. To alleviate this problem, the approximation:

$$
\begin{equation*}
\left.\overline{\mathrm{e}(\mathrm{kt}) \mathrm{y}^{*}\left(\mathrm{kT} \mathrm{~s}_{\text {sym }}-\mathrm{nT} \mathrm{~T}_{\text {taps }}\right.}\right) \approx \mathrm{e}(\mathrm{kt}) \mathrm{y}^{*}\left(\mathrm{kT} \mathrm{~s}_{\text {sym }}-\mathrm{nT} \mathrm{t}_{\text {taps }}\right) \tag{6}
\end{equation*}
$$

is used instead of time-averaging. This simplification of the steepest descent algorithm greatly reduces the amount of computation. It is very popular and is generally referred to as the least mean square (LMS) algorithm.

An LMS algorithm updates the equalizer tap weights according to

$$
\begin{equation*}
C_{n}(k+1)=C_{n}(k)+\beta e(k t) y^{*}\left(k T_{\text {sym }}-n T_{\text {taps }}\right) \tag{7}
\end{equation*}
$$

Listing 5.2, on the following page, shows an LMS algorithm implemented on the ADSP-2100 family.

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\{ Complex SG Update LMS Subroutine.
This routine updates the complex taps according to the relation:
$\operatorname{Cn}(k+1)=\operatorname{Cn}(k)-\operatorname{Beta} \cdot E(k) \cdot Y^{*}(n-K)$
where: <Beta> = Adaptation step size
$\langle E(k)\rangle=$ estimation error at time $k$
$\left\langle Y^{*}(n-k)\right\rangle=$ Received signal complex
conjugated \& sampled at time ( $n-k$ )

Calling Parameters
IO $\rightarrow$ Oldest data value in real delay line LO $=N$
I1 $\rightarrow$ Oldest data value in imag. delay line $\quad \mathrm{L} 1=\mathrm{N}$ I4 $\rightarrow$ Beginning of real coefficient table $L 4=N$ I5 $\rightarrow$ Beginning of imag coefficient table L5 $=\mathrm{N}$ MXO $=$ real part of Beta * Error MX1 = imag part of Beta * Error M0, M5 = 1 M1 $=-1$ M6=0 CNTR $=$ Filter length ( N )

Return Values
Coefficients updated
IO $\rightarrow$ Oldest data value in real delay line I1 $\rightarrow$ Oldest data value in imag delay line I4 $\rightarrow$ Beginning of real coefficient table I5 $->$ Beginning of imag coefficient table

Altered Registers
MY0, MY1, MR, SR, AY0, AY1, AR
Computation Time
$6 * N+10$ cycles
All coefficients and data values are assumed to be in 1.15 format.
\}
upd_taps:

MYO $=\mathrm{DM}(\mathrm{IO}, \mathrm{MO})$;
MR=MX0*MY0 (SS), MY1=DM(I1,M0); DO adaptc UNTIL CE; $M R=M R+M X 1 * M Y 1$ (RND) , $A Y 0=P M(I 4, M 6)$; $A R=A Y 0-M R 1, A Y 1=P M(I 5, M 6)$; $\operatorname{PM}(I 4, M 5)=A R, \quad M R=M X 1 * M Y O(S S) ;$ $M R=M R-M X 0 * M Y 1$ (RND), $M Y 0=D M(I 0, M 0)$; AR=AY1-MR1, MY1 =DM (I1, M0) ; $\operatorname{PM}(I 5, M 5)=A R, \quad M R=M X 0 * M Y 0(S S) ;$ MODIFY (IO,M1); MODIFY (I1,M1); RTS;
\{Get Xr\}
\{Er*Xr, get Xi\}
\{Ei*Xi, get Cr\}
\{Cr-(Er*Xr+Ei*Xi), get Ci\}
\{Store new Cr, Ei*Xr\}
\{Er*Xi, get Xr\}
\{Ci-(Ei*Xr-Er*Xi), get Xi\}
\{Store new Ci, Er*Xr\}
\{point back to start\}
\{of complex delay line\}

## Listing 5.2 LMS Routine

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### 5.7 PROGRAM STRUCTURE

The flowchart shown in Figure 5.7 depicts the sequence of operations of an equalizer program. Each program section is discussed below.


Figure 5.7 Adaptive Equalizer Flowchart

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### 5.7.1 Input New Sample

The equalizer program is interrupt-driven. The arrival of a new complex sample causes the equalizer to start executing. The sample_in port in Listing 5.3 holds the new sample (real, then imaginary). Index registers I0 and I1 point to the complex delay line.

The V. 32 modem recommendation specifies a fractionally spaced equalizer. The delay line therefore consists of delays that are spaced at one-half the symbol rate. This means that the output (at 2400 symbols/s) is only computed for every two input samples (at 4800 symbols $/ \mathrm{s}$ ). The variable decimator_flag is used to decide whether to get another sample or to start computing the output.

```
{ input_new_sample routine
This part will read a new sample from the port 'sample_in' and
place it on the delay line. This new complex sample will overwrite
the oldest value on the delay line (complex also).
}
start: AR=DM(sample in); {read in real & imag. values}
    DM(IO,MO)=AR; {of new sample and store them}
    AR=DM(sample_in); {in delay line}
    DM(I1,MO)=AR;
    AR=DM(decimator_flag); {check flag to see if filtering}
    {is required this time through.}
    AR=NOT AR; {Then toggle the flag}
    DM(decimator_flag)=AR; {to ensure that we filter}
    {every other sample}
    IF EQ RTS; {as required in an FSE}
```


## Listing 5.3 Input Routine

### 5.7.2 Filtering (Equalizing)

The actual filtering is performed in the subroutine in Listing 5.4. The calling parameters for the filter are initialized, and after the subroutine is called the return values are stored in data memory.

# Adaptive Equalizer 5 

```
{ do the fir filtering (equalization)
Performs the actual fir filtering. Takes the input sample
from the receiver front end & produces an output value
(fir_out_real & fir_out_imag )
}
    AXO = no_of_taps - 1;
    CNTR= no_of_taps - 1;
    CALL fir;
    DM(fir_out_real)=SR1; {save return values of}
    {subroutine in}
    DM(fir_out_imag)=MRI; {their designated var names:}
    {fir_out_real & fir_out_imag}
```


## Listing 5.4 Filter Routine

### 5.7.3 Training Sequence

Initially the tap weights of the equalizer are at some arbitrary state (possibly zero) that is typically far from the optimum state. The receiver decisions based on the output of the equalizer are therefore incorrect with a high probability. Decision-directed adaptation is not guaranteed to work because of the initial high error rate. The equalizer might be unable to move into the error-free region and the adaptation would diverge or stop (MSE neither increasing nor decreasing significantly).

To train the equalizer through this blind stage, a data sequence that is known at the receiver is used for initial transmission. If the locally generated reference is properly synchronized to the received signal, this training brings the equalizer to its optimum state. After training, slow channel variations are tracked using decision-directed adaptation.

The stored training sequence at the receiver is read at the training_list port (real, then imaginary) in Listing 5.5. The received signal is read in from the filter outputs fir_out_real and fir_out_imag. A complex error value which is equal to the Euclidean distance between the two samples is generated. The estimation error is stored in data memory (error_real and error_imag).

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```
{ estimate the transmitted symbol ( training )
Given fir_out_real & fir_out_imag, we compute the error value
(real and complex) using the training sequence as a reference.
This estimate for error is used only initially to train the
equalizer. Following the training, decision directed adaptation
would take over.
}
```

```
AXO=DM(fir_out_real); {inputs are fed in directly}
```

AXO=DM(fir_out_real); {inputs are fed in directly}
AX1=DM(fir_out_imag); {from output of fir}
AX1=DM(fir_out_imag); {from output of fir}
AYO=DM(training list);
AYO=DM(training list);
AY1=DM(training list);
AY1=DM(training list);
CALL est_error_train;

```
CALL est_error_train;
```



```
{
Est_error_train subroutine: Returns the equalizer output minus the
ideal value available from the training sequence.
    AX0 = fir_out_real
    AX1 = fir_out_imag
    AYO = ideal_symbol_real
    AY1 = ideal_symbol_imag
    Returns:
        error_real
        error_imag
}
est_error_train: AR=AXO-AY0;
        DM(error_real)=AR;
        AR=AX1-AY1;
        DM(error_imag)=AR;
        RTS;
```


## Listing 5.5 Training Sequence Routine

### 5.7.4 Decision-Directed Adaptation

Once the equalizer is trained, decision-directed adaptation is possible. In this mode, symbols estimated at the receiver are used as the reference from which to measure the deviation error and subsequently adjust the taps. With the equalizer trained, low decision-error rates make it possible to continue to adapt to small changes in channel conditions.

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In Listing 5.6, the estimated symbol is chosen as the symbol geometrically closest to the received coordinates. A 15-instruction loop (worst case) computes the distance to each of the 32 symbols in the symbol table and determines the nearest one. The routine returns a pointer to the estimated symbol in the table as well as the real and imaginary values of the error.

```
{ estimate the transmitted symbol ( no training )
Given fir_out_real & fir_out_imag, we compute the error value (real and complex) using
a Euclidean distance routine (decision directed adaptation). In this mode the estimated
symbol is the geometrically closest to the received coordinates. This routine also
returns the complex error value.
}
    AXO=DM(fir out real); {these inputs are fed in directly}
    AXI=DM(fir_out_imag); {from the output of the fir}
    CALL est_error_eucl;
{-}
{Estimate_error_euclidean Symbol Subroutine
(normal mode, i.e. no training):
Maps input sample onto an ideal symbol in the constellation table This routine also
returns the value of the error measured as the Euclidean distance between received
signal and its ideal value.
    Calling Parameters
    AXO contains Xr
    AX1 contains Xi
    MO = 1
    M1 = -1
Return Values
    SI = decision index j
    (position with respect to end of table)
    AF = minimum distance (squared)
    I2 -> Beginning of constellation table
Altered Registers
    AY0, AY1, AF , AR, MX0, MY0, MY1, MR, SI
    AR_SAT mode enabled
Computation Time
    15*N + 5 (maximum)
}
```


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```
est_error_eucl: I2 = ^constellation_table;
    L2 = 3; {number of symbols in}
    {constellation table}
    AYO=32767; {Initialize minimum distance to}
    {largest possible value}
    ENA AR_SAT; {put ALU in saturation mode to}
    {prevent overflow}
    AF=PASS AYO, AYO=DM(I2,M0); {Get Cr}
    CNTR =32;
DO ptloop UNTIL CE;
    AR=AX0-AY0, AY1=DM(I2,M0); {Xr-Cr, Get Ci}
    MY0=AR, AR=AX1-AY1; {Copy Xr-Cr, Xi-Ci}
    MY1=AR; {Copy Xi-Ci}
    MR=AR*MY1 (SS), MX0=MY0; {(Xi-Ci)^2, }
    {Copy Xr-Cr}
    MR=MR+MX0*MYO (SS);
    IF MV SAT MR; {clip result to max value}
    AR=MR1-AF; {Compare with previous minimum}
    IF GE JUMP ptloop;
    AF=PASS MR1; {New minimum if MR1<AF}
    AR=AXO-AYO; {error is euclidean distance}
    DM(error_real)=AR; {between actual received}
    AR=AX1-A}\overline{Y}1; {signal and ideal symbol
    DM(error_imag)=AR; {coordinates}
    SI=CNTR; {Record constellation index}
ptloop:
    AYO=DM(I2,M0);
MODIFY(I2,M1); {Point to beginning of table}
RTS;
```


## Listing 5.6 Decision-Directed Adaptation Routine

### 5.7.5 Tap Update (LMS Algorithm)

Once an estimate error is computed, it is possible to adapt the equalizer coefficients to a new set of values closer to the optimum vector. The LMS routine in Listing 5.7 performs the computation. The estimation error is first scaled down by the adaptation step size ( $\beta$ ). This constant provides a mechanism to trade off convergence speed against the amount of jitter in the steady state value of the tap vector.

## Adaptive Equalizer 5

```
{ update the taps
Takes the estimation error values previously computed multiply
them by the step size (beta). The upd taps routine is then called
to update coefficients of the equalizer.
}
```

```
MYO=DM(error_real); {MX0=beta x error_real }
```

MYO=DM(error_real); {MX0=beta x error_real }
MX0=beta;
MX0=beta;
MR =MXO*MYO(SS);
MR =MXO*MYO(SS);
MXO=MR1;
MXO=MR1;
MY1=DM(error_imag);
MY1=DM(error_imag);
MX1=beta
MX1=beta
MR =MX1*MY1 (SS);
MR =MX1*MY1 (SS);
MX1=MR1;
MX1=MR1;
CNTR=NO_of_taps;
CNTR=NO_of_taps;
CALL upd_taps;

```
CALL upd_taps;
```


## Listing 5.7 Tap Update Routine

### 5.7.6 Output

These equalizer routines can be integrated into other modules to form the receiver block of a V. 32 modem (see Chapter 2). As specified in the V. 32 recommendation, the equalized sample is decoded using the Viterbi algorithm. The equalizer output (real and the imaginary) is therefore written to an I/O port sample_out.

```
{ output the resulting sample of the equalizer}
AR=DM(fir_out_real); {output the equalizer output}
DM(sample_out)=AR; {to the outport port}
AR=DM(fir_out_imag);
DM(sample_out)=AR;
RTS; {return from equalizer routine and}
    {wait for a new sample interrupt}
```


## Listing 5.8 Output Routine

## 5 Adaptive Equalizer

### 5.8 PRACTICAL CONSIDERATIONS

This section describes considerations for using and modifying the routines in this chapter.

### 5.8.1 Viterbi Decoder

In the implementation of decision-directed adaptation, the received sample is matched to the nearest symbol and the error is used to adjust the taps. A few wrong decisions could cause the equalizer to wander off temporarily, but because right decisions have a proportionately larger effect, convergence is ensured.

If a sophisticated algorithm such as Viterbi decoding is used to improve the decision, the signal sample and error are not available until several symbol intervals after the input time. This Viterbi delay requires a modified LMS updating routing with delayed coefficient adaptation (DLMS). It can be shown that the DLMS adaptation has the same steady state behavior as the LMS adaptation, provided the adaptation constant is within a certain range (Long et al, 1989).

### 5.8.2 Pseudo-Random Training Sequence

The routines in this chapter have been validated with a pseudo-random training sequence. This training sequence consists of a set of symbol values with a repetition period that is much longer than the convergence time of the equalizer. The benefit of using such a sequence is that the approximation of the gradient vector $\partial \mathrm{E} / \partial \mathrm{C}_{\mathrm{k}}$ is less noisy. Noisy estimates of the gradient vector can cause the tap coefficients to wander a long way from the path of the steepest descent (Bingham, 1988).

### 5.8.3 Delay Line Length

If the exact source of the channel's distortion is known and the impulse response can be modeled precisely, it is possible to calculate the minimum order of the equalizer transfer function needed to reduce the MSE to an acceptable level. In general, the only practical method of deciding the length of the delay line is to derive a theoretical length based on several worst-case channel characteristics. The equalizer is then designed slightly longer than the theoretical minimum to compensate for the cumulative effects of finite precision arithmetic in the ADSP-2100 family processor. For a discussion of quantization effects in the LMS algorithm, see Bershad, 1989.

## Adaptive Equalizer 5

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# Continuous Phase Frequency-Shift Keyed Modulation 

### 6.1 INTRODUCTION

Constant phase modulation (CPM) techniques find applications in satellite communications. Because of power amplifier considerations, satellite communications require a modulation technique with a constant or nearly constant envelope versus time (no amplitude modulation). Technological and regulatory limitations also require low error probability for a given signal-to-noise ratio and high bits per second of transmitted information for a given bandwidth. The technique of multi- $h$ CPM, which combines encoding and modulation, achieves all of these goals.

This chapter describes an implementation of continuous phase frequencyshift keying (CPFSK), a sub-class of multi- $h$ CPM, on the ADSP-2100 family of processors. Only modulation is described here; demodulation is usually performed with the Viterbi algorithm, described in Chapter 2.

Fast frequency-shift keying (FFSK) is a special case of CPFSK with $\mathrm{h}=1 / 2$.

### 6.2 CPFSK METHODOLOGY

The general form for a multi- $h$ CPM signal is:

```
\(\mathrm{s}(\mathrm{t} ; \alpha)=\sqrt{ }\left(2 \mathrm{E}_{\mathrm{s}} / \mathrm{T}_{\mathrm{s}}\right) \cos \left[2 \pi \mathrm{f}_{0} \mathrm{t}+\varphi(\mathrm{t} ; \alpha)+\varphi_{0}\right]\)
\(\mathrm{E}_{\mathrm{s}} \quad=\) symbol energy
\(\mathrm{T}_{\mathrm{s}}=\) symbol duration
\(\mathrm{f}_{0} \quad=\) carrier frequency \((\mathrm{Hz})\)
\(\varphi_{0} \quad=\) carrier phase (arbitrary)
\(\varphi(t ; \alpha)=\) information-carrying phase function, expressed as:
```

$\underset{-\infty}{\mathrm{t}} \int_{\mathrm{i}=-\infty}^{\infty} \mathrm{h}_{\mathrm{i}} \mathrm{a}_{\mathrm{i}} \mathrm{g}\left(\tau-\mathrm{i} \mathrm{T}_{\mathrm{s}}\right) \mathrm{d} \tau \quad-\infty<\mathrm{t}<\infty$

## 6 CPFSK

where:
$\alpha \quad=\left(\ldots, a_{-2}, a_{-1}, a_{0}, a_{1}, a_{2}, \ldots\right)$, representing the data sequence
$h_{i} \quad=$ set of $K$ modulation indices cycled through periodically,
i.e., $h_{i+K}=h_{i}$
$\mathrm{g}(\mathrm{t})=$ frequency pulse-shape function

For CPFSK, all $h_{i}$ are equal and the pulse-shape function is:

$$
\mathrm{g}(\mathrm{t})=\mathrm{T}_{\mathrm{s}} / 2 \quad \text { for } 0 \leq \mathrm{t} \leq \mathrm{T}_{\mathrm{s}^{\prime}} \text { otherwise } 0
$$

### 6.3 ADSP-2100 FAMILY IMPLEMENTATION

Figure 6.1 shows a flowchart of the CPFSK program implemented on the ADSP-2100 family of processors. This particular implementation uses the ADSP-2101 to take advantage of its on-chip serial ports and timer. The timer generates a clock at the symbol rate ( 2400 baud) for reading input data. The ADSP-2101 outputs CPFSK modulated data to a digital-toanalog converter (DAC) at the rate of 8 kHz .

The CPFSK program is shown in Listing 6.1. This program sets up a buffer of dummy data for demonstrations; in actual use, the data would come from an input device and could be read from the FI (Flag In) input of the ADSP-2101.

The CPFSK routine calls two external routines not shown here. The cntlreg_inits routine initializes the ADSP-2101's control registers. The boot_sin routine computes the sine of the input in AX0, returning the output in AR. See Chapter 4 for a similar routine.

After setup (initializing variables, etc.) the processor waits for one of two interrupts. The SPORT0 interrupt causes the processor to calculate the next output sample by adding the current phase increment to the phase accumulator and computing the sine of the result. The output samples are transmitted from SPORT0 and are also sent to a DAC for display (for demonstration).

The timer interrupt causes the processor to select a new phase increment based on the value of the input data. Because the data is binary ( 1 or 0 ) it could be input through the flag input (FI) pin instead of data memory as shown. The code would have to be modified to use the state of the input flag as a condition for selecting the phase increment.

$\varphi \quad=$ current phase value (stored in "phase accumulator")
$\Delta \varphi \quad=$ current phase increment
$\Delta \varphi_{\mathrm{a}}=$ phase increment for tone a
$\Delta \varphi_{b}=$ phase increment for tone $b$

Figure 6.1 CPFSK Flow Diagram

## 6 CPFSK



## CPFSK 6

```
\begin{tabular}{|c|c|c|}
\hline make demo_data: & SI=lo_tone; & DM (hertz0) =SI; \\
\hline & SI=hi_tone; & DM (hertzi) =SI; \\
\hline & SI=logic_one; & DM (demo_input_data) =SI; \\
\hline & SI=logic_zero; & DM (demo_input_data+1)=SI; \\
\hline & & DM (demo_input_data+2) =SI; \\
\hline & & DM (demo_input_data+3) =SI; \\
\hline & & DM (demo_input_data+4)=SI; \\
\hline & & DM (demo_input_data+5) =SI; \\
\hline & & DM (demo_input_data+6) =SI; \\
\hline & I0 0 ^demo_input_data; & \\
\hline & L0 0 \%demo_input_data; & \\
\hline
\end{tabular}
```



```
load tone1: SI=DM(hertz1);
    SR=ASHIFT SI BY 3(HI);
    MY0=H#4189; {mult Hz by .512*2}
    MR=SR1*MY0 (RND); {i.e. mult by 1.024}
    SR=ASHIFT MR1 BY 1(HI);
    DM(phase_incr_1)=SR1;
load_tone0: SI=DM(hertz0);
    SR=ASHIFT SI BY 3(HI);
    MY0=H#4189; {mult Hz by .512*2}
    MR=SR1*MYO (RND); {i.e. mult by 1.024}
    SR=ASHIFT MR1 BY 1(HI);
    DM(phase_incr_0)=SR1;
SI=0;}
        DM(phase accumulator)=SI; {clear phase accumulator on startup}
        CALL symb̄ol; {start with first symbol}
        ICNTL=B#01111;
        IMASK=B#001001; {enable SPORT0_RX, TIMER now}
        ENA TIMER; {start baud_clock now}
{——伍}
here: JUMP here; {wait for symbol and sample interrupts}
```


## 6 CPFSK

```
{===================================================================== }
{=========== P R O C E S S A N E W S A M P L E =========== }
{===================================================================}
sample: AX0=DM(phase_accumulator);
        AY0=DM(phase_increment);
        AR=AX0+AY0;
        DM(phase_accumulator)=AR;
        AX0=AR;
        CALL boot_sin;
sound: DM(write_\overline{dac0)=AR; {"display" CPFSK on oscilloscope}}
    DM(load_dac)=AR;
    SR=ASHIFT AR BY -2(HI);
    TX0=SR1; {"hear" CPFSK from speaker (PCM out) }
    RTI;
{===================================================================}
{=========== P R O C E S S A N E W S Y M B O L ============}
{==================================================================}
symbol: AX1=DM(I0,M0); {get input data (could be FLAG_IN)}
    DM(write_dacl)=AX1; {echo input data stream for demo}
    DM(load_dac)=AR;
    AF=PASS AX1;
    IF EQ JUMP zero;
one: SI=DM(phase_incr_1); DM(phase_increment)=SI; RTS;
zero: SI=DM(phase_incr_0); DM(phase_increment)=SI; RTS;
```

. ENDMOD;

## Listing 6.1 CPFSK Program (ADSP-2101)

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